Control of mobile robots
Problems n°1
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PID-based angular position control of a DC servo-motor

One of the most common devices for actuating a control system is DC motors. A DC motor is any of a class of rotary electrical motors that converts direct current electrical energy into mechanical energy. Small DC motors are commonly used in tools, toys, appliances and in computer-related equipment such as disk drives or printers. Larger DC motors are currently used in propulsion of electric vehicles, robot systems, elevators, cranes and hoisting devices. Large DC motors are also widely used in the industry such as in drives for steel rolling mills. DC motors that are used in servo systems are called DC servo-motors. In DC servo-motors, the rotor inertias have been made very small. They are frequently used in robot control systems and other angular position or speed control system.
We study the angular position control of a DC servo-motor in this problem as they appear for each rotary joint of robotic arms such as the Canadarm2 as shown in Figure 1.1 for example.

Figure 1.1: Astronaut Stephen Robinson anchored to the end of Canadarm2 during STS-114, 2005. By NASA - http://spaceflight.nasa.gov/gallery/images/shuttle/sts-114/html/s114e6647.html

1. DC servo-motor modelling

The inductance in the armature circuit of a DC servo-motor is usually small and can therefore be neglected. In this case, the motor voltage-to-angular position transfer function for the DC servo-motor takes the following form

$$G(s) = \frac{\Theta(s)}{U(s)} = \frac{K}{s(1+Ts)} \tag{1}$$

- $\Theta(s) = \mathscr{L}[\theta(t)]$, where $\theta(t)$ is the angular position of the motor shaft in rad;
- $U(s) = \mathscr{L}[u(t)]$, where $u(t)$ is the applied motor voltage in V;
- K is the motor gain in rad/(V.s);
- T is the motor time-constant in second.

1.a. Define the input and the output of the DC servo-motor and their unit.

1.b. From equation (1), it can be seen that the transfer function involves a pure integrator term $\frac{1}{s}$. The transfer function model can also be seen as the cascade of a pure integrator and a simple first-order model.
The angular velocity $\omega(t)$ being the time-derivative of the angular position $\theta(t)$

$$\omega(t) = \frac{d\theta(t)}{dt}$$

both variables are linked in the Laplace domain by a pure derivator (or pure integrator) so that (1) can be expressed as:

$$\frac{\Theta(s)}{U(s)} = \frac{\Theta(s)}{\Omega(s)} \times \frac{\Omega(s)}{U(s)} = \frac{1}{s} \times \frac{K}{1+Ts} \tag{2}$$

$\Omega(s) = \mathscr{L}[\omega(t)]$, where $\omega(t)$ is the motor angular velocity (or speed) in rad/s.
From the analysis above, complete the block-diagram in Figure 1.2.

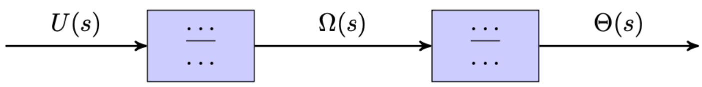

Figure 1.2: Block-diagram of the DC motor

1.c. Identifying a system having a pure integrator from a step response is tricky since the response is diverging. The angular position (in rad) and velocity (in rad/sec) responses for a positive step from 0 to 2 V sent after 1s followed by a negative step from 2 V to 0 applied to the motor voltage after 3 seconds are plotted in Figure 1.3.
The angular position response of the motor starts out slowly due to the time constant, but once that is out of the way the motor position ramps at a constant velocity. It is easier when the motor speed is also measured to identify the response between the motor angular velocity and the input voltage since the voltage-to-angular velocity transfer function has the well-known first-order model form whose parameters can be easily estimated from the step response (the *a priori* knowledge about the pure integrator is then added in the final model):

$$\frac{\Omega(s)}{U(s)} = \frac{K}{1+Ts}, \tag{3}$$

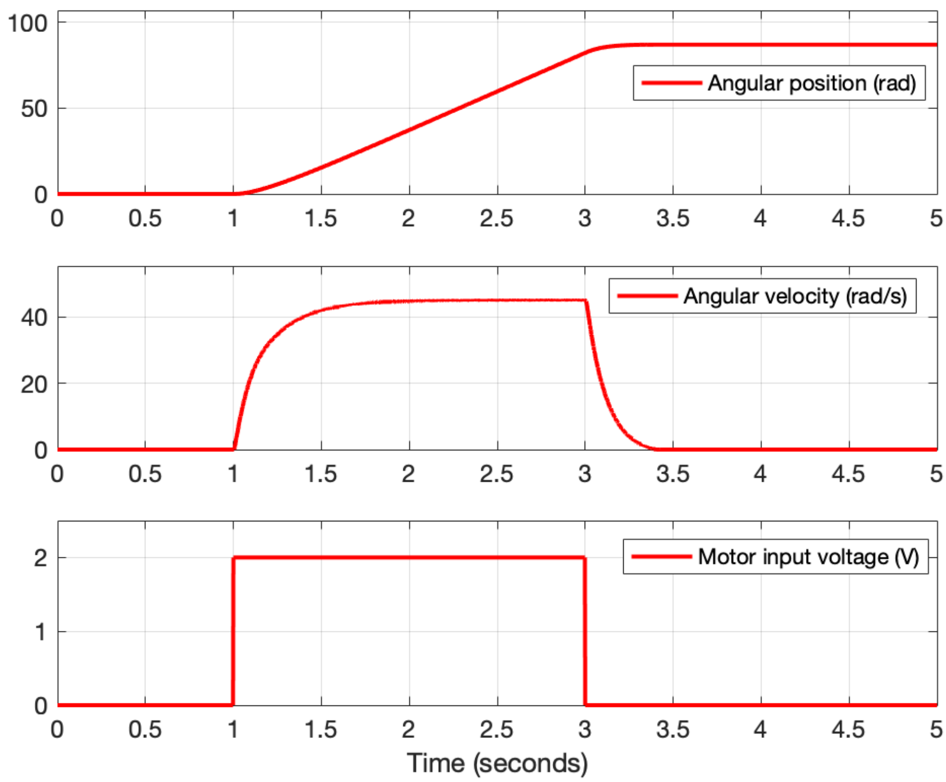

Figure 1.3: Angular position and velocity responses to a positive and negative step input sent to the motor voltage

The performance requirements for the angular position control design are described in Table 1.

Requirement	Assessment criteria	Level
Control the position	position reference tracking	No steady-state error
	motor input voltage	limited to the range [-10V ; +10 V]
	Percent Overshoot	$D_1 = 4.3$ %
	Settling time at 5 %	$T_s^{5\%} = 0.05$ s or as short as possible
	Disturbance rejection	Rejection of load effects

Table 1: Performance requirements for angular position control

We assume in the following that the numerical values of the model parameters are:

- $K = 23$ rad/(V.s);
- $T = 0.2$ s

2. Servo-motor control using simple proportional feedback

Figure 1.4 shows a simple proportional feedback configuration of the positional servo system. This basic configuration has been used in industry for many years.

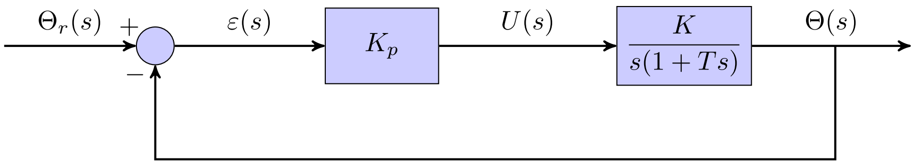

Figure 1.4: Block-diagram of the simple proportional feedback configuration of the positional servo system

2.a. Determine the open-loop transfer function $F_{OL}(s)$ in terms of K, K_p and T.

2.b. Express the closed-loop transfer function $F_{CL}(s)$ in terms of K, K_p and T.

2.c. Determine the range of values for K_p that ensures the stability of the feedback control system.

2.d. Calculate the steady-state tracking error in response to a step $\theta_r(t) = A\Gamma(t)$, *i.e.* :

$$\lim_{t\to+\infty} \varepsilon(t) = \lim_{t\to+\infty} (\theta_r(t) - \theta(t))$$

2.e. Show that the value for K_p that makes the closed-loop transfer function to have a percent overshoot $D_{1\%} = 4.3$ % is

$$K_p = \frac{1}{4z^2KT} = \frac{1}{2KT}$$

The following formula will be useful:

$$z = \sqrt{\frac{(\ln(D_1))^2}{\pi^2 + (\ln(D_1))^2}}$$

It is also recalled that the transfer function of a standard second-order system is

$$G(s) = \frac{K_{ss}}{\frac{s^2}{\omega_n^2} + 2\frac{z}{\omega_n}s + 1} = \frac{K_{ss}\omega_n^2}{s^2 + 2z\omega_n s + \omega_n^2}$$

where

- K_{ss}: steady-state gain
- z : damping ratio
- ω_n : undamped natural frequency (in rad/sec)

2.f. Calculate the percent overshoot, the peak time T_{D_1}, the settling time at 5 %, $T_s^{5\%}$, and the angular position value at the peak time $\theta(T_{D_1})$. The following formulas will be useful:

$$T_s^{5\%} \approx \frac{3}{z\omega_n}; \qquad T_{D_1} = \frac{\pi}{\omega_n\sqrt{1-z^2}}$$

2.g. Plot the shape of the closed-loop P control response to a unit step setpoint.

2.h. Are the performance requirements satisfied?

3. Servo-motor control using proportional and derivative feedback

We now consider the performance of a proportional and derivative (PD) control which involves a velocity feedback loop as shown in Figure 1.5. This control system represents a high-speed, high precision positional servo system. The positional servomotor systems of this type are used frequently in today's angular position control systems.

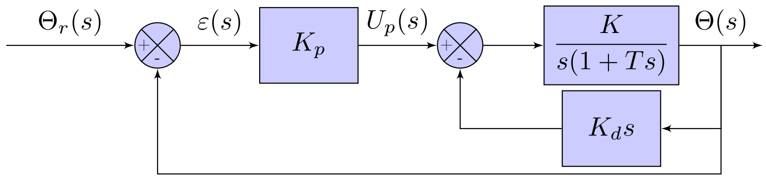

Figure 1.5: Block-diagram of the PD feedback configuration of the positional servo system

3.a. What is the advantage of implementing the derivative term on the output rather than on the error signal $\varepsilon(s)$?

3.b. Determine the internal closed-loop transfer function $F_i(s) = \dfrac{\Theta(s)}{U_p(s)}$ in terms of K, K_d and T.

3.c. Plot the simplified closed-loop block-diagram.

3.d. Determine the closed-loop transfer function $F_{CL}(s)$ in terms of K, K_p, K_d and T.

3.e. Determine the range of values for K_p and K_d that ensure the stability of the feedback control system.

3.f. Calculate the steady-state tracking error in response to a step $\theta_r(t) = A\Gamma(t)$, *i.e.* :

$$\lim_{t\to+\infty} \varepsilon(t) = \lim_{t\to+\infty} (\theta_r(t) - \theta(t))$$

3.g. Show that the value for K_p and K_d that make the closed-loop transfer function step response to have a percent overshoot $D_{1\%} = 4.3$ % and a settling time at 5% $T_s^{5\%} = 0.05$ s are:

$$\begin{cases} K_p = \dfrac{\omega_n^2 T}{K} \\ K_d = \dfrac{2zK_p}{\omega_n} - \dfrac{1}{K} \end{cases}$$

3.h. Calculate the peak time T_{D_1} along with the angular position at the peak time $\theta(T_{D_1})$.

3.i. Plot the shape of the closed-loop PD control response to a unit step setpoint. Are the performance requirements satisfied? Compare your plot with the closed-loop PD control response to a unit step setpoint displayed in Figure 1.6.

3.j. In practice, the derivative action is not implemented directly but it requires the use of a low-pass filter $\dfrac{\omega_f}{s+\omega_f}$ to reduce the measurement noise effect of the angular position.

$$K_d s \approx K_d s \times \frac{\omega_f}{s+\omega_f} \tag{4}$$

The low-pass filter transfer function on the right approximates the pure derivative well at low frequencies. The filter cut-off frequency ω_f is usually chosen to reduce the sensor noise effects.
Modify the block-diagram of the closed-loop control to make appear the low pass-filter.

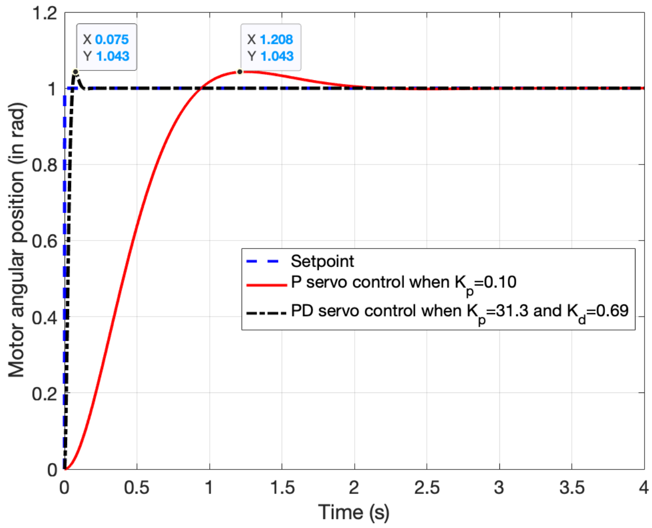

Figure 1.6: Closed-loop P and PD control responses to a unit step setpoint. Both controls satisfy the overshoot requirement but the PD control response is much faster.