Table of common unilateral z-transforms

The table below provides a number of z-transforms and their region of convergence (ROC).

Signal	z-transform	ROC
$\delta(n)$	1	All z
$\delta(n-i)$	z^{-i}	$z \neq 0$
$u(n)$	$\frac{z}{z-1}$	$\|z\| > 1$
$nu(n)$	$\frac{z}{(z-1)^2}$	$\|z\| > 1$
$n^2u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\|z\| > 1$
$a^n u(n)$	$\frac{z}{z-a}$	$\|z\| > \|a\|$
$na^n u(n)$	$\frac{az}{(z-a)^2}$	$\|z\| > \|a\|$
$n^2 a^n u(n)$	$\frac{az(z+a)}{(z-a)^3}$	$\|z\| > \|a\|$
$cos(\omega_0 n)u(n)$	$\frac{z(z-cos(\omega_0))}{z^2-2cos(\omega_0)z+1}$	$\|z\| > \|1\|$
$sin(\omega_0 n)u(n)$	$\frac{sin(\omega_0)z}{z^2-2cos(\omega_0)z+1}$	$\|z\| > \|1\|$

Remarks:

• The notation for z found in the table above may differ from that found in other tables. For example, the basic z-transform of a unit discrete-time step $u(n)$ can be written as either of the following two expressions, which are equivalent:

$$Z(u(n)) = U(z) = \frac{z}{z-1} = \frac{1}{1-z^{-1}}$$

• The ROC for a given sequence $x(n)$, is defined as the range of z for which the z-transform converges. Since the z-transform is a power series, it converges when $x(n)z^{-n}$ is absolutely summable. Stated differently,

$$\sum_{n=0}^{+\infty} |x(n)z^{-n}| < \infty \quad \text{must be satisfied for convergence.}$$

Property 1. if $x(n)$ is of finite duration, then the ROC is the entire z-plane, except possibly $z = 0$ and/or $z = \infty$.

Property 2. The ROC does not contain any poles.

Useful properties of the unilateral z-transform

Some useful properties which have found practical use in signal processing are summarized below.

Property	signal	z-transform
linearity	$ax(n)+by(n)$	$aX(z)+bY(z)$
delays (or shifts)	$x(n-1)$	$z^{-1}X(z)+x(-1)$
	$x(n-2)$	$z^{-2}X(z)+x(-2)+z^{-1}x(-1)$
	$x(n-i)$	$z^{-i}X(z)+x(-i)+z^{-1}x(-i+1)+\dots$
		$\dots+\mathrm{z}^{-i+1}x(-1)$
convolution	$y(n)=[h\star e](n)$	$Y(z)=H(z)E(z)$
	$y(n)=\sum_{k=-\infty}^{\infty}h(k)e(n-k)$	
differentiation	$nx(n)$	$-z\frac{dX(z)}{dz}$
accumulation	$\sum_{k=0}^{n}x(k)$	$\frac{1}{1-z^{-1}}X(z)$
initial value theorem	if $x(n)=0$ for $n<0$	$x(0)=\lim_{z\to+\infty}X(z)$
final value theorem	$\lim_{n\to+\infty}x(n)=\lim_{z\to1}(z-1)X(z)$	if the limit exists

Useful properties of geometric series

Some useful properties of geometric series are summarized below.

finite sum of geometric series	$\sum_{n=0}^{N}q^n=N+1$	if $q=1$
	$\sum_{n=0}^{N}q^n=\frac{1-q^{N+1}}{1-q}$	if $q\neq1$
infinite sum of geometric series	$\lim_{N\to+\infty}\sum_{n=0}^{N}q^n=\frac{1}{1-q}$	if $\lvert q\rvert<1$