# *Data-driven model learning of dynamical systems*

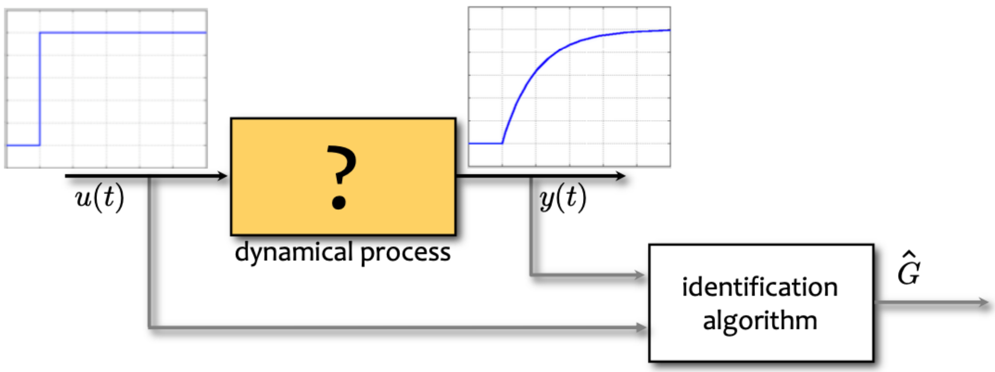
------

*Review of basic step response-based methods*


Hugues GARNIER

hugues.garnier@univ-lorraine.fr

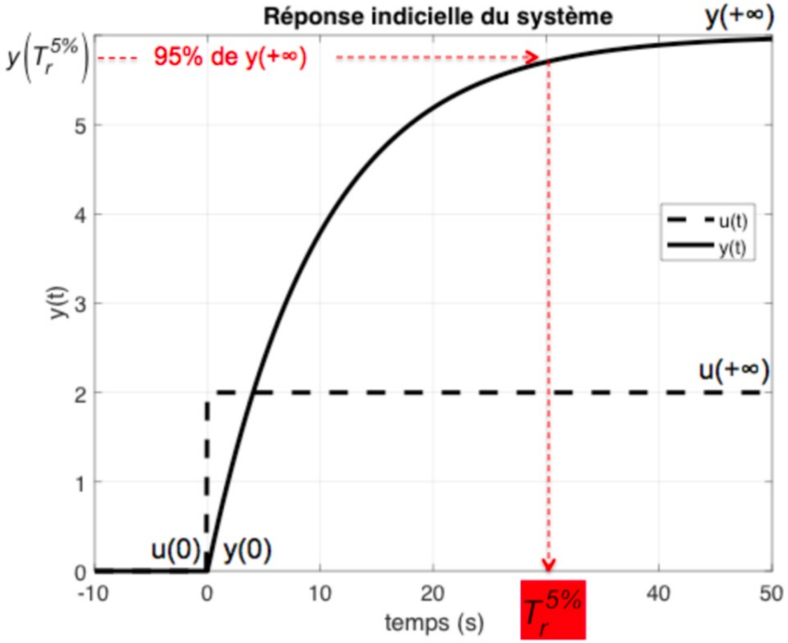
# Step response-based system identification




- Excite the process with a step $u(t)$ , record output response $y(t)$
- Observe the shape of $y(t)$ and reconstruct $G$
  (1st-order response ? 2nd-order undamped response ? Any delay ? ...)
- Mostly used in process control: excitation experiment is easily done, superposition of effects can be used in the multivariable case to identify each entry $G_{ij}$ of the **transfer matrix** $G$ , one at the time

# Identification of a first-order model plus delay from step response data

$$G(s) = \frac{K\, e^{-\tau s}}{1 + T s}$$




There is no delay here τ=0
Otherwise, it is determined from visual inspection

1. Find the final and initial values of the response and of the step.

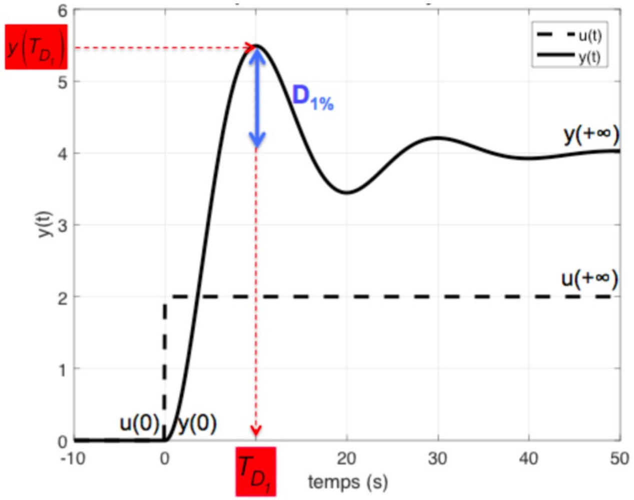
$$K = \frac{y(+\infty) - y(0)}{u(+\infty) - u(0)}$$

2. Find $y(T_r^{5\%})$, deduce from it $T_r^{5\%}$ then $T$:

$$T = \frac{T_r^{5\%}}{3}$$

# Identification of a second-order undamped model plus delay from the step response

$$G(s) = \frac{K\,e^{-\tau s}}{\frac{s^2}{\omega_0^2} + 2\frac{z}{\omega_0}s + 1}$$




There is no delay here $\tau$=0
Otherwise, it is determined from visual inspection

1. Find the final and initial values of the response and of the step. Deduce $K$ from:

$$K = \frac{y(+\infty) - y(0)}{u(+\infty) - u(0)}$$

2. Find the final and initial values of the response and that of the first overshoot $y(t_{D_1})$ Deduce from it $D_1$, then $z$:

$$D_1 = \frac{y(T_{D_1}) - y(+\infty)}{y(+\infty) - y(0)}$$

$$z = \sqrt{\frac{(\ln(D_1))^2}{(\ln(D_1))^2 + \pi^2}}$$

3. Find the time-instant of the first overshoot $T_{D_1}$. Deduce from it $\omega_0$:

$$\omega_0 = \frac{\pi}{T_{D_1}\sqrt{1 - z^2}}$$


H. Garnier

UNIVERSITÉ DE LORRAINE

POLYTECH NANCY

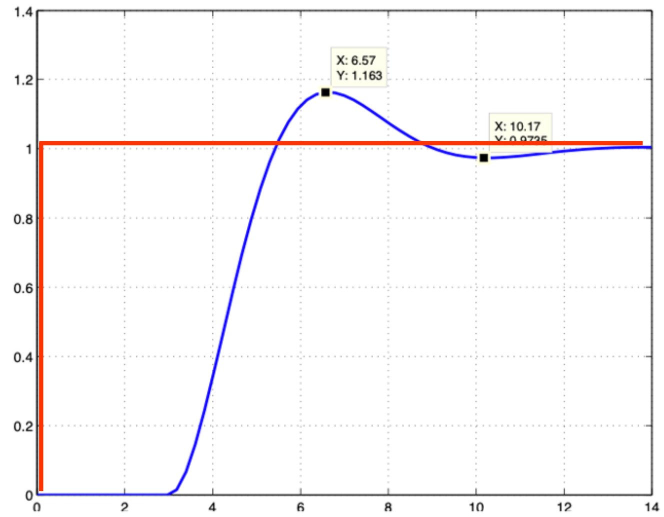
# Identification of a second-order undamped model plus time-delay from step response - Example

The step has been sent at time t=0s

The step response looks like a second-order undamped model with a clear time-delay

$$G(s) = \frac{K\, e^{-\tau s}}{\dfrac{s^2}{\omega_0^2} + 2\dfrac{z}{\omega_0}s + 1}$$

From visual inspection: $\tau \approx 3$s




1. Find the final and initial values of the response and of the step. Deduce $K$ from:

$$K = \frac{y(+\infty) - y(0)}{u(+\infty) - u(0)} = \frac{1-0}{1-0} = 1$$

2. Find the final and initial values of the response and that of the first overshoot $y(t_{D_1})$
   Deduce from it $D_1$, then $z$:

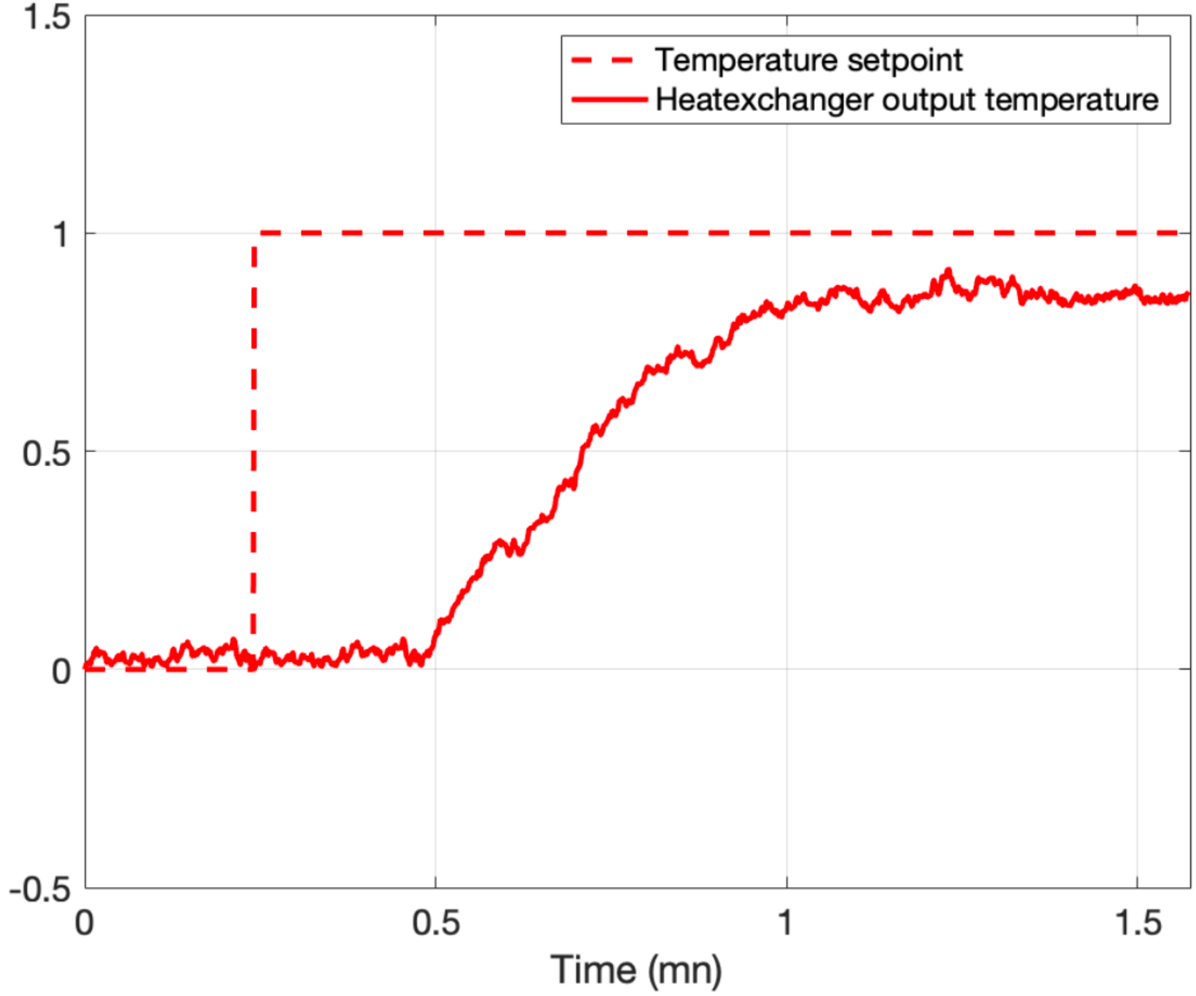
$$D_1 = \frac{y(T_{D_1}) - y(+\infty)}{y(+\infty) - y(0)} = \frac{1.163-1}{1-0} = 0.163$$

$$z = \sqrt{\frac{(\ln(D_1))^2}{(\ln(D_1))^2 + \pi^2}} \approx 0.5$$

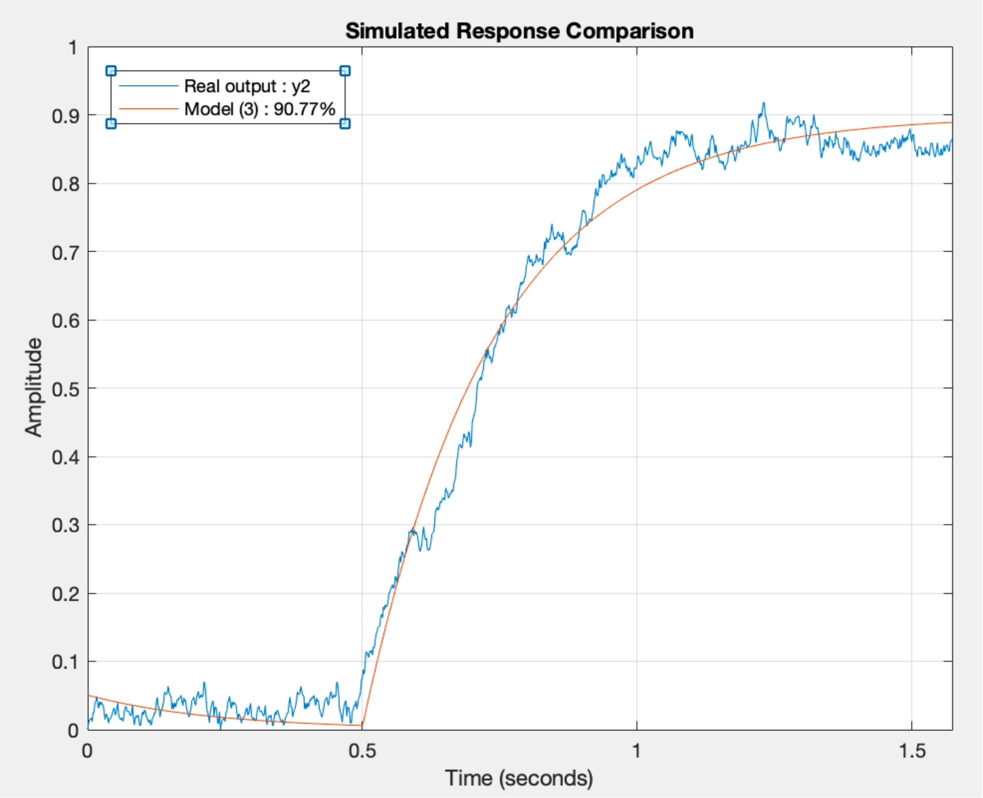
3. Find the time-instant of the first overshoot $T_{D_1}$. Deduce from it $\omega_0$:

$$\omega_0 = \frac{\pi}{T_{D_1}\sqrt{1-z^2}} = \frac{\pi}{6.57\sqrt{1-0.5^2}} \approx 0.55 \text{ rad/s}$$

# Identification of a heat exchanger model from step response data



# Identification of a heat exchanger model from step response data




There are nowadays methods that directly determine more general transfer function model parameters from time-response data not limited to step responses

➡ Objectives of the rest of the course …