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Abstract

The simplified refined instrumental variable method for continuous-time model identification is extended to multiple input-single output systems where the characteristic polynomials of the transfer functions associated with each input are not constrained to be identical. Some experimental data are analyzed to indicate the applicability and the properties of the proposed approach.

1 Introduction

System identification is an established field in the area of system analysis and control. It aims at determining particular models for dynamical systems based on observed inputs and outputs. Although dynamical systems in the physical world are native to continuous-time (CT) domain, most system identification schemes have been based in the past on discrete-time (DT) models without concern for the merits of the native continuous-time models. The development of CT model based system identification techniques originated in the middle of the last century but was overshadowed by the overwhelming developments of DT methods. This was mainly due to the 'go completely digital' trend that was spurred by parallel developments in digital computers. Interest in CT approaches to system identification has however been growing in the last decade (see [1] and [2], [3], [4], [5], [6] for more recent references). Recent publications ([7], [8]) have drawn attention to difficulties that can be encountered when utilizing DT estimation algorithms under conditions that are non-standard, such as rapidly sampled data and systems with widely different natural frequencies.

Moreover, practical applications of estimation methods to real-life processes lead to consider multi-input single output (MISO) systems. In DT model estimation, the proposed approaches combine either extensions of linear regression techniques like pseudo-linear, multi-linear regression, filtering, instrumental variable, or non linear optimization techniques [9]. The latter techniques often require a good initial parameter set to converge to the global minimum of the cost function. For the CT case, as far as the authors are aware, the parameter estimation procedures for MISO systems have usually been developed by a straightforward extension of procedures devoted to SISO systems, which only allows transfer function estimation with common denominator. Since this case is not very realistic in many applications, this paper aims at presenting a new method to estimate MISO models written in terms of CT transfer functions with different denominators.

Amongst the different estimation procedures available, a so-called simplified refined instrumental variable (IV), denoted as *srivc* from hereon, has been chosen. This IV-type method has indeed often proved to be particularly useful in practical applications (see e.g. [10]). It was first proposed for DT model estimation [11, 12] and then extended for DT MISO systems with different denominators [13]. The approach has also been extended for SISO CT model estimation [14]. It has recently been revisited [15] and adapted to handle the case of non uniformly sampled data [16]. These IV approaches are known, for discrete-time model identification, to be robust against the properties of the noise [12], as well as against the number of samples [17] and the initial parameter set. Moreover, it has been shown by Monte Carlo simulations and also on real-life processes that these refined IV approaches present interesting performances. The aim of this paper is to present how the *srivc* method can be extended to identify multiple input systems described by multiple CT transfer functions.

The paper is organized in the following way. Section 2 states the problem. The proposed method and derivation of the algorithm are described in section 3. The properties of the proposed algorithm are illustrated through Monte Carlo simulation in section 4. Section 5 presents the results of the identification of a pilot winding process. Finally, section 6 gives some concluding remarks.

2 Problem statement

Consider a MISO CT linear time-invariant causal system that can be described by the following multi-input transfer functions (MITF)[1]

$$\mathcal{S}:\begin{cases} y_{u_i}(t) = \frac{B_i^o(p)}{F_i^o(p)} u_i(t-\tau_i^o), \\ y_u(t) = \sum_{i=1}^{n_u} y_{u_i}(t), \\ y(t) = y_u(t) + v(t) \end{cases} \quad (1)$$

with

$$G_i^o(p) = \frac{B_i^o(p)}{F_i^o(p)} \quad (2)$$

$$B_i^o(p) = b_{i,0}^o + b_{i,1}^o p + \ldots + b_{i,m_i}^o p^{m_i} \quad (3)$$

$$F_i^o(p) = f_{i,0}^o + f_{i,1}^o p + \ldots + f_{i,n_i}^o p^{n_i} \quad (4)$$

$$f_{i,n_i}^o = 1, \quad n_i \geq m_i, \quad i = 1, \ldots, n_u,$$

where $u(t) = [u_1(t) \ldots u_{n_u}(t)]$ is the vector of uncorrelated input signals, $y_u(t)$ the system response to $u(t)$. p is the differential operator, *i.e.* $px(t) = \frac{dx(t)}{dt}$.

The polynomials $F_i^o(p)$ and $B_i^o(p)$ are assumed to be relatively prime and the roots of the polynomials $F_i^o(p)$ are assumed to have negative real parts; the system under study is therefore assumed to be asymptotically stable.

The first equation in (1) describes the output at all values of the continuous-time variable t and can also be written as a set of ordinary differential equations

$$\begin{aligned} & f_{i,0}^o y_{u_i}(t) + f_{i,1}^o y_{u_i}^{(1)}(t) + \cdots + y_{u_i}^{(n_i)}(t) \\ & = b_{i,0}^o u_i(t-\tau_i^o) + \cdots + b_{i,m_i}^o u_i^{(m_i)}(t-\tau_i^o), \end{aligned} \quad (5)$$

where $x^{(l)}(t)$ denotes the l^{th} time-derivative of the continuous-time signal $x(t)$. τ_i^o denotes the time-delay between the output and the *ith* corresponding input.

$G_i^o(p)$ describes the true dynamics of each system which is subject to an arbitrary set of initial conditions

$$u^0 = \begin{bmatrix} u_1^0 & \cdots & u_{n_u}^0 \end{bmatrix}, \; y_u^0 = \begin{bmatrix} y_{u_1}^0 & \cdots & y_{u_{n_u}}^0 \end{bmatrix}, \quad (6)$$

$$u_i^0 = \begin{bmatrix} u_i(0) & u_i^{(1)}(0) & \cdots & u_i^{(m_i-1)}(0) \end{bmatrix} \in \mathbb{R}^{m_i}, \quad (7)$$

$$y_{u_i}^0 = \begin{bmatrix} y_{u_i}(0) & y_{u_i}^{(1)}(0) & \cdots & y_{u_i}^{(n_i-1)}(0) \end{bmatrix} \in \mathbb{R}^{n_i}. \quad (8)$$

It is furthermore assumed that the disturbances that cannot be explained from the input signal can be lumped into the additive term $v(t)$ (1). The disturbance term $v(t)$ is assumed to be independent of the input $u(t)$, *i.e.* the case of the open-loop operation of the system is considered. For the identification problem, it is also assumed that the continuous-time signals $u(t)$ and $y(t)$ are sampled at regular time-interval T_s.

[1] with slight abuse of notation, we refer to $G_i^o(p)$ as the transfer function of the system

The goal is then to build a model of equation (1) based on sampled input and output data. Models of the following form are considered

$$\mathcal{G}:\begin{cases} y_{u_i}(t_k,\theta_i) = G_i(p,\theta_i)u(t_k-\tau_i) \\ y_u(t_k,\theta_i) = \sum_{i=1}^{n_u} y_{u_i}(t_k,\theta_i) \\ y(t_k) = y_u(t_k,\theta_i) + v(t_k) \end{cases} \quad (9)$$

where $x(t_k)$ denotes the sample of the continuous-time signal $x(t)$ at time-instant $t = kT_s$ and $G_i(p,\theta_i)$ is the *ith* plant model transfer function given by

$$G_i(p,\theta_i) = \frac{B_i(p)}{F_i(p)} = \frac{b_{i,0} + b_{i,1}p + \cdots + b_{i,m_i}p^{m_i}}{f_{i,0} + f_{i,1}p + \cdots + f_{i,n_i}p^{n_i}}, \quad (10)$$

$$f_{i,n_i} = 1, \quad n_i \geq m_i, \quad i = 1, \ldots, n_u,$$

and $\theta_i = [b_{i,m_i} \ldots b_{i,0} \; f_{i,n_i-1} \; \ldots \; f_{i,0}]^T \in \mathbb{R}^{n_{p_i}}$, with $n_{p_i} = n_i + m_i + 1$, where n_i and m_i denote the denominator and numerator orders of $G_i(p,\theta_i)$ respectively. Therefore, the sought parameter vector is

$$\theta = \begin{bmatrix} \theta_1^T & \ldots & \theta_{n_u}^T \end{bmatrix}^T \in \mathbb{R}^{n_p \times 1} \quad (11)$$

with $n_p = \sum_{i=1}^{n_u} n_{p_i}$.

Note that estimation methods presented in this paper focus on identifying the parameters of each plant transfer function $G_i(p,\theta_i)$ rather than the additive noise appearing in (1). The disturbance term is modelled here as a zero-mean discrete-time noise sequence denoted as $v(t_k)$. Moreover, the pure time-delays are supposed to be multiple integers of the sampling period $\tau_i = n_{k_i}T_s$.

The identification problem can now be stated as follows: determine the orders n_i and m_i and estimate the parameter vector $\theta = \begin{bmatrix} \theta_1^T \cdots \theta_{n_u}^T \end{bmatrix}^T$ of the continuous-time plant model from N sampled measurements of the input and the output $Z^N = \{y(t_k)u_1(t_k) \ldots u_{n_u}(t_k)\}_{k=1}^N$.

3 *srivc* for MISO systems

3.1 CT model identification

There are two ways to obtain a CT model. The first is to estimate from the sampled data an initial DT model and then convert it into a CT model. The second approach consists in identifying directly a CT model from the DT data. In comparison with the DT counterpart, CT model identification raises several technical issues. The first point is related to implementation. Unlike the difference equation model, the differential equation model is not a linear combination of samples of only the measurable process input and output signals. It also contains input and output time-derivatives which are not available as measurement data in most practical cases. Various types of continuous-time filters have been devised to circumvent the need to reconstruct these time-derivatives [1], [6]. As recently shown, these filters can be regarded as regularizing the problem of the numerical differentiation of noisy signals [18]. Furthermore, the advantage of these filters is to allow the specification of a frequency band where the model is desired to well-fit the system [19]. The CONtinuous-Time System IDentification (CONTSID) toolbox[2] has been developed on the basis of these methods [20].

Most of the CT system identification methods have been developed for the SISO or MISO common denominator (CD) models. To make the presentation clearer, the SISO case is considered first in this section (the extension to MISO CD is straightforward). Most of the methods presented in [6] aim at identifying a model of the following form

$$A(p,\theta)y(t_k) = B(p,\theta)u(t_k-\tau) + \varepsilon(t_k) \tag{12}$$

with

$$A(p,\theta) = \sum_{l=0}^{n-1} a_l p^l + p^n \quad \text{and } B(p,\theta) = \sum_{l=0}^{m} b_l p^l \tag{13}$$

Suppose that a causal stable analog filter with Laplace transfer function $L(s)$ of minimal order n is selected for the identification procedure. By passing both excitation and output measurements $u(t)$ and $y(t)$ through this filter, the time-derivatives of the filtered signals may be obtained. This operation when applied to system (12) for time-instant $t = t_k$ yields

$$\sum_{l=0}^{n-1} a_l y_f^{(l)}(t_k) + y_f^{(n)}(t_k) = \sum_{l=0}^{m} b_l\, u_f^{(l)}(t_k-\tau) + \varepsilon_f(t_k) \tag{14}$$

where $\varepsilon_f(t_k)$ denotes the equation error, $x^{(l)}(t_k)$ represents the l^{th} derivative of the signal $x(t)$ at time-instant $t = t_k$ and

$$y_f^{(l)}(t) = \mathcal{L}^{-1}[s^l L(s)Y(s)] \tag{15}$$

$$u_f^{(l)}(t) = \mathcal{L}^{-1}[s^l L(s)U(s)] \tag{16}$$

with $\mathcal{L}^{-1}$ denoting the inverse Laplace transform. For simplicity, it has been assumed that the differential equation model is initially at rest. Note however that in the general case the initial condition terms do not vanish in equation (14). Whether they require estimation or they can be neglected depends upon the selected pre-filtering method. The parameters can then be estimated by using an IV approach using an auxiliary model. There is a multitude of choice for the pre-filter (see e.g. [6] for a detailed study). It has been shown that the pre-filter $L(s)$ should be designed in such a way that it has frequency response characteristics close to the system to be identified. Therefore, the *srivc* method originally suggested in [14] develops optimal approaches to CT transfer function model estimation, in which the pre-filters are automatically selected in an iterative manner.

[2]see http://www.cran.uhp-nancy.fr/

3.2 *srivc* for SISO and MISO CD models

This method is based on the *Maximum Likelihood* (ML) approach where the error function is given by

$$v(t_k,\theta) = y(t_k) - \frac{B(p,\theta)}{F(p,\theta)}u(t_k-\tau), \tag{17}$$

$$= \frac{1}{F(p,\theta)}\left[F(p,\theta)y(t_k) - B(p,\theta)u(t_k-\tau)\right] \tag{18}$$

with

$$F(p,\theta) = \sum_{l=0}^{n-1} f_l p^l + p^n \quad \text{and } B(p,\theta) = \sum_{l=0}^{m} b_l p^l \tag{19}$$

Equation (18) can also be written as follows

$$v(t_k,\theta) = F(p,\theta)y_f(t_k) - B(p,\theta)u_f(t_k-\tau) \tag{20}$$

where $y_f(t_k)$ and $u_f(t_k)$ are the variables pre-filtered by $L(p,\theta) = \frac{1}{F(p,\theta)}$. The problem with this formulation is that θ and therefore $F(p,\theta)$ are unknown *a priori*. To cope with this drawback, a 'relaxation' optimization procedure is used. This latter consists in adaptively adjusting an initial estimate θ^0 of θ iteratively until it converges on an optimal estimate. Therefore, at each step, a linear in the unknown parameter θ equation has to be solved

$$y_f^{(n)}(t_k,\hat{\theta}^j) = \phi_f^T(t_k,\hat{\theta}^j)\theta^{j+1} + \varepsilon_f(t_k,\hat{\theta}^j) \tag{21}$$

$$\phi_f^T(t_k,\hat{\theta}^j) = \Big[u_f^{(m)}(t_k-\tau,\hat{\theta}^j)\ \dots\ u_f(t_k-\tau,\hat{\theta}^j) \\ -y_f^{(n-1)}(t_k,\hat{\theta}^j)\ \dots\ -y_f(t_k,\hat{\theta}^j)\Big] \tag{22}$$

where $\hat{\theta}^j$ is the parameter vector estimated at the j^{th} step of the algorithm, and θ^{j+1} is the parameter vector to be estimated.

Moreover, the noise is usually colored and this problem is solved in the second stage of the CT identification procedure, by exploiting IV estimation within the iterative optimization algorithm. The instrumental variable is generated from the following auxiliary model

$$y_u(t_k,\hat{\theta}) = \frac{B(p,\hat{\theta})}{F(p,\hat{\theta})}u(t_k-\tau) \tag{23}$$

The algorithm presenting the main steps of the *srivc* method dedicated to SISO model (or MISO CD models) is given in [15].

3.3 *srivc* for MISO DD models

The proposed method derives from the equivalent approach for DT model [13]. It aims at identifying MISO model with different denominators (DD) for each input (9), which is more realistic than the previous CD case. The model is however no longer linear in the parameters and the proposed MISO version of *srivc* lies therefore in the multi-linear regression. This consists in converting the MISO model (9) into n_u SISO models as follows

$$v(t_k,\theta) = \xi_{if}(t_k,\theta) - y_{u_i f}(t_k,\theta_i) \quad (24)$$

$$\xi_{if}(t_k,\theta) = y_f(t_k) - \sum_{j=1,j\neq i}^{n_u} y_{u_j f}(t_k,\theta_j) \quad (25)$$

The parameter vector θ is partitioned into classes $\theta_1,\ldots,\theta_{n_u}$ such that the error is affine with respect to the parameters of any of these classes when the parameters of all others are fixed [21]. It is then possible to search for $\hat{\theta}$ by applying successively the SISO version of the *srivc* to estimate the parameters of each class in turn, with a cyclic exploration of all classes. This is achieved by following the same type of 'relaxation' procedure described in section 3.2 and used in [13]. It is then possible to search for

$$\xi_{if}^{(n_i)}(t_k,\hat{\theta}^j) = \phi_{if}^T(t_k,\hat{\theta}^j)\theta_i^{j+1} + \varepsilon_{if}(t_k,\hat{\theta}^j) \quad (26)$$

$$\phi_{if}^T(t_k,\hat{\theta}^j) = \Big[u_{if}^{(m_i)}(t_k-\tau_i,\hat{\theta}^j)\ \ldots\ u_{if}(t_k-\tau_i,\hat{\theta}^j)\ -\xi_{if}^{(n_i-1)}(t_k,\hat{\theta}^j)\ \ldots\ -\xi_{if}(t_k,\hat{\theta}^j)\Big] \quad (27)$$

where the filter is $L_i(p,\hat{\theta}_i) = \frac{1}{F_i(p,\hat{\theta}_i)}$. This equation is solved by applying an IV technique. At each step, the value of the cost decreases toward some constant value. Nothing guarantees that the estimate of θ thus obtained corresponds to a global minimim of the cost function. However, the tests carried out in simulation and on data stemming from industrial processes show the relevance of this MISO version of the *srivc*.

3.4 Algorithm

The proposed iterative *srivc* algorithm summarizes the method developed section in 3.3.

1. $i = 1\ldots n_u$

   Estimate initial parameter vectors

   $$\hat{\theta}_i^0 = \Big[\hat{b}_{i,m_i}^0\ \ldots \hat{b}_{i,0}^0\ \hat{f}_{i,n_i-1}^0\ \ldots\ \hat{f}_{i,0}^0\Big]^T$$

   between $u_i(t)$ and $y(t)$ by a SISO estimation method.

   Generate the auxiliary model outputs by

   $$y_{u_i}(t_k,\hat{\theta}_i^0) = \frac{B_i(p,\hat{\theta}_i^0)}{F_i(p,\hat{\theta}_i^0)}u_i(t_k-\tau_i).$$

2. $j = 0\ldots N_{Iter}-1$, $i = 1\ldots n_u$

   (a) Generate an estimate of the noisy response to u_i to apply the multi-linear regression,

   $$\xi_{u_i}(t_k,\hat{\theta}_i^j) = y(t_k) - \sum_{l=1,l\neq i}^{n_u} y_{u_l}(t_k,\hat{\theta}_l^j).$$

   Filter the latter variable, the input signal and the auxiliary model output to overcome the unmeasurable derivative signals

   $$u_{i_f}(t_k,\hat{\theta}_i^j) = \frac{1}{F_i(p,\hat{\theta}_i^j)}u_i(t_k),$$
   $$\xi_{u_{i_f}}(t_k,\hat{\theta}_i^j) = \frac{1}{F_i(p,\hat{\theta}_i^j)}\xi_{u_i}(t_k,\hat{\theta}_i^j),$$
   $$y_{u_{i_f}}(t_k,\hat{\theta}_i^j) = \frac{1}{F_i(p,\hat{\theta}_i^j)}y_{u_i}(t_k,\hat{\theta}_i^j).$$

   (b) Generate the regressor (27) and the instruments

   $$Z_{i_f}^T(t_k,\hat{\theta}_i^j) = \quad (28)$$
   $$\big[u_{i_f}^{(m_i)}(t_k-\tau_i,\hat{\theta}_i^j)\ldots u_{i_f}(t_k-\tau_i,\hat{\theta}_i^j),\ -y_{u_{i_f}}^{(n_i-1)}(t_k,\hat{\theta}_i^j)\ldots - y_{u_{i_f}}(t_k,\hat{\theta}_i^j)\big], \quad (29)$$

   Calculate the IV estimate of the parameter vector $\hat{\theta}_i^{j+1}$:

   $$\hat{\theta}_i^{j+1} = \left[\sum_{i=1}^{N} Z_{i_f}(t_k,\hat{\theta}_i^j)\phi_{i_f}^T(t_k,\hat{\theta}_i^j)\right]^{-1} \sum_{i=1}^{N} Z_{i_f}^T(t_k,\hat{\theta}_i^j)y_{i_f}^{(n_i)}(t_k,\hat{\theta}_i^j), \quad (30)$$

   Use $\hat{\theta}_i^{j+1}$ to generate the auxiliary model output

   $$y_{u_i}(t_k,\hat{\theta}_i^{j+1}) = \frac{B_i(p,\hat{\theta}_i^{j+1})}{F_i(p,\hat{\theta}_i^{j+1})}u_i(t_k-\tau_i).$$

   Repeat step 2 until the relative variation on the parameters is sufficiently small :

   $$\sum_{i=1}^{n_u}\sum_{l=1}^{n_i+m_i+1}\left|\frac{\hat{\theta}_{il}^{j+1}-\hat{\theta}_{il}^j}{\hat{\theta}_{il}^j}\right| < \varepsilon, \quad (31)$$

   where $\hat{\theta}_{il}^j$ denotes the l^{th} item of the parameter $\hat{\theta}_i^j$, ε is a given tolerance and N_{Iter} is the final iteration number.

3. For $i = 1\ldots n_u$, $\hat{\theta}_i = \hat{\theta}_i^{N_{Iter}}$. Generate an estimate of the parametric covariance $\hat{P}_i$ matrix using the symmetric version of the IV algorithm and assuming a white noise

   $$\hat{P}_i = \hat{\sigma}_\epsilon^2\left[\sum_{k=1}^{N} Z_{i_f}(t_k,\hat{\theta}_i)Z_{i_f}^T(t_k,\hat{\theta}_i)\right]^{-1} \quad (32)$$

where $\hat{\sigma}_\epsilon^2$ denotes the empirical variance of the simulation error

$$\epsilon(t_k,\hat{\theta}) = y(t_k) - y_u(t_k,\hat{\theta}).$$

The global parameter estimation and parametric covariance matrix are given by

$$\hat{\theta} = [\hat{\theta}_1^T \dots \hat{\theta}_{n_u}^T]^T \tag{33}$$

$$\hat{P} = \begin{pmatrix} \hat{P}_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \hat{P}_{n_u} \end{pmatrix} \tag{34}$$

The initial step of the above algorithm is performed by a SISO Generalized Poisson Moment Functionals (GPMF) approach coupled with an auxiliary model IV estimation procedure (*ivgpmf*). This version is implemented in the CONTSID toolbox.

3.5 Properties

This method is an IV-type estimation technique. If the algorithm converges, it therefore gives consistent estimates when the model belongs to the system class. Moreover, some other interesting properties can be noticed:

- this method can be implemented recursively [14];
- the convergence of the algorithm is not sensitive to the choice of the initial parameter vector;
- an indication on the parameter uncertainties is given which makes it possible to assess the model quality.

3.6 Model order estimation

A key point to be solved in the identification procedure concerns the model order selection. The method available for SISO systems (see e.g. [15]) has been extended in the CONTSID toolbox (*srivcstruc* routine) for MISO systems represented by MITF with common and different denominators. While models are estimated from an estimation data set, two statistical measures are calculated on a validation data set and used to choose between a range of model orders. These are R_T^2 and YIC, which are defined as follows,

$$\begin{aligned} R_T^2 &= 1 - \frac{\hat{\sigma}_\epsilon^2}{\hat{\sigma}_y^2}, \\ YIC &= log_e \left\{ \frac{\hat{\sigma}_\epsilon^2}{\hat{\sigma}_y^2} \right\} + log_e \frac{1}{p} \sum_{j=1}^{p} \frac{\hat{\sigma}_\epsilon^2 p_{jj}}{\hat{a}_j^2} \end{aligned} \tag{35}$$

where $\hat{\sigma}_y^2$, ${\hat{\sigma}_\epsilon}^2$ denote respectively the variance of the measured output and the variance of the error between the measured output and simulated model output, $\hat{a}_j^2$ is the squared value of the j^{th} estimated parameter; p_{jj} is the j^{th} diagonal element of the *srivc* estimated parametric error covariance matrix $\hat{P}$ and p is the number of parameters. R_T^2 is recognized as the coefficient of determination based on the simulation error. It is a measure of how well the model output fit to the system output and must be close to 1. However, R_T^2 tends to overestimate the model orders. The Young's Information Criterion (YIC) is more complex and provides a measure of how well the parameters are defined statistically: the more negative the YIC, the better the definition. However it tends to underestimate the model orders. Both criteria are inspected to find the orders for which R_T^2 is sufficiently close to 1 and YIC sufficiently negative.

4 Numerical simulations

In this section Monte Carlo simulations are used to illustrate the properties of the proposed algorithm on a simulation example. The CONTSID toolbox, version 3.0 is used to perform the following simulations.

4.1 Simulation example presentation

The system considered is a two inputs one output system, with second-order non-minimum phase transfer functions. The first transfer function presents a resonant mode, with a damping of 0.3 and a natural frequency of 1 rad/s, while the second has two time constants equal to 1 and 3 s. The data generating system is given by

$$\begin{aligned} y_u(t) &= \frac{-0.5p+1}{p^2+0.6p+1}u_1(t) + \frac{-3p+2}{p^2+4p+3}u_2(t) \\ y(t_k) &= y_u(t_k) + v(t_k). \end{aligned} \tag{36}$$

The measured output $y(t_k)$ consists of a noise free output $y_u(t)$ sampled at time t_k on which is added an additive perturbation term $v(t_k)$. $\{v(t_k)\}_{k=1}^N$ is a zero-mean independent identically distributed Gaussian sequence.

The system is excited by two multi-sines of 25 frequencies with starting and end frequencies at 0.1 rad/s and half of the Nyquist frequency $\frac{1}{2}\frac{\pi}{Ts}$. Moreover, the two inputs are uncorrelated. The sampling period is equal to $T_s = 50$ ms.

Monte-Carlo simulations are used for a signal-to-noise (SNR) of 10 dB. The SNR is defined as

$$SNR = 10log\frac{P_{y_u}}{P_v}, \tag{37}$$

where P_{y_u} denotes the average power of the noise-free output fluctuations while P_v represents the average power of the zero-mean additive noise.

4.2 Monte Carlo simulation results

The system orders are first assumed known and Monte-Carlo simulations of 1000 realizations are used to study the sensitivity of the method to the initialisation, to the nature of the additive noise, and to the number of samples. The model order selection procedure described in section 3.6 is then evaluated.

The following criteria are selected for the performance evaluation

- the bias energy ($\|\hat{b}_{\bar{\boldsymbol{\theta}}}\|^2$),
- the energy of the estimated parameter standard deviation $\|\hat{\sigma}_{\bar{\boldsymbol{\theta}}}\|^2$,
- the energy of the mean square error ($\|\hat{b}_{\bar{\boldsymbol{\theta}}}\|^2 + \|\hat{\sigma}_{\bar{\boldsymbol{\theta}}}\|^2$),
- the energy of the parameter standard deviation estimated for a maximum-likelihood minimum variance estimator $\|\hat{\sigma}^{ML}_{\bar{\boldsymbol{\theta}}}\|^2$, obtained by the theoretical Cramér-Rao lower bound; in the case where $v(t)$ is white noise, this value is given by matrix $\hat{P}$ (34).

These criteria are applied to the normalized parameters. The number of iterations N_{Iter} of the *srivc* algorithm is also considered in the performance evaluation.

Sensitivity to the initialisation: The SISO *ivgpmf* routine [6] is used to produce an initial estimate. This approach requires to specify *a priori* the cut-off frequency λ of the first-order cascaded filter. Four different values for λ have been tested for a number of samples $N = 2000$. The results are shown in Table 1. It can be noticed that whatever the quality of the initialisation, the *srivc* routine still gives similar accurate estimates, with a slight larger number of iterations, when the initial estimate is far from the true one.

	IVGPMF SISO			
λ	0.5	2	10	15
$\|\hat{b}_{\bar{\boldsymbol{\theta}}}\|^2 + \|\hat{\sigma}_{\bar{\boldsymbol{\theta}}}\|^2$	5.433822	2.625535	0.676442	1.047361
	SRIVC MISO DD			
$\|\hat{b}_{\bar{\boldsymbol{\theta}}}\|^2 + \|\hat{\sigma}_{\bar{\boldsymbol{\theta}}}\|^2$	0.113991	0.084859	0.087825	0.089527
N_{iter}	10.124	8.253	7.266	7.232

Table 1: Sensitivity to the initialisation

Sensitivity to the sample number: The *srivc* routine is applied to 3 data sets of different size. The results are given in Table 2. Obviously, the bias, the variance and the number of iterations decrease when the sample number increases. The variance of the parameters is close to the theoretical minimum ($\|\hat{\sigma}^{ML}_{\bar{\boldsymbol{\theta}}}\|^2$). This adequacy is stronger when N is large, but remains acceptable for smaller N.

N	$\|\hat{b}_{\bar{\boldsymbol{\theta}}}\|^2$	$\|\hat{\sigma}_{\bar{\boldsymbol{\theta}}}\|^2$	$\|\hat{\sigma}^{ML}_{\bar{\boldsymbol{\theta}}}\|^2$	N_{Iter}
2000	0.001571	0.086242	0.080235	8.2
5000	0.000376	0.033893	0.033739	5.4
10000	0.000101	0.018380	0.018184	4.9

Table 2: Sensitivity to the sample number

Sensitivity to the additive noise type: Three different types of noise are chosen to illustrate the robustness of the method when the working hypotheses are no longer verified. $\{v(t_k)\}_{k=1}^{N}$ is then either a zero-mean independent identically distributed (i.i.d.) Gaussian sequence (g.w.n.), or an i.i.d. uniformly distributed sequence (u.w.n.) or an i.i.d. Gaussian sequence colored (g.c.n.) by

$$v(t_k) = \frac{0.1944q^{-1} - 0.1673q^{-2}}{1 - 1.792q^{-1} + 0.8187q^{-2}} e(t_k).$$

The results for $N = 10000$ are given in Table 3. It can be seen that the performance of the method are, as expected, a slightly less good when a colored noise is applied but they remain very acceptable for N large enough. Moreover, the results are accurate whatever the type of noise, therefore the method does not depend strongly on the type of additive noise. Note that in the case of colored noise, the estimated covariance matrix estimate given by $\hat{P}$ (34) does not hold true.

noise	$\|\hat{b}_{\bar{\boldsymbol{\theta}}}\|^2$	$\|\hat{\sigma}_{\bar{\boldsymbol{\theta}}}\|^2$	$\|\hat{\sigma}^{ML}_{\bar{\boldsymbol{\theta}}}\|^2$	N_{Iter}
g.w.n.	0.000101	0.018380	0.018184	4.9
u.w.n.	0.000200	0.018093	0.018268	5.0
g.c.n.	0.000108	0.179917	0.018548	7.6

Table 3: Sensitivity to the noise

Model order selection: The model order selection procedure presented in section 3.6 is applied to the simulation example.

The *srivcstruc* algorithm is applied to search all models in the range $[n_1, n_2, m_1, m_2, nk_1, nk_2] = [1, 1, 0, 0, 0, 0]$ to $[3, 3, 2, 2, 0, 0]$. This procedure is applied on the previous data set in the case of a white Gaussian noise. The first half is used for parameter estimation while the criteria (R_T^2 and YIC) are computed on the second half of the set. Table 4 shows the best 15 model orders sorted in increasing YIC.

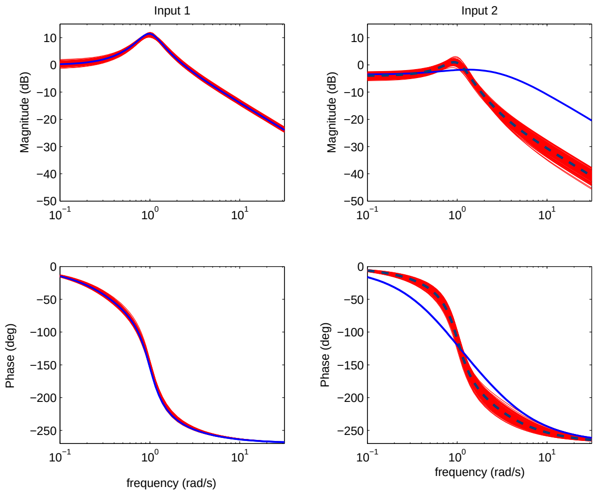
The best obtained model is indeed of orders $[2, 2, 1, 1, 0, 0]$. It presents the most negative $YIC = -9.52$ with a $R_2^T = 0.905$ very close to the best one.

Comparison *srivc* CD - *srivc* DD: Both *srivc* algorithms for CD and DD are applied to identify system (36). To investigate the performance of the algorithms, Monte Carlo simulation of 200 runs have

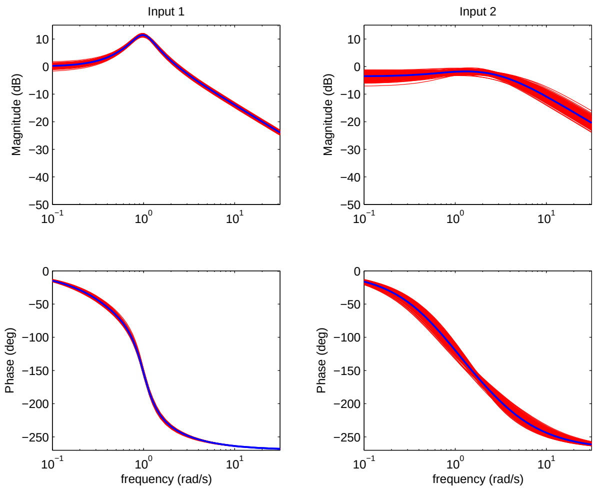
n_1	n_2	m_1	m_2	R_T^2	YIC
2	2	1	1	0.905	−9.52[1]
3	2	0	1	0.889	-8.71
2	2	0	1	0.844	-8.49
2	2	2	1	0.905	-5.88
3	2	1	1	0.904	-4.99
3	3	0	2	0.842	-3.84
2	2	0	2	0.844	-3.68
3	3	2	2	0.896	-3.46
2	3	0	1	0.843	-3.15
3	3	1	2	0.867	-1.77
2	3	1	2	0.867	-1.68
2	3	2	2	0.864	-1.68
2	3	2	1	0.903	-1.33
3	3	0	1	0.888	-1.26
2	3	1	1	0.907	-1.25

Table 4: Best 15 model orders

been performed when a Gaussian white noise is added to the output and $N = 2000$. Bode diagrams of the 200 estimated CT models for both methods are plotted in Figure 1. These plots illustrate, as expected, the relevance of the proposed version of the *srivc* algorithm dedicated to DD models. Indeed, it can be seen that the method considering common denominators fails to give a good estimate because the same dynamic is used for the two transfer functions. On the contrary, the *srivc* algorithm which considers different denominators give very accurate result with no bias and a very slight variance.



(a) CD models



(b) DD models

Figure 1: Bode plots of the 200 estimated CD and DD models together with the true system (thicker line)

5 Winding process application

5.1 Process description

The main part of this MIMO pilot plant is a winding process composed of a plastic web and three reels. Each reel is coupled with a direct-current motor via gear reduction. The angular speed of each reel (S_1, S_2, S_3) is measured by a tachometer while the tensions between the reels (T_1, T_3) are measured by tension meters. At a second level, each motor is driven by a local controller. Two PI control loops adjust the motor currents (I_1) and (I_3) and a double PI control loop drives the angular speed (S_2). The set-points of the local controllers (I_1^*, S_2^*, I_3^*) constitute the manipulated inputs of the winding system $u(t) = \begin{bmatrix} I_1^*(t) & S_2^*(t) & I_3^*(t) \end{bmatrix}^T$. Driving a winding process essentially comes down to controlling the web linear velocity and the web tensions (T_1) and (T_3) around a given operating point. Consequently, the output variables of the winding system are $y(t) = \begin{bmatrix} T_1(t) & T_3(t) & S_2(t) \end{bmatrix}^T$. The process is described in more detail in [22].

5.2 Experiment design

It has been shown that the validity of a linear model of the winding machine depends of the magnitude of the radius of the reels [22]. The weaker these variations are, the more the model can be reasonably considered as linear. The excitation sequence duration has to be sufficiently long to contain information about all the dynamics of the system but relatively short to neglect the radius variations. A solution consists to choose a very short sampling period (Ts=10 ms) in order to get an important number of samples (N=6000) over a short observation period (1 mn). Discrete-time internal binary sequences were used as excitation signals, with a sampling period set to 10 ms. Mean and linear trend of the signals have been removed. The input-output data set can be found in the Matlab file *winding.mat* of the *CONTSID* toolbox.

5.3 Model order selection

The *srivc* estimator presented in section 3.4 has been used to determine a CT multiple transfer function of the winding process. For each output a large number of models have been estimated for a wide range of model orders and time-delays. Best model structures according to validation criteria YIC and R_T^2 are given in Table 5. Each model presented in this table respects two conditions

- $YIC < \min(YIC) + \log(2)$,
- $R_T^2 < \max(R_T^2) - 0.01$.

The model set defined in this way is rational according to YIC and R_T^2 criteria. Models which present the smaller number of parameters and better YIC have been chosen for cross validation purpose. They are referenced by [1] in Table 5. Results have been obtained with the routines *srivcstruc.m* and *selcstruc.m* of the *CONTSID* toolbox.

m_1	m_2	m_3	n_1	n_2	n_3	R_T^2	YIC
			Output 1, $T_1(t_k)$				
2	2	2	0	0	0	0.962	-8.39[1]
2	2	2	0	0	1	0.969	-8.24
2	1	2	0	0	1	0.967	-8.21
2	2	2	0	1	1	0.970	-8.11
2	1	2	0	1	1	0.968	-8.00
1	1	2	1	0	1	0.964	-7.94
			Output 2, $T_3(t_k)$				
2	2	1	1	0	0	0.879	-7.86[1]
2	2	1	1	0	1	0.881	-7.47
2	2	1	1	1	0	0.882	-7.28
2	1	1	1	0	1	0.875	-7.22
			Output 3, $S_2(t_k)$				
2	2	2	0	1	0	0.993	-11.01
2	1	2	1	0	1	0.991	-10.99
2	2	1	0	0	0	0.991	-10.98[1]
2	1	2	1	1	1	0.992	-10.96
2	1	1	0	1	0	0.990	-10.96
2	2	2	1	0	1	0.993	-10.89
2	2	2	0	1	1	0.994	-10.81

Table 5: Best model structures according to YIC et R_T^2

5.4 Cross validation results

Cross-validation results are plotted on Figure 2 where it may be observed that there is a very good agreement with quite high values for the coefficient of determination. These identification results are comparable with those previously obtained by using a subspace-based method [23].

6 Conclusion

In this paper, the identification problem of continuous-time models of linear dynamic MISO systems has been

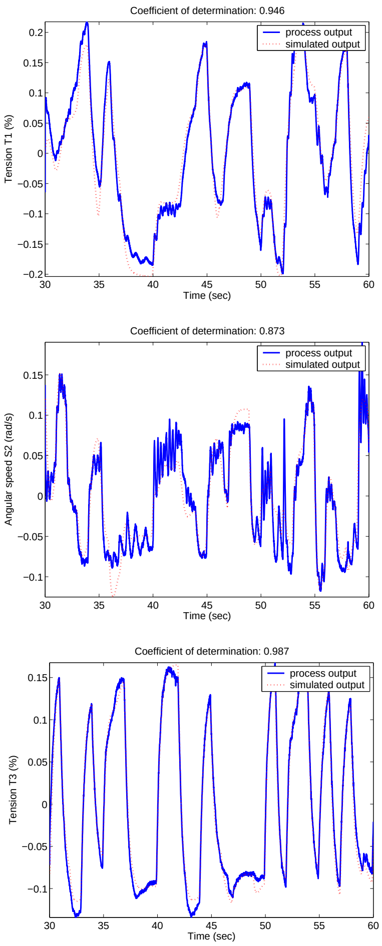


Figure 2: Cross validation results for the winding process

addressed by using a particular IV-type estimation method. A new method has been proposed to estimate multi-input transfer functions with different denominators. The performances of the method and its principal properties have been illustrated on the basis of a simulated example through Monte Carlo simulations. Finally the application of the proposed method to a pilot plant which simulates industrial material transport control problems has proved its efficiency in the case of real-life data.

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