



*Identification of linear models for dynamical  
MIMO systems*

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## Categories of systems

Systems can be categorized by the number of inputs and outputs they have

- Single Input Single Output (SISO)
- Single Input Multiple Output (SIMO)
- Multiple Input Single Output (MISO)
- Multiple Input Multiple Output (MIMO)



(a) QUBE-Servo 2 with Inertia Disc Module



## Linear continuous-time models

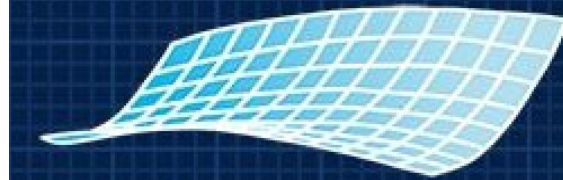
TRANSFER FUNCTION

$$G(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$$

STATE SPACE

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}$$

LINEAR PARAMETER  
VARYING



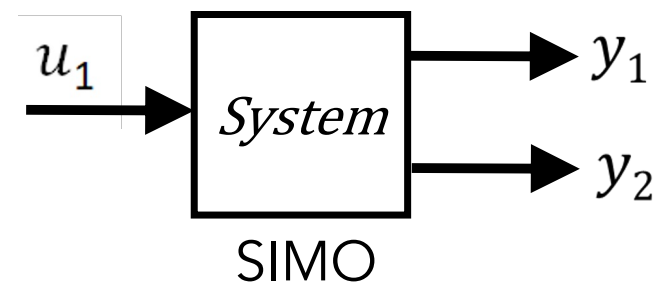
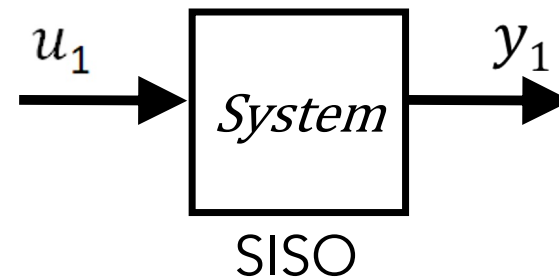
$$G(s, v(t)) = \frac{K(v(t))}{1 + T(v(t))s}$$

## Single input systems

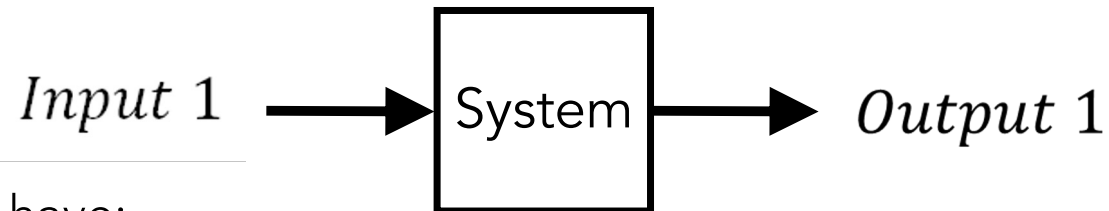
Single-input systems have a single control input

Depending on the number of outputs, sub-categories include:

- Single-input, single-output (SISO)
- Single-input, multi-output (SIMO)



## Single Input Single Output (SISO) systems



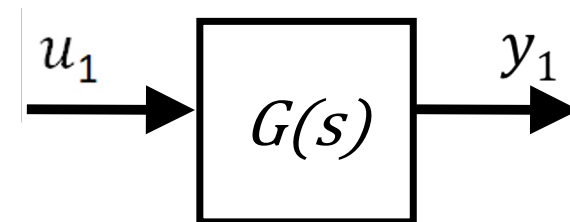
SISO systems have:

- A single actuating element
- A single sensing element

Typically modeled using *transfer functions*

Relatively simple control system

- Mainly *PID control*, but also on-off control

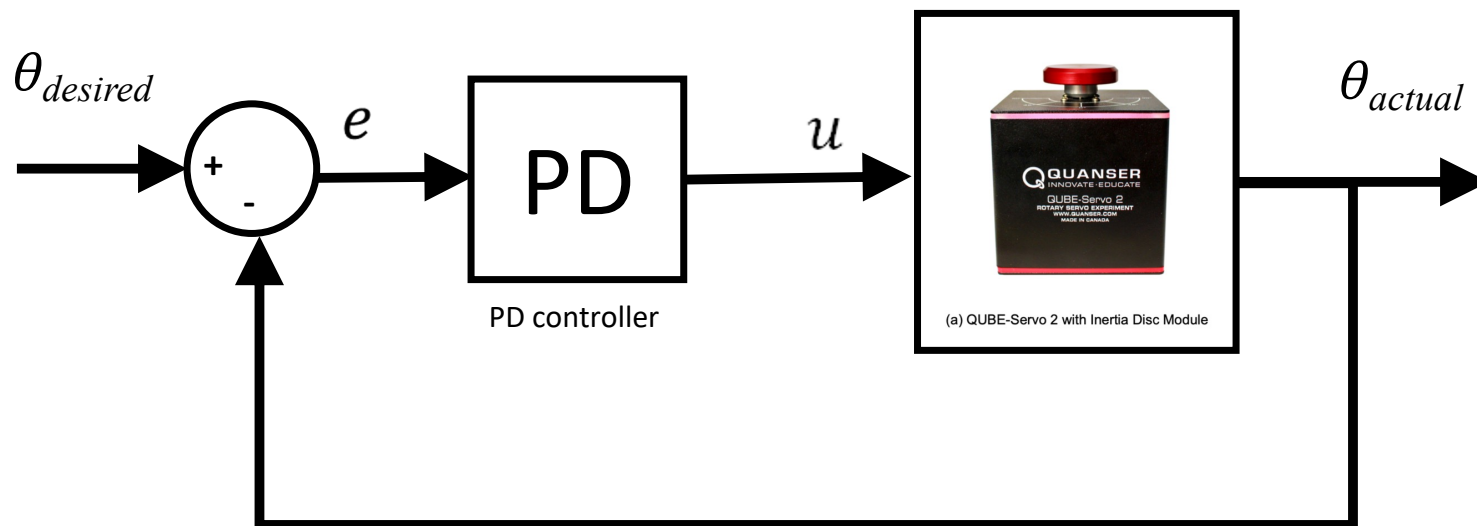


Examples include:

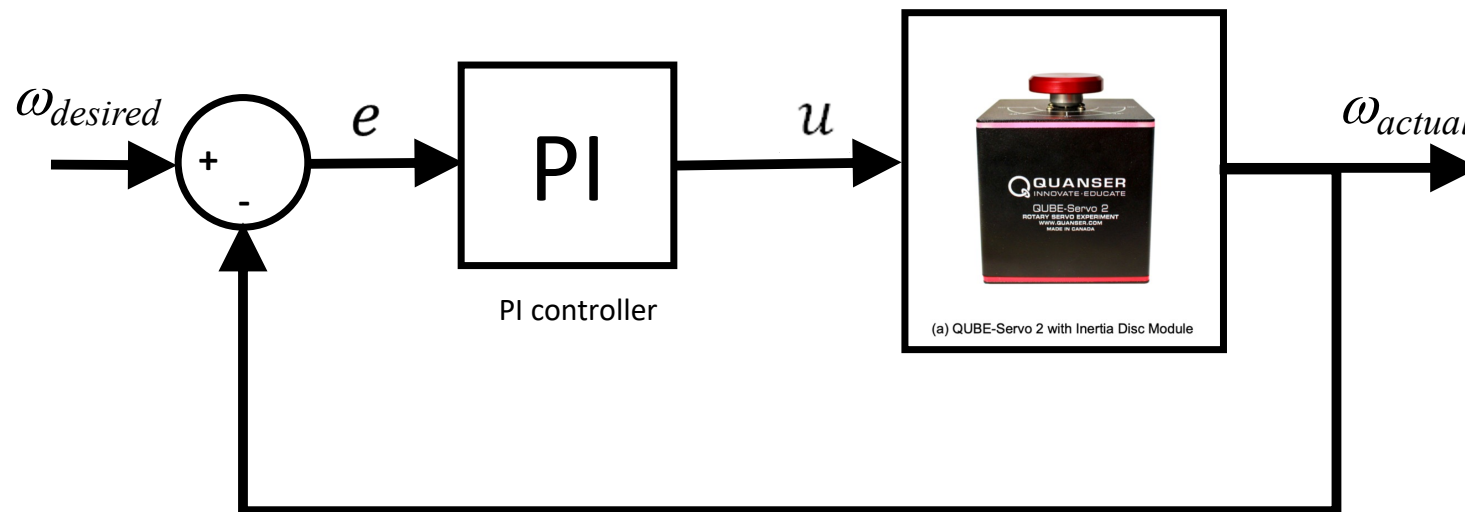
- Angular position control of the QUBE servo 2 inertia disc
- Angular velocity control of the QUBE servo 2 inertia disc



## Example: PD control of the inertia disc angular position



## Example: PI control of the inertia disc angular velocity

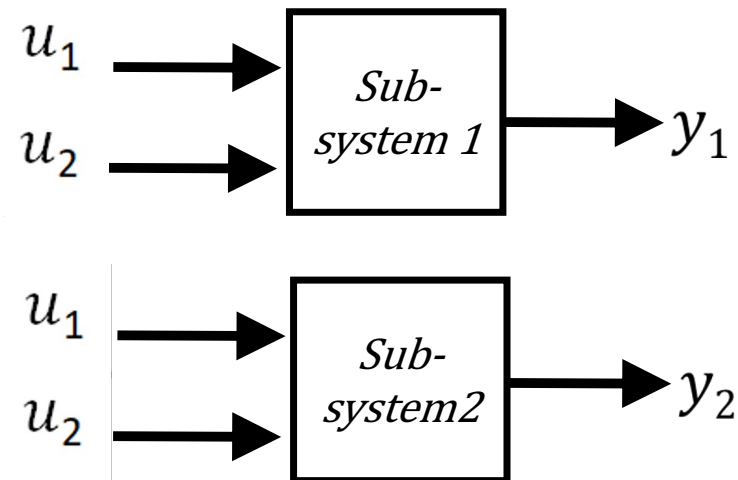
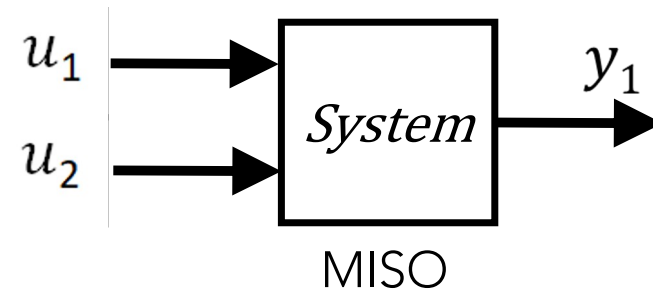
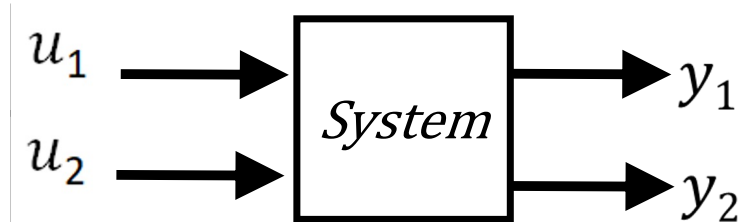


## Multi input Single output (MISO) systems

Multi-input single output systems have multiple control inputs and one single output

➤ *quite rare*

In certain cases, MISO systems can be modeled as multiple SISO systems





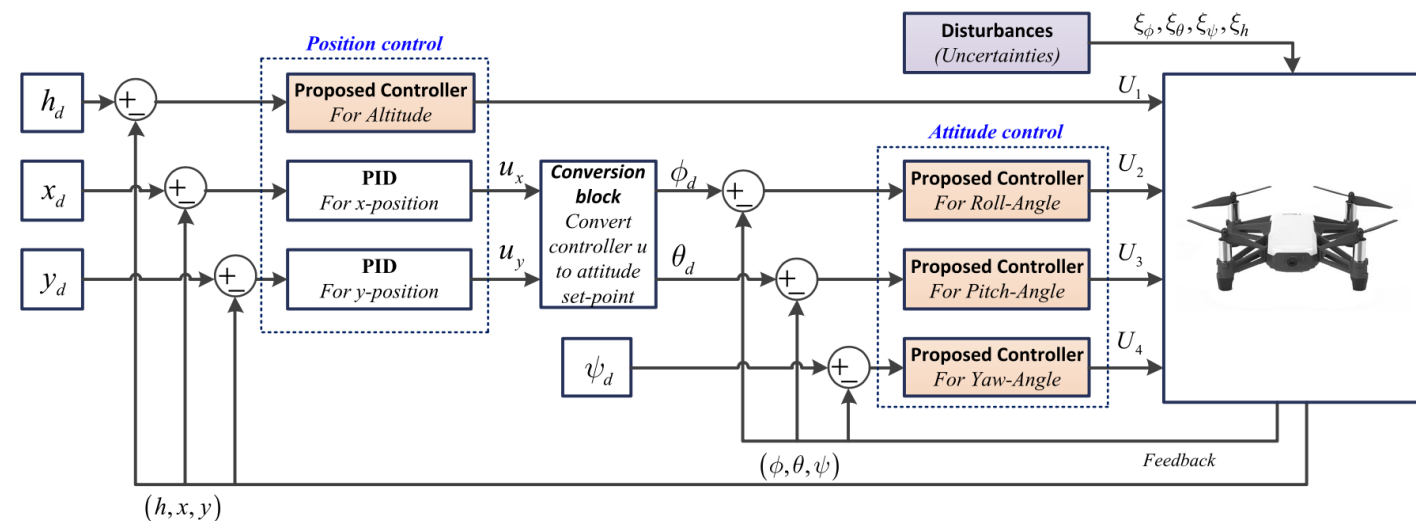
## SISO vs. MIMO

In certain cases, MIMO systems can be modeled as multiple SISO systems

- Why? SISO systems are easier to model and control !
- Easier to tune multiple SISO systems, instead of a single MIMO system

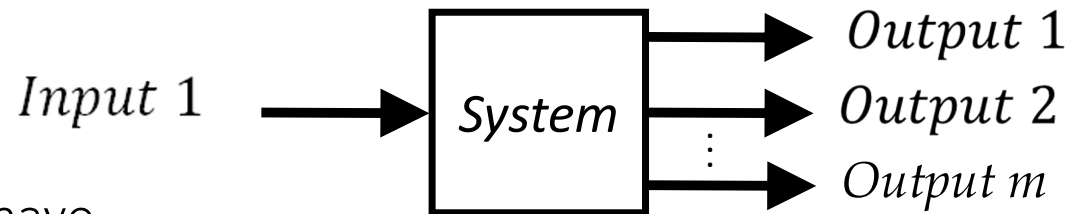
Examples include:

➤ Drone attitude control



When? Nonexistent or **negligible coupled dynamic** – Not always possible !

## Single Input Multi Output (SIMO) systems



SIMO systems have

- Single actuating element
- Multiple sensing elements
- Known as *under-actuated systems*

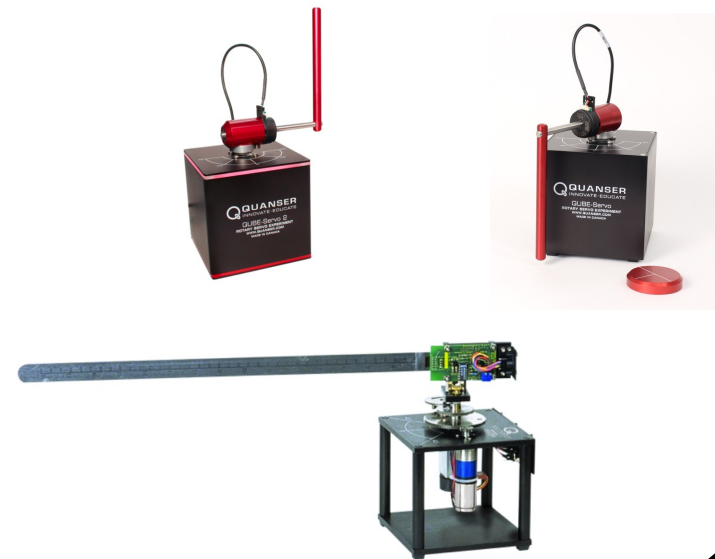
Typically modeled using *state-space models*

More advanced control systems

- *LQR control*

Examples include:

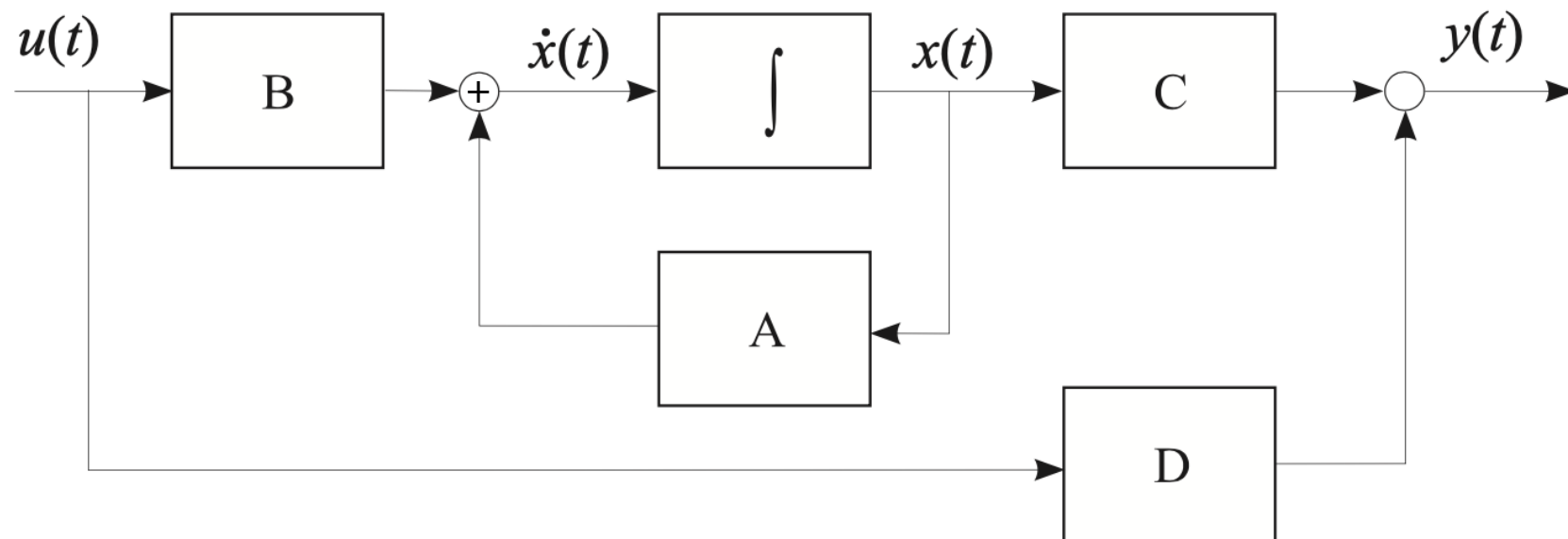
- Inverted pendulum, tower crane
- Flexible robotic arm



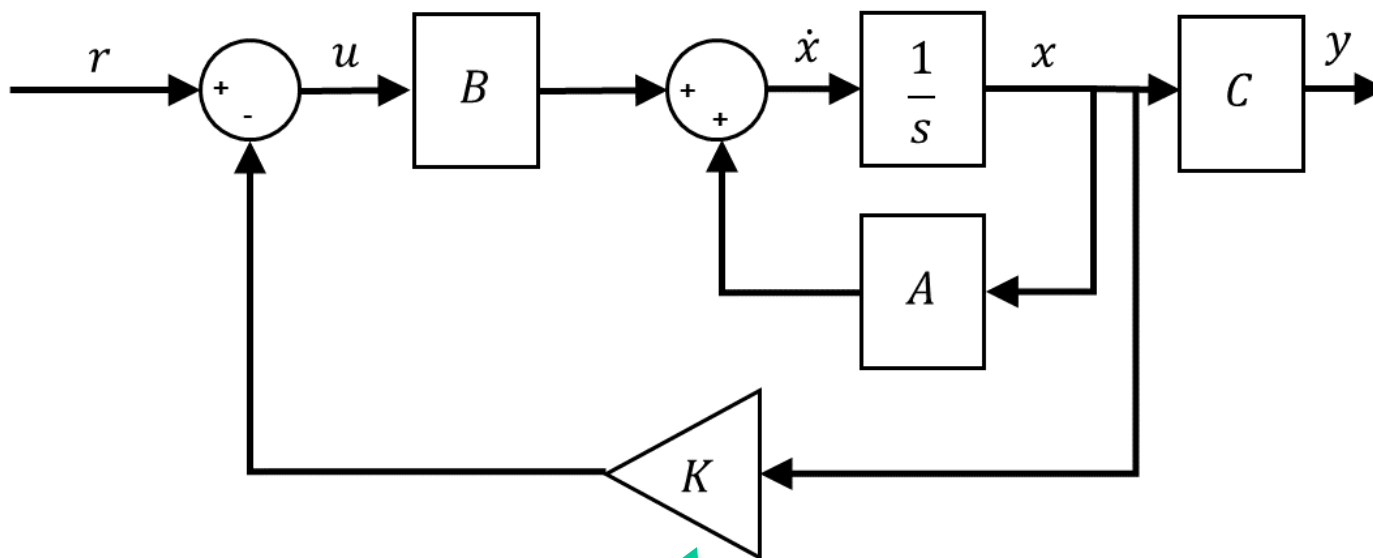
## Continuous-time linear state-space model

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$



# State-feedback control



Use pole placement or LQR control design

## Example: Canadarm

Arm is made of several *flexible* links and motorized joints

- Each link/joint represents a SIMO system modeled using *state-space*

Coupled dynamics

- Joint movement/flexible link dynamically affect each other

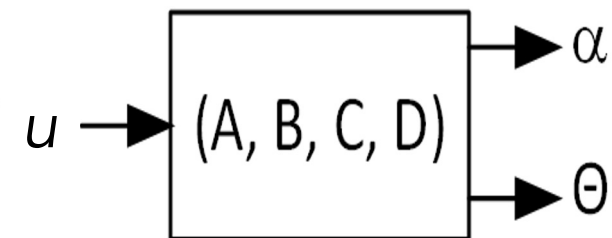
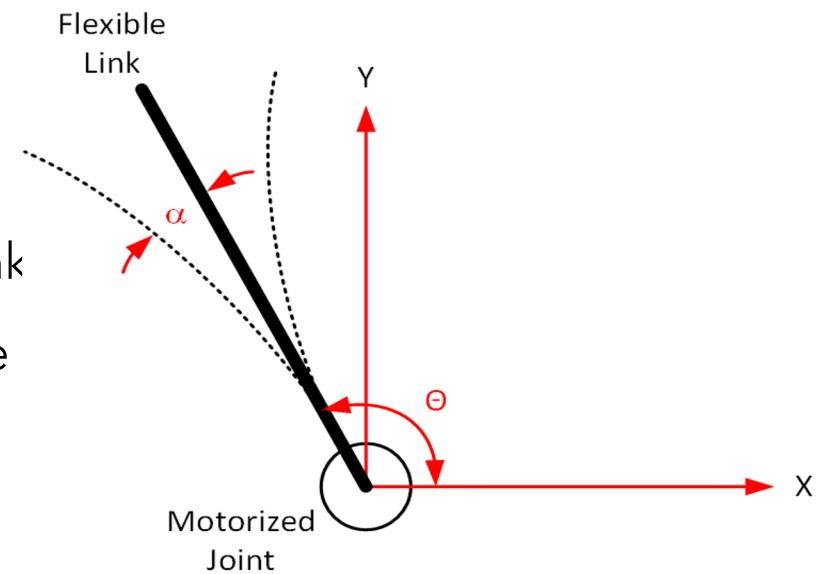
We will study the identification and control of a flexible robot link



## Identification of a state-space model for a SIMO flexible link

It is a 1 input - 2 outputs (SIMO) system

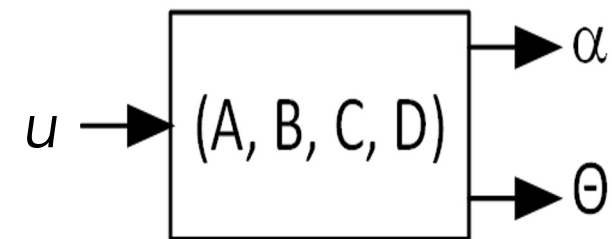
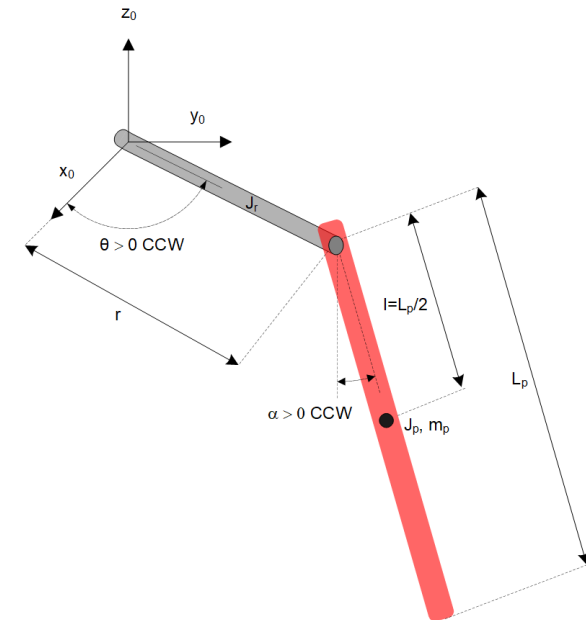
- $u(t)$ : the motor voltage
- $\alpha(t)$ : the angle deflection of the flexible link
- $\theta(t)$ : the angular position of the servo base



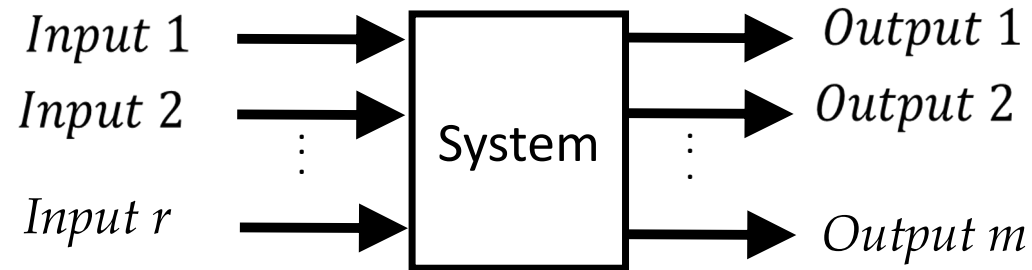
# Identification of a state-space model for a tower crane

Motions of a rotary pendulum are similar to that of a tower crane. It is a 1 input - 2 outputs system

- $u(t)$ : the motor voltage
- $\alpha(t)$ : the angular position of the pendulum
- $\theta(t)$ : the angular position of the servo base/arm



## Multi-Input Multi-Output (MIMO) Systems



MIMO systems have

- Multiple actuating elements
- Multiple sensing elements

Typically modeled using *state-space models*

Advanced control systems are required

- *LQR control*,...

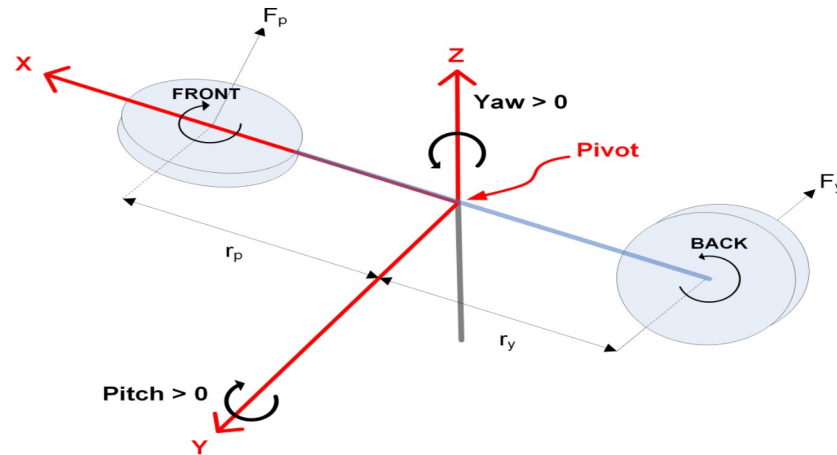
Examples include:

- Dual-rotor AERO helicopter

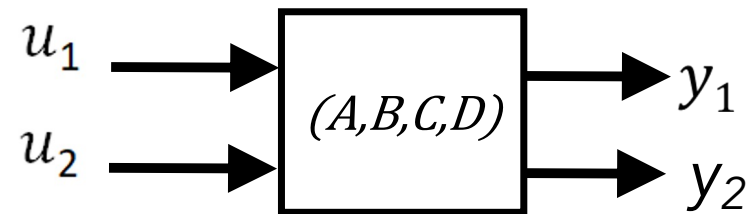




# Example: 2 inputs - 2 outputs AERO helicopter



- Two inputs
  - Front rotor thrust
  - Rear rotor thrust
- Two outputs (*no roll*)
  - Pitch
  - Yaw
- Coupled dynamics
  - Pitch/yaw affect each other



We will study the identification and control of the AERO helicopter