

1 Presentation

The Quanser Aero Experiment experiment can be configured as a conventional dual-rotor helicopter, as shown in Figure 1.1. The front rotor that is horizontal to the ground predominantly affects the motion about the pitch axis while the back or tail rotor mainly affects the motion about the yaw axis (about the shaft).



Figure 1.1: Quanser Aero Experiment

The tail rotor in helicopters is also known as the anti-torque rotor because it is used to reduce the torque that the main rotor generates about the yaw. Without this, the helicopter would be difficult to stabilize about the yaw axis. Because the rotors on the Quanser Aero Experiment are the same size and equidistant from each other, the tail rotor also generates a torque about the pitch axis. As a result, both the front and back/tail rotors generate a torque on each other.

Note: The dynamic coupling between the pitch and yaw axes is **only seen when using the low-efficiency rotors**. It is not witnessed with the high-efficiency rotors provided. See the Quanser Aero User Manual for more information about the two different types of rotors supplied.

Topics covered:

- Derive linear equations of motion for the 2 DOF Helicopter configuration.
- Find the transfer function and state-space representation models.
- Identify the viscous damping coefficients about the pitch and yaw axes experimentally.
- Estimate the various torque thrust constants experimentally.
- Design a de-coupled PD control to control the pitch and yaw axes.
- Simulate the closed-loop system and implement on the Quanser Aero Experiment .
- Design state-feedback control using LQR optimization.
- Simulate the closed-loop system and implement on the Quanser Aero Experiment .
- Design and implement an Kalman-based LQG observer and controller for the Aero 1 DOF attitude configuration.

2 Modeling

2.1 Background

2.1.1 Equations of Motion

The free-body diagram of the Quanser Aero Experiment is illustrated in Figure 2.1.

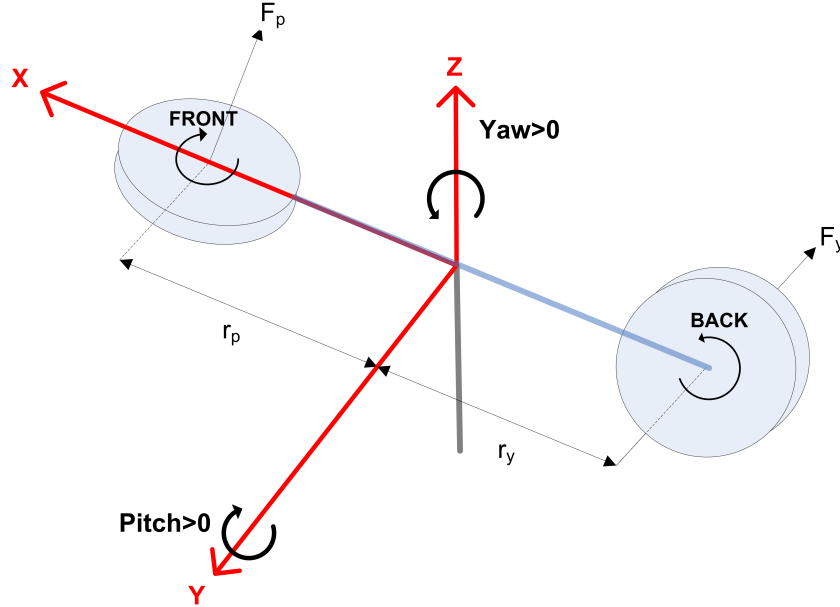


Figure 2.1: Simple free-body diagram of Quanser Aero Experiment

The following conventions are used for the modeling:

1. The helicopter is horizontal and parallel with the ground when the pitch angle is zero, i.e. $\theta = 0$.
2. The pitch angle increases positively, $\dot{\theta}(t) > 0$, when the front rotor is moved upwards and the body rotates **counter-clockwise (CCW)** about the Y axis.
3. The yaw angle increases positively, $\dot{\psi}(t) > 0$, when the body rotates **counter-clockwise (CCW)** about the Z axis.
4. Pitch increases, $\dot{\theta} > 0$, when the front rotor voltage is positive $V_p > 0$.
5. Yaw increases, $\dot{\psi} > 0$, when the back (or tail) rotor voltage is positive, $V_y > 0$.

When voltage is applied to the pitch motor, V_p , the speed of rotation results in a force, F_p , that acts normal to the body at a distance r_p from the pitch axis. The rotation of the propeller generates a torque about the pitch rotor motor shaft which is in turn seen about the yaw axis. Thus rotating the pitch propeller does not only cause motion about the pitch axis but also about the yaw axis. As described earlier, that's why conventional helicopters include a tail, or anti-torque, rotor to compensate for the torque generated about the yaw axis by the large, main rotor.

Similarly, the yaw motor causes a force F_y that acts on the body at a distance r_y from the yaw axis as well as a torque about the pitch axis.

We can develop a simple linear model that takes this coupling into account, and represents the motions of the Quanser Aero about the horizontal, i.e. when the body is parallel with the ground. The equations of motion are:

$$J_p \ddot{\theta} + D_p \dot{\theta} + K_{sp} \theta = \tau_p, \quad (2.1)$$

$$J_y \ddot{\psi} + D_y \dot{\psi} = \tau_y \quad (2.2)$$

where the torques acting on the pitch and yaw axes are

$$\begin{aligned}\tau_p &= K_{pp}V_p + K_{py}V_y, \text{ and} \\ \tau_y &= K_{yp}V_p + K_{yy}V_y.\end{aligned}$$

The parameters used in the EOMs above are:

- J_p is the total moment of inertia about the pitch axis,
- D_p is the damping about the pitch axis,
- K_{sp} is the stiffness about the pitch axis,
- K_{pp} is torque thrust gain from the pitch rotor,
- K_{py} is the cross-torque thrust gain acting on the pitch from the yaw rotor,
- V_p is the voltage applied to the pitch rotor, and
- V_y is the voltage applied to the yaw rotor motor.

Similarly, the total moment of inertia and damping about the yaw axis is J_y and D_y , respectively, K_{yy} is the torque-thrust gain from the yaw rotor, and K_{yp} is the cross torque-thrust gain acting on the yaw axis from the pitch rotor. Some of these model parameters are given in the Quanser Aero Experiment User Manual.

Remark the coupling between the pitch and yaw rotors. The total torque acting on each axis is generated from both rotors. Thus the total torque acting on the pitch equals $\tau_p = K_{pp}V_p + K_{py}V_y$ and the total torque on the yaw is $\tau_y = K_{yp}V_p + K_{yy}V_y$.

The total moment of inertia acting about the pitch and yaw axes are

$$\begin{aligned}J_p &= J_{body} + 2J_{prop} \\ J_y &= J_{body} + 2J_{prop} + J_{yoke}\end{aligned}$$

Expressing the rotor as a single-point mass, the inertia acting about the pitch or yaw axis from a single rotor is $J_{prop} = m_{prop}r_{prop}^2$. Modeling the helicopter body as a cylinder rotating about its center, the inertia is $J_{body} = m_{body}L_{body}^2/12$. Finally the forked yoke that rotates about the yaw axis can be approximated as cylinder rotating about its center as well and expressed as $J_{yoke} = m_{yoke}r_{fork}^2/2$. Evaluating the moment of inertia using the parameters listed in the Quanser Aero Experiment User Manual gives:

$$\begin{aligned}J_p &= 0.0219 \text{ kg-m}^2 \\ J_y &= 0.0220 \text{ kg-m}^2\end{aligned}$$

which are closed to the moment of inertia values listed in the User Manual that were derived from the CAD model.

2.1.2 Transfer Function Model

Taking the Laplace transform of the equations of motion given in Equation 2.2

$$J_p \left(\Theta(s)s^2 - \theta(0^-)s - \dot{\theta}(0^-) \right) + D_p \left(\Theta(s)s - \theta(0^-) \right) + K_{sp}\Theta(s) = K_{pp}V_p(s) + K_{py}V_y(s)$$

and

$$J_y \left(\Psi(s)s^2 - \psi(0^-)s - \dot{\psi}(0^-) \right) + D_y \left(\Psi(s)s - \psi(0^-) \right) = K_{yp}V_p(s) + K_{yy}V_y(s)$$

Because this is a MIMO system with two outputs and two inputs, the system is represented as a set of four transfer functions: $\Theta(s)/V_p(s)$ and $\Theta(s)/V_y(s)$ for pitch and $\Psi(s)/V_p(s)$ and $\Psi(s)/V_y(s)$ for yaw. Using this and assuming all

the initial conditions are zero, i.e. $\theta(0^-) = 0$, $\dot{\theta}(0^-) = 0$, $\psi(0^-) = 0$, and $\dot{\psi}(0^-) = 0$, we obtain the following transfer functions describing the system motions relative to the different inputs:

$$\frac{\Theta(s)}{V_p(s)} = \frac{K_{pp}}{J_p s^2 + D_p s + K_{sp}} \text{ and } \frac{\Psi(s)}{V_p(s)} = \frac{K_{yp}}{J_y s^2 + D_y s} \quad (2.3)$$

and

$$\frac{\Theta(s)}{V_y(s)} = \frac{K_{py}}{J_p s^2 + D_p s + K_{sp}} \text{ and } \frac{\Psi(s)}{V_y(s)} = \frac{K_{yy}}{J_y s^2 + D_y s}. \quad (2.4)$$

2.1.3 Linear State-Space Representation

Given the linear state-space equations: $\dot{x} = Ax + Bu$ and $y = Cx + Du$, we define the state for the Quanser Aero Experiment as

$$x^T = [\theta(t), \psi(t), \dot{\theta}(t), \dot{\psi}(t)], \quad (2.5)$$

the output vector as

$$y^T = [\theta(t), \psi(t)]$$

and the control variables as

$$u^T = [V_p(t) \ V_y(t)]$$

where θ and ψ are the pitch and yaw angles, respectively, and V_p and V_y are the motor voltages applied to the pitch and yaw rotors (i.e. the main and tail rotors). Using the equations of motion in Equation 2.2, the state-space matrices are

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K_{sp}/J_p & 0 & -D_p/J_p & 0 \\ 0 & 0 & 0 & -D_y/J_y \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ K_{pp}/J_p & K_{py}/J_p \\ K_{yp}/J_y & K_{yy}/J_y \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \text{ and } D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$