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H. Garnier, G.C. Goodwin, J. Welsh

Centre de Recherche en Automatique de Nancy
CRAN UMR 7039 CNRS-INPL-UHP
http ://www.cran.uhp-nancy.fr
E-mail : hugues.garnier@cran.uhp-nancy.fr

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A TIME-DOMAIN APPROACH TO CONTINUOUS-TIME MODEL IDENTIFICATION OF HIGHLY RESONANT WIDE-BAND SYSTEMS

H. Garnier†∗, G.C. Goodwin†, J.S. Welsh†

† *Centre for Complex Dynamic Systems and Control, University of Newcastle, Australia.*
∗*On leave from: Centre de Recherche en Automatique de Nancy CNRS UMR 7039, Université Henri Poincaré, Nancy, France.*

Abstract: The purpose of this paper is to present some issues that arise when applying a traditional continuous-time model identification technique to highly resonant systems over a large bandwidth. A scheme based on the use of frequency localising basis functions to improve the numerical conditioning of the identification problem is examined. Examples are presented showing the superior performance of the proposed method when applied to wide-band estimation problems.

Keywords: Basis functions, continuous-time, linear systems, resonant systems, wide-band systems, system identification, time-domain.

1. INTRODUCTION

Many mechatronic systems exhibit resonant behavior (see for example Akcay and Ninness (1999), Moheimani (2000)). Control of such systems typically relies upon the availability of high precision models. In principle, such models could be obtained from phenomenological considerations but usually this type of model is too complex to be accurately described using physical parameters. Hence, one needs to obtain the models using data collected from the system. However, several challenges arise including the fact that the systems are highly resonant and the fact that the model may be required over a wide frequency band. These factors make the problem challenging.

This paper is aimed at examining the associated estimation problems for continuous-time systems. In particular, we extend the frequency-domain method of Welsh and Goodwin (2003) to the time-domain. Comparison results with a traditional continuous-time model identification method are also presented.

The remainder of the paper is organized in the following way. Section 2 states the problem. The traditional state-variable filter approach is reviewed in section 3. The proposed new method is then described in section 4. The performance of the proposed algorithm is illustrated through a simulation example in section 5. Section 6 presents the results of the identification of a flexible robot arm. Finally, section 7 gives concluding remarks.

2. PROBLEM STATEMENT

Consider a single-input single-output continuous-time linear time-invariant causal system whose input $u(t)$ and output $y(t)$ are related by a constant coefficient differential equation of order n

$$y^{(n)}(t) + a_{n-1}y^{(n-1)}(t) + \ldots + a_0 y^{(0)}(t) = b_m u^{(m)}(t) + \ldots + b_0 u^{(0)}(t) \quad (1)$$

where $x^{(i)}(t)$ denotes the ith time-derivative of the continuous-time signal $x(t)$.

The system is assumed to be subject to an arbitrary set of initial conditions

$$u_0 = \left[u(0)\ u^{(1)}(0)\ \cdots\ u^{(m-1)}(0)\right],$$
$$y_0 = \left[y(0)\ y^{(1)}(0)\ \cdots\ y^{(n-1)}(0)\right].$$

Equation (1) can also be written as

$$A(p)y(t) = B(p)u(t), \tag{2}$$

with

$$B(p) = b_m p^m + \cdots + b_1 p + b_0,$$
$$A(p) = p^n + a_{n-1}p^{n-1} + \cdots + a_1 p + a_0, \quad n \geq m,$$

where p is the differential operator, *i.e.* $px(t) = \frac{dx(t)}{dt}$.

The polynomials $A(p)$ and $B(p)$ are assumed to be relatively prime and the roots of the polynomial $A(p)$ are assumed to have negative real parts; the system under study is therefore assumed to be asymptotically stable. It is also assumed that the continuous-time signals $u(t)$ and $y(t)$ are sampled at a regular time interval, T_s. The sampled signals will be denoted as $\{u(t_k); y(t_k)\}$.

The identification problem can then be stated as follows: estimate the parameters of the differential equation model from N sampled measurements of the input and output $Z^N = \{u(t_k); y(t_k)\}_{k=1}^N$.

3. THE TRADITIONAL SVF APPROACH

There are two main time-domain approaches to estimate a continuous-time model from discrete-time data. The first is to estimate from the sampled data, an initial discrete-time model and then convert it into a continuous-time model. The second approach consists in identifying directly a continuous-time model from the discrete-time data. In comparison with the discrete-time counterpart, the direct method raises several technical issues. Unlike the difference equation model, the differential equation model is not a linear combination of the sampled process input and output signals, i.e. it also contains time-derivative terms which are not available as measurement data in most practical cases.

Various types of methods have been devised to deal with the need to reconstruct these time-derivatives. Each method is characterized by specific advantages such as mathematical convenience, simplicity in numerical implementation and computation, physical insight, accuracy, etc (Garnier *et al.*, 2003*b*). The CONtinuous-Time System IDentification (CONTSID) toolbox has been developed on the basis of these methods (Garnier *et al.*, 2003*a*). One traditional approach that dates from the days of analog computers (Young, 1964) is known as the state variable filter (SVF) method. We begin by a short review of this method with the objective to highlight the difference with the newly proposed method.

3.1 *Outline of the SVF approach*

Consider the Laplace transform of the differential equation defined in (1),

$$A(s)Y(s) = B(s)U(s) + C(s), \tag{3}$$

with

$$C(s) = c_{n-1}s^{n-1} + \cdots + c_1 s + c_0 \tag{4}$$

where s represents the Laplace variable and $Y(s)$ and $U(s)$ are the Laplace transforms of $y(t)$ and $u(t)$ respectively. The coefficients c_i depend on the unknown parameters a_i and b_i as well as the unknown initial conditions. Assume now that a filter has a Laplace transform $L(s) = 1/E(s)$. Applying this filter to both sides of (3) yields

$$\frac{A(s)}{E(s)}Y(s) = \frac{B(s)}{E(s)}U(s) + \frac{C(s)}{E(s)}, \tag{5}$$

or

$$\frac{s^n}{E(s)}Y(s) + \sum_{i=0}^{n-1} a_i \frac{s^i}{E(s)}Y(s) = \sum_{i=0}^{m} b_i \frac{s^i}{E(s)}U(s) + \sum_{i=0}^{n-1} c_i \frac{s^i}{E(s)}. \tag{6}$$

The minimum-order SVF filter is typically chosen to have the following form

$$L(s) = \frac{1}{E(s)} = \left(\frac{p_n}{s+p_n}\right)^n \tag{7}$$

where p_n is the breakpoint frequency. This latter quantity can be chosen in order to emphasize the frequency band of interest and it is advised, in general, to choose it slightly larger than the bandwidth of the system to be identified (Young, 1964). Let $L_k(s)$, for $k = 0, 1, 2, \ldots, n$, be a set of filters defined as

$$L_k(s) = \frac{s^k}{E(s)} = \frac{(p_n)^n s^k}{(s+p_n)^n} \tag{8}$$

and $l_k(t)$ be their corresponding functions in the time-domain. By using the filters defined in (8), (6) can be rewritten as

$$\begin{aligned}&\big(L_n(s) + a_{n-1}L_{n-1}(s) + \ldots + a_0 L_0(s)\big)Y(s)\\ &\quad = \big(b_m L_m(s) + \ldots + b_0 L_0(s)\big)U(s)\\ &\qquad + \big(c_{n-1}L_{n-1}(s) + \ldots + c_0 L_0(s)\big).\end{aligned} \tag{9}$$

In terms of time-domain signals, (9) can be written as

$$\begin{aligned}&[L_n y](t) + a_{n-1}[L_{n-1}y](t) + \ldots + a_0[L_0 y](t)\\ &\quad = b_m[L_m u](t) + \ldots + b_0[L_0 u](t)\\ &\qquad + c_{n-1}l_{n-1}(t) + \ldots + c_0 l_0(t)\end{aligned} \tag{10}$$

where

$$[L_i y](t) = l_i(t) * y(t)$$
$$[L_i u](t) = l_i(t) * u(t)$$

and $*$ denotes the convolution operator. The filter outputs $[L_i y]$ and $[L_i u]$ will then provide time-derivatives of the inputs and outputs in the bandwidth of interest, which may be exploited for linear regression and other parameter estimation techniques.

At time-instant $t = t_k$, equation (10) can be rewritten in standard linear regression form as

$$[L_n y](t_k) = \phi^T(t_k)\theta \tag{11}$$

where

$$\begin{aligned}\phi^T(t_k) = \big[-[L_{n-1}y](t_k)\ \ldots\ -[L_0 y](t_k)\\ [L_m u](t_k)\ \ldots\ [L_0 u](t_k)\\ l_{n-1}(t_k)\ \ldots\ l_0(t_k)\big]\end{aligned} \tag{12}$$

$$\theta = [a_{n-1}\ \ldots\ a_0\ b_m\ \ldots\ b_0\ c_{n-1} \ldots c_0]^T. \tag{13}$$

Now, from N samples of the input and output signals observed at discrete times $t_1, \ldots, t_N$, the least-squares (LS)-based SVF estimates are given by

$$\hat{\theta}_N = \left[\sum_{k=1}^{N} \phi(t_k)\phi^T(t_k)\right]^{-1} \sum_{k=1}^{N} \phi(t_k)\ [L_n y](t_k).$$

The SVF technique can also be associated with a basic instrumental variable (IV) method when the output signal is contaminated with noise. This technique belongs to the six methods which have proven to exhibit very good performance in extensive Monte Carlo simulation studies (Garnier *et al.*, 2003*b*).

3.2 *Implementation and numerical issues*

Initial conditions. The SVF approach makes it possible to estimate the initial condition terms c_i along with the model parameters. However, treating them as an additional set of unknowns complicates the parameter estimation. From (10), it can be seen that although the initial condition terms do not vanish, the impulse responses of the low-pass filters decay exponentially provided the SVF filters are stable and hence become insignificant quite quickly. Thus, if the SVF-based algorithm is used with a large observation time T, the terms related to the initial conditions may be neglected after a time $T_0 = k_0 T_s$. The estimation algorithm is then applied over $[T_0, T]$, where T_0 has to be chosen comparable to the settling time of the filter (7). The number of parameters to be estimated can, in this way, be reduced substantially and this is advantageous with regard to computation effort and numerical properties. Note however, that this is not recommended for highly resonant wide-band systems since the transient response of such systems (and therefore of the SVF filter impulse responses) can be significant.

Digital implementation of the SVF filter. Since only the sampled versions of the continuous-time signals are available, the output of the state-variable filters can only be computed from a discrete approximation of these filters. This problem is well known and should be treated in a proper manner since errors generated by the digital implementation can have a severe influence on the quality of the estimated model (Chou *et al.*, 1999). Using a control canonical form, the state-space representations of the continuous-time SVF filter can be either integrated by the Runge-Kutta method or discretized by using an appropriate method provided the intersample nature of the continuous-time input signal is known. A high sampling frequency is often required to obtain an accurate simulation.

Numerical conditioning of the estimation problem. The normal matrix can be identified in (3.1) as $R = \left[\sum_{k=1}^{N} \phi(t_k)\phi^T(t_k)\right]$. If the normal matrix is 'near singular' then the solution for the least-squares method becomes ill-conditioned. The condition number of the normal matrix is defined as

$$\kappa(R) = \frac{\lambda_{max}(R)}{\lambda_{min}(R)} \tag{14}$$

where $\lambda_{max}(R)$ and $\lambda_{min}(R)$ denote the largest and smallest singular values, respectively (Golub and Loan, 1996). It is known that the relative error of the solution, $\hat{\theta}_N$, is proportional to the condition number and it is therefore of interest to have the normal matrix well conditioned (Gevers and Li, 1993). There are two main reasons for the normal matrix to become ill-conditioned, and hence lead to erroneous parameter estimates:

- when the difference between the magnitude of the diagonal elements becomes large, the identification becomes off balanced and the parameters corresponding to the small elements are almost negligible;
- when the rows of the normal matrix become approximately aligned, the matrix will be close to singular (ill-conditioned) which will cause the estimation to 'blow up'.

The normal matrix formed using the traditional SVF method is susceptible to extreme ill-conditioning when the system is highly resonant over several decades and/or when the system order is high and/or when the data are rapidly sampled. Indeed, for a large bandwidth, as the order of the system increases the difference between the magnitude of the term with the highest order and that of the term of the lowest order increases dramatically. As a result, the last term in the regression vector receives little information and the normal matrix becomes ill-conditioned. One way to improve the condition number of the normal matrix consists in normalising the entries by the L2-norm of the corresponding entry. This leads to a scaled normal matrix with identical diagonal elements. The parameters are then deduced from the known scaling matrix after the least-squares estimation. This scaling technique is used in all methods considered in this paper. However, this scaling strategy is often insufficient, as we will see in the examples, using the traditional SVF-based approach for highly resonant wide-band systems.

4. THE PROPOSED FLBF APPROACH

In this section we present a scheme to improve the numerical conditioning of the estimation problem. The proposed method consists of two steps. In the first, the model is re-parameterised using frequency localising basis functions (FLBF's). This is equivalent to filtering the measured input and output through a bank of passband filters. Note that the proposed method differs with the traditional SVF approach or the scheme suggested by Johansson (1994) based on a bank of low-pass filters and the approach taken by Chou *et al.* (1999) based on a bank of all-pass filters. The filtered input, filtered output and the impulse responses of the passband filter are related by a linear equation with constant coefficients of which can be estimated using LS-type of techniques. Once these coefficients have been determined, the parameters of the original differential equation can be computed via a linear transformation in the second step. Note that the idea of using this kind of band-splitting filters was first mentioned in Young (1970).

4.1 *Outline of the FLBF approach*

The basis functions that have been recently proposed for wide-band frequency-domain identification, take the following form (Welsh and Goodwin, 2003):

$$F_k(s) = \prod_{l=1}^{k} \frac{s^{k-1} p_k}{s+p_l}, \quad k = 1, \dots, n. \tag{15}$$

Let $f_k(t)$ be their corresponding functions in the time-domain. Note that these filters have approximately 0dB gain in the range $[p_{k-1}, p_k]$ for basis function F_k. Outside of this range the gain decreases by at least 20 dB per decade.

4.1.1. *First step* The model (5) is first parameterised as

$$\frac{A(s)}{E(s)} = 1 + \sum_{l=1}^{n} a^{\perp}_{l-1} F_l(s) \tag{16}$$

$$\frac{B(s)}{E(s)} = \sum_{l=1}^{n} b^{\perp}_{l-1} F_l(s) \tag{17}$$

$$\frac{C(s)}{E(s)} = \sum_{l=1}^{n} c^{\perp}_{l-1} F_l(s) \tag{18}$$

where

$$E(s) = \prod_{l=1}^{n} (s+p_l) \tag{19}$$

$$= s^n + e_{n-1}s^{n-1} + \cdots + e_0. \tag{20}$$

With the new model parameterisation, (5) can be rewritten as

$$\begin{aligned}&\big(1 + a^{\perp}_{n-1}F_n(s) + \ldots + a^{\perp}_0 F_1(s)\big)Y(s)\\ &\quad = \big(b^{\perp}_{n-1}F_n(s) + \ldots + b^{\perp}_0 F_1(s)\big)U(s)\\ &\qquad + \big(c^{\perp}_{n-1}F_n(s) + \ldots + c^{\perp}_0 F_1(s)\big).\end{aligned} \tag{21}$$

In terms of time-domain signals, (21) can be written as

$$\begin{aligned}&y(t) + a^{\perp}_{n-1}[F_n y](t) + \ldots + a^{\perp}_0 [F_1 y](t)\\ &\quad = b^{\perp}_{n-1}[F_{n-1}u](t) + \ldots + b^{\perp}_0 [F_1 u](t)\\ &\qquad + c^{\perp}_{n-1} f_n(t) + \ldots + c^{\perp}_0 f_1(t)\end{aligned} \tag{22}$$

where

$$[F_i y](t) = f_i(t) * y(t)$$
$$[F_i u](t) = f_i(t) * u(t).$$

The key point regarding these filters is that they are non-zero essentially for only a small set of frequencies that lie in their passband. Hence the least-squares normal matrix will take on a near block diagonal form. The filtered input, filtered output and the impulse responses of the passband filter are again related by a linear equation with constant coefficients denoted as $a^{\perp}_i, b^{\perp}_i, c^{\perp}_i$, which are also be estimated by the least-squares method.

At time-instant $t = t_k$, equation (22) can be rewritten in standard linear regression form as

$$y(t_k) = \psi^T(t_k)\theta^{\perp}, \tag{23}$$

with

$$\begin{aligned}\psi^T(t_k) = \big[&-[F_n y](t_k) \ \ldots \ -[F_1 y](t_k)\\ &[F_{n-1}u](t_k) \ \ldots \ [F_1 u](t_k)\\ &f_n(t_k) \ \ldots \ f_1(t_k)\big]\end{aligned} \tag{24}$$

$$\theta^{\perp} = \left[a^{\perp}_{n-1} \ \ldots a^{\perp}_0 \ b^{\perp}_{n-1} \ \ldots b^{\perp}_0 \ c^{\perp}_{n-1} \ \ldots c^{\perp}_0\right]^T. \tag{25}$$

Now, with N samples of the input and output signals, the LS-based FLBF estimates are given by

$$\hat{\theta}^{\perp}_N = \left[\sum_{k=1}^{N} \psi(t_k)\psi^T(t_k)\right]^{-1} \sum_{k=1}^{N} \psi(t_k)\, y(t_k). \tag{26}$$

4.1.2. *Second step* The model parameterisation can be easily expressed in terms of the original polynomials by using the following transformation. Let

$$\begin{aligned}M^l(s) &= F_l(s)E(s)\\ &= m^l_{n-1}s^{n-1} + \ldots + m^l_{l-1}s^{l-1}, \quad l = 1, \dots, n\end{aligned} \tag{27}$$

where l represents the order of lth basis function, then

$$\mathbf{a} = M\mathbf{a}^{\perp} + \mathbf{e}, \quad \mathbf{b} = M\mathbf{b}^{\perp}, \quad \mathbf{c} = M\mathbf{c}^{\perp} \tag{28}$$

where $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{e}$ are the parameter vectors of $A(s)$, $B(s)$, $C(s)$ and $E(s)$ respectively, $\mathbf{a}^{\perp}$, $\mathbf{b}^{\perp}$ and $\mathbf{c}^{\perp}$ are the vectors of parameters in (16) to (18) and

$$M = \begin{bmatrix} m^1_0 & 0 & \ldots & 0 \\ m^1_1 & m^2_1 & \ddots & \vdots \\ \vdots & \vdots & & 0 \\ m^1_{n-1} & m^2_{n-1} & \ldots & m^n_{n-1} \end{bmatrix} \tag{29}$$

4.2 *Remarks*

The basis function representation given in (16) to (18) is specifically for a strictly proper model. If the model is bi-proper then the re-parameterisation for $A(s)$ and $C(s)$ remain the same, however re-parameterisation for $B(s)$ becomes:

$$\frac{B(s)}{E(s)} = \sum_{l=1}^{n} b_{l-1}^{\perp} F_l(s) + \frac{b_n^{\perp} s}{s + p_n} \tag{30}$$

where p_n is the high breakpoint frequency.
We have found that initial condition effects are important with the FLBF's when they are utilized in the time-domain. Hence inclusion of the terms $c_i^{\perp}$ in the parameter vector seems important in this case.
In the presence of a noisy output signal, the proposed FLBF method can be associated to a basic IV algorithm to reduce the LS estimation bias (see Garnier *et al.* (2003*b*) for example).

5. SIMULATION EXAMPLE

Consider the identification problem of an 8th order highly resonant system that spans several decades of frequency described by

$$G(s) = \frac{1}{\alpha} \sum_{k=1}^{4} \frac{b_k \omega_k^2 (s + \omega_0)}{(s^2 + 2\zeta_k \omega_k s + \omega_k^2)} \tag{31}$$

where $[b_1,\, b_2,\, b_3,\, b_4] = [1,\, 3,\, 1,\, 1]$, $\alpha = 1320$ and $\omega_0 = 225$ rad/sec. This system is characterized by four highly resonant modes with $[\omega_1,\, \omega_2,\, \omega_3,\, \omega_4] = [4,\, 40,\, 400,\, 4000]$ and $[\zeta_1,\, \zeta_2,\, \zeta_3,\, \zeta_4] = 0.0008 \times [10,\, 1,\, 1,\, 1]$.

A magnitude Bode plot of the true system is shown in Figure 1 as a dash line. The system is excited by an aperiodic multisine of 25 logarithmically spaced frequencies with starting and end frequency at 2 and 6000 rad/sec respectively which spans the entire frequency range of the system. No noise was added to both input/output signals for this comparison. The time window for observation is fixed at 4 s and the sampling frequency is set to $50 \times max(\omega_u)$ rad/sec. This large value has been chosen in order to minimize the numerical errors in the digital simulation of the continuous-time filtering. The model was then estimated using the following methods:

(1) LS-based SVF method with filter breakpoint frequency $p_n = 4000$ chosen equal to the highest natural frequency of the system;
(2) LS-based FLBF method with breakpoint frequencies chosen logarithmically spaced between 2rad/sec and $2\times$the highest natural frequency of the system.

Note that the initial condition terms were estimated along with the transfer function parameters in both SVF and FLBF methods. Table 1 compares the condition number of the normal matrix for the two methods.

Method	Condition number
SVF	1.4e+14
FLBF	4120

Table 1. Condition number for the two compared LS-based methods

In each of the estimation methods, we utilised the QR factorisation (Golub and Loan, 1996) to improve the numerical properties of the least-squares routines. It may be noticed from Table 1 that the condition number for the LS-based SVF method is much poorer than the LS-based FLBF method. Note furthermore that the estimated LS-based SVF model is unstable. The magnitude Bode plot is nevertheless plotted for comparison purposes. Figure 1 compares the magnitudes of the estimated frequency responses with that of the true system. It is observed that the LS-based FLBF method provides a very good model fit.

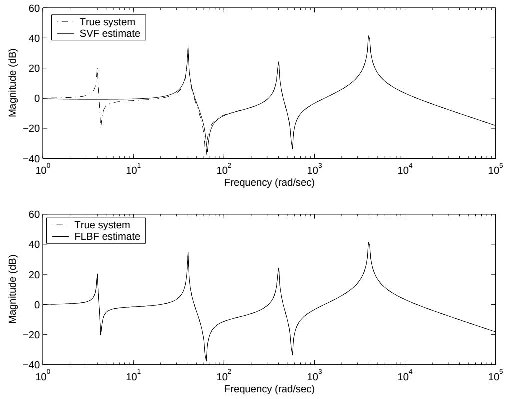

Fig. 1. Magnitude Bode of the LS-based SVF and FLBF estimates for the 8th order system.

6. REAL DATA FROM A ROBOT ARM

In this section, we consider the modelling of a flexible robot arm based on experimental data [1]. The input is the controlled torque applied to the vertical axis at one end of the arm, while the output is the tangential acceleration of the other end. The excitation signal was a multisine. The sampling period was set to 2 ms. Measurements were made using anti-alias filters. The data set consisted of 10 periods each of length 4096 were exactly measured. The process and experiment are described in more detail in Kollar *et al.* (1994).

For the estimation algorithm, the numerator and denominator orders of the transfer function have to be given. From the empirical transfer function estimate (Kollar *et al.*, 1994), it may be noticed that the system has at least two complex pole pairs and two complex zero pairs. The best results have been obtained here for a $5/5$ transfer function model. The process identification is performed with the IV-based SVF and FLBF al-

[1] The authors are most grateful to Istvan Kollar for kindly providing access to the data.

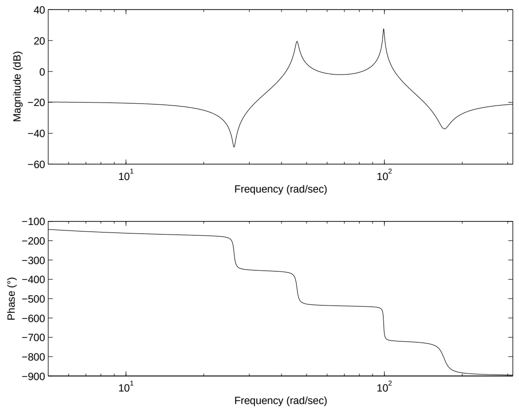

Fig. 2. Bode plot for the estimated model

gorithms on the fifth period data set. As in the simulation example case, the traditional SVF-based method was not able to estimate a reasonable model from this data. The Bode plot for the estimated FLBF-based model is displayed in Figure 2. It may be noticed that the resonant mode around 40 rad/sec is slightly more damped than in the case of the estimated models discussed in Kollar *et al.* (1994). Figure 3 compares the simulated model output (dotted line) with the measured output (full line), over a short interval of 0.6 seconds of the 7-th period data set. It may be seen that the simulated output matches the measured data quite well, with a coefficient of determination of 0.95.

Remark: The authors acknowledge that other researchers have been able to obtain excellent fits by using frequency domain approaches (Kollar *et al.*, 1994). We are only able to state that the proposed time-domain FLBF method appears to give a reasonable model while the traditional SVF technique fails with this data.

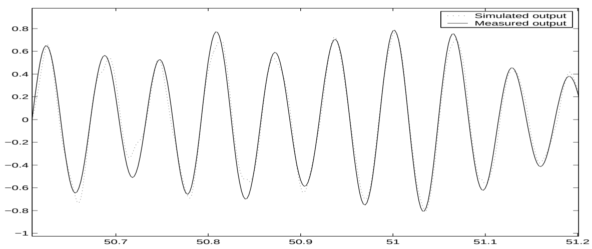

Fig. 3. Cross-validation results for the robot arm

7. CONCLUSIONS

In this paper we have extended a recently proposed frequency-domain identification technique based on the use of frequency localising basis functions to the time-domain. This method is aimed at improving the numerical conditioning of least-squares estimation problem. The performance of the proposed approach has been compared with a traditional continuous-time model identification method using a simulation example and real data from a flexible robot arm.

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