Identification of dynamical systems

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Identification de systèmes dynamiques

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Course organization and prerequisites

- Organization
  - 8h00 of lectures
  - 8h00 of tutorials
  - 8h00 of lab/mini-project

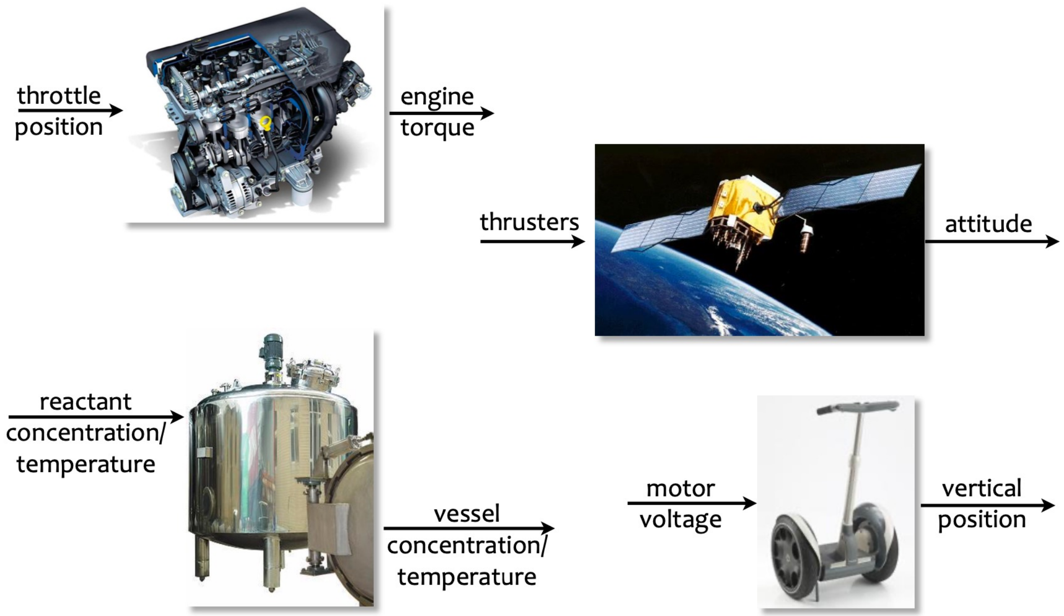
- Prerequisites
  - System theory and control
  - A sound knowledge about probability and statistics
  - Regression analysis
  - Optimization methods
  - Programming proficiency in Matlab/Simulink

- Skill assessments

  In pairs, you will work on a data-driven modelling problem using real-life data
  - 1 scientific report that gathers your analysis (0,4)
  - 1 oral presentation of a data-driven modelling mini-project (0,6)

Dynamical systems

- A dynamical system is a physical object that evolved over time usually under the effect of external excitations



- The way the system evolves over time is called the *dynamics* of the system

Dynamical models

- A dynamical model of a system is a set of mathematical laws that explain how the system evolves over time, usually under the effect of external excitations

- Example of linear continuous-time dynamical models

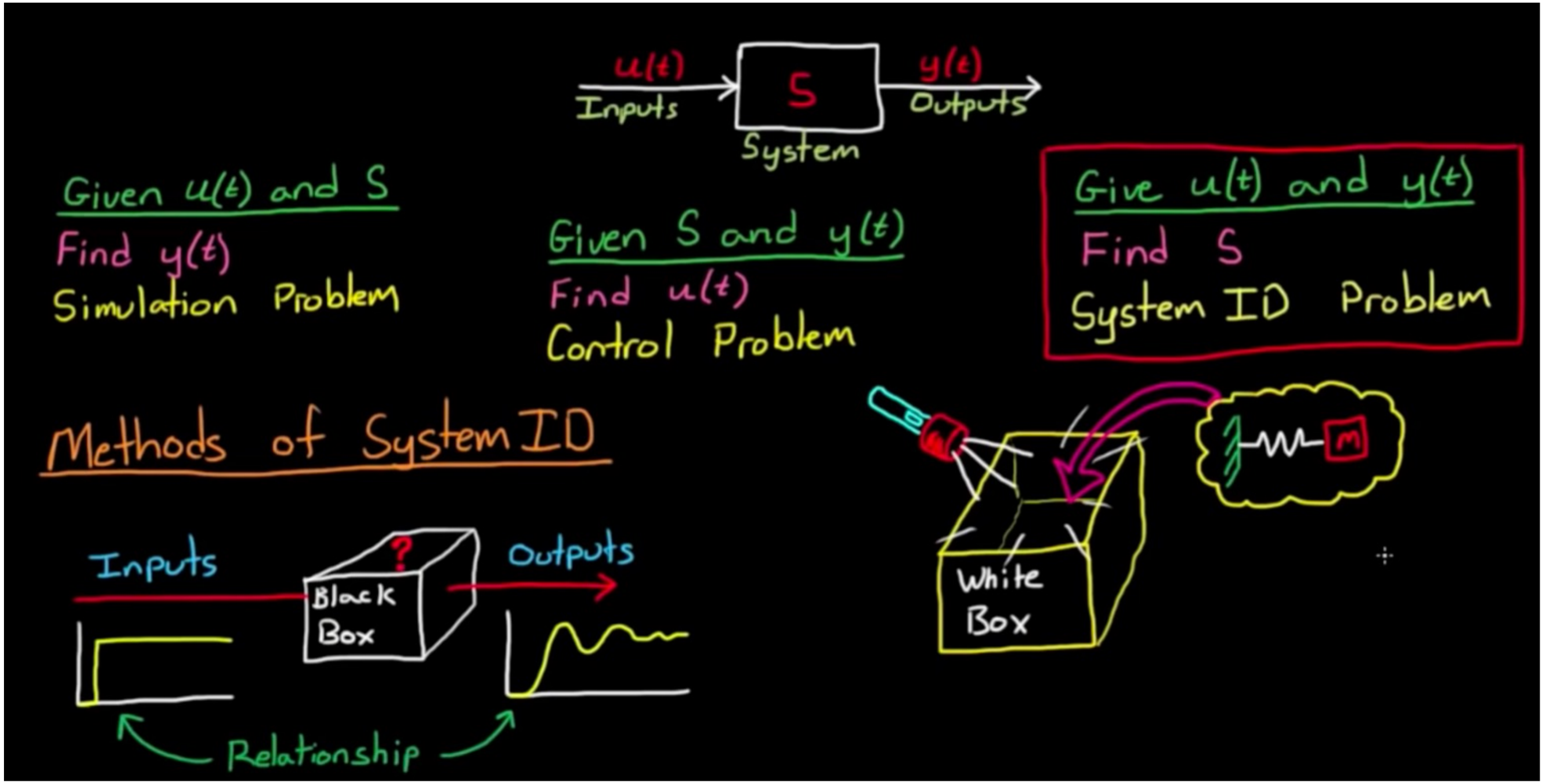
$$\frac{dy^{(n)}(t)}{dt^n} + a_{n-1}\frac{dy^{(n-1)}(t)}{dt^{n-1}} + \cdots + a_1\dot{y}(t) + a_0 y(t)$$
$$= b_{n-1}\frac{du^{(n-1)}(t)}{dt} + b_{n-2}\frac{du^{(n-2)}(t)}{dt} + \cdots + b_1\dot{u}(t) + b_0 u(t)$$

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

$$\begin{cases} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= x_3(t) \\ \quad\vdots & \quad\vdots \\ \dot{x}_n(t) &= -a_0x_1(t) + \ldots - a_{n-1}x_n(t) + u(t) \\ y(t) &= b_0x_1(t) + \ldots + b_{n-1}x_n(t) \end{cases} \Rightarrow A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} b_0 & b_1 & b_2 & \ldots & b_{n-1} \end{bmatrix},\ D = 0$$

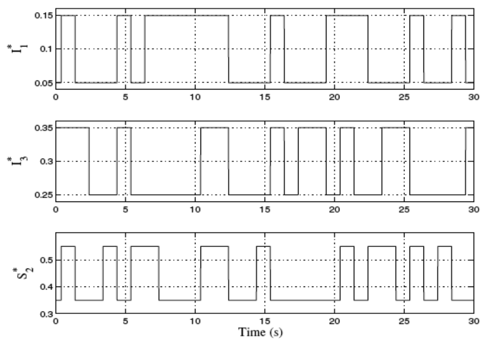
The 3 general problems
of dynamical systems and control

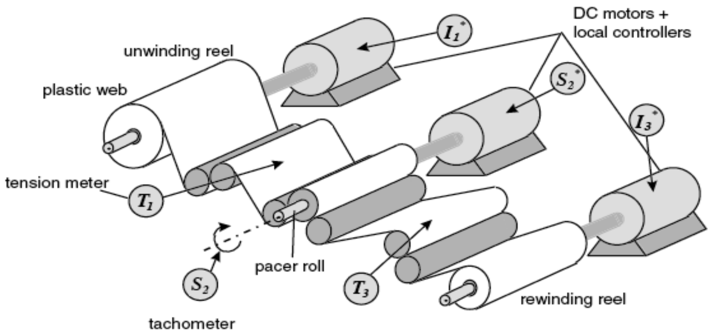


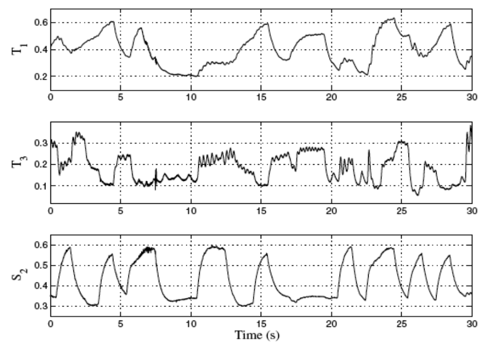
From Brian Douglas

Data-driven learning of dynamical models also known as *System identification*

- Aims at building mathematical models that describe the I/O behavior of a dynamic system based on **observed data**

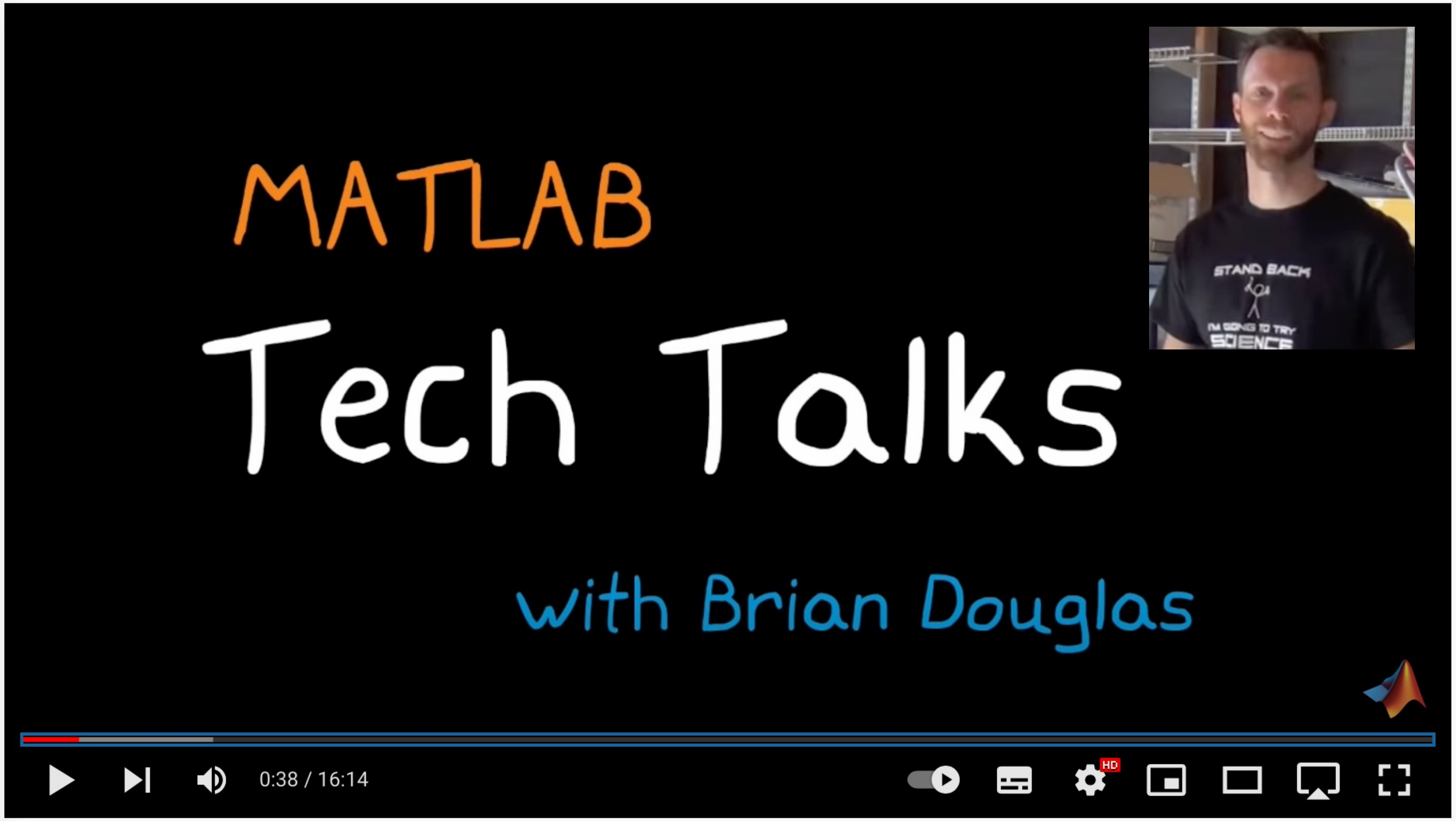






- Common use of dynamical models
  - better understanding
  - simulation
  - prediction
  - formal analysis
  - control design
  - predictive maintenance
  - control performance assessment
  - process monitoring and diagnosis

An overview of System Identification

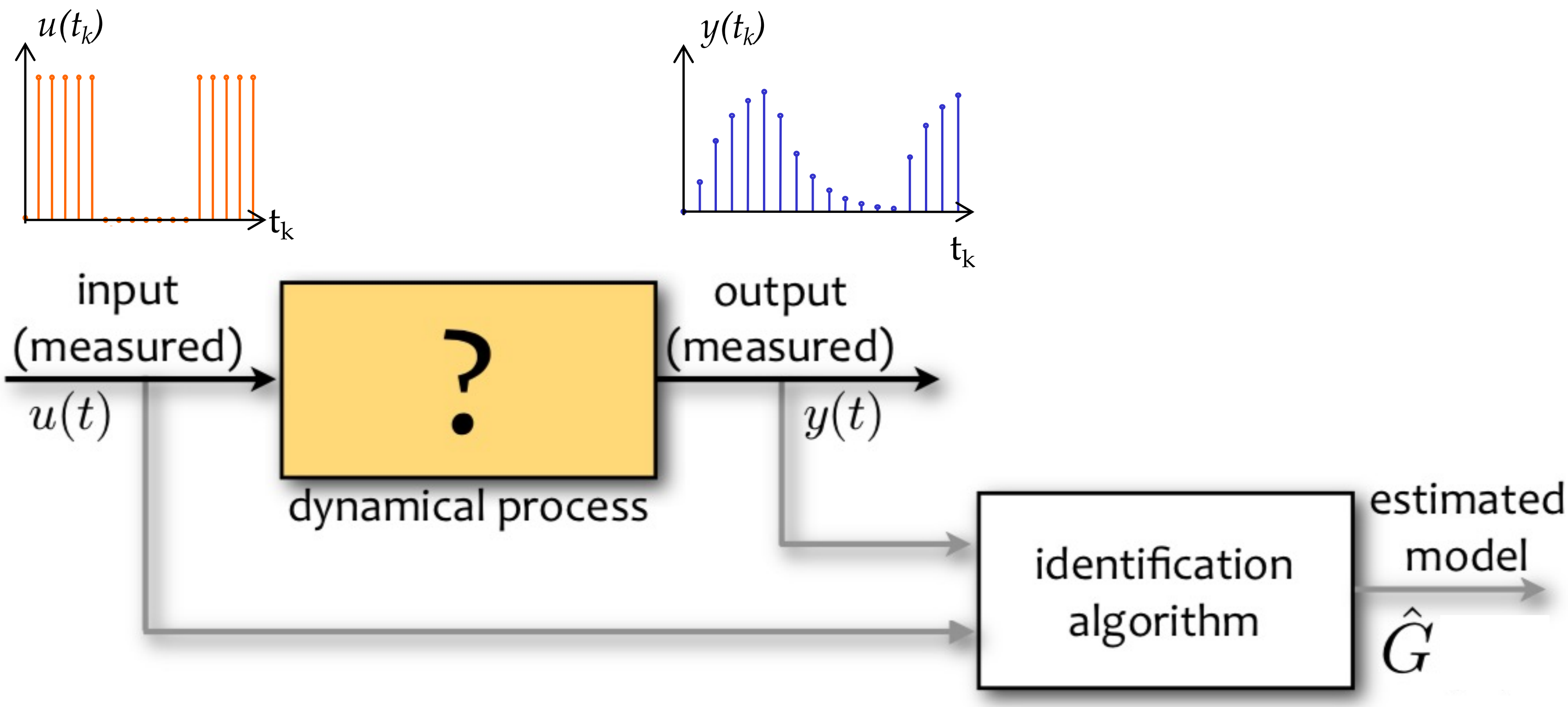


What Is System Identification? | System Identification, Part 1

www.youtube.com/watch?v=Z1QS6FsxrJI

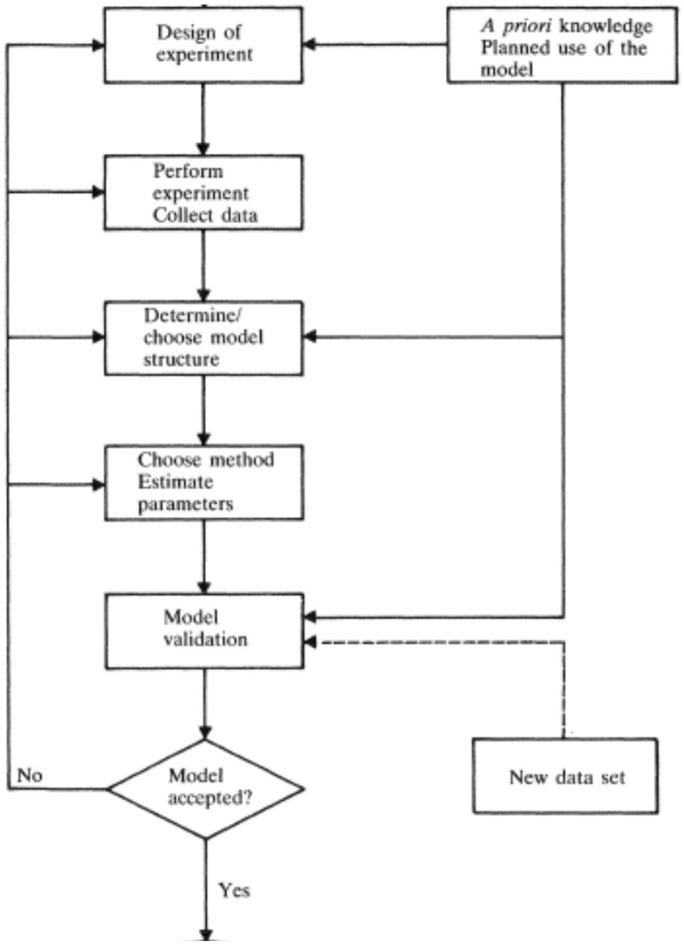
16 mn

Data-driven system identification



The data-driven system identification procedure

- The ***iterative*** procedure involves the following steps:
  1. Experiment design. Choose an input to enable a good model fit
  2. Collect time-domain input/output data from the system
  3. *Examine and prefilter the data. Remove trends and outliers, and select useful portions of the original data (often a crucial step)*
  4. Select and define a ***model structure*** (a set of candidate system descriptions) within which a model is to be estimated
  5. Estimate the parameters of the chosen model structure according to the input/output data and a given criterion of fit
  6. Validate the model by examining its properties



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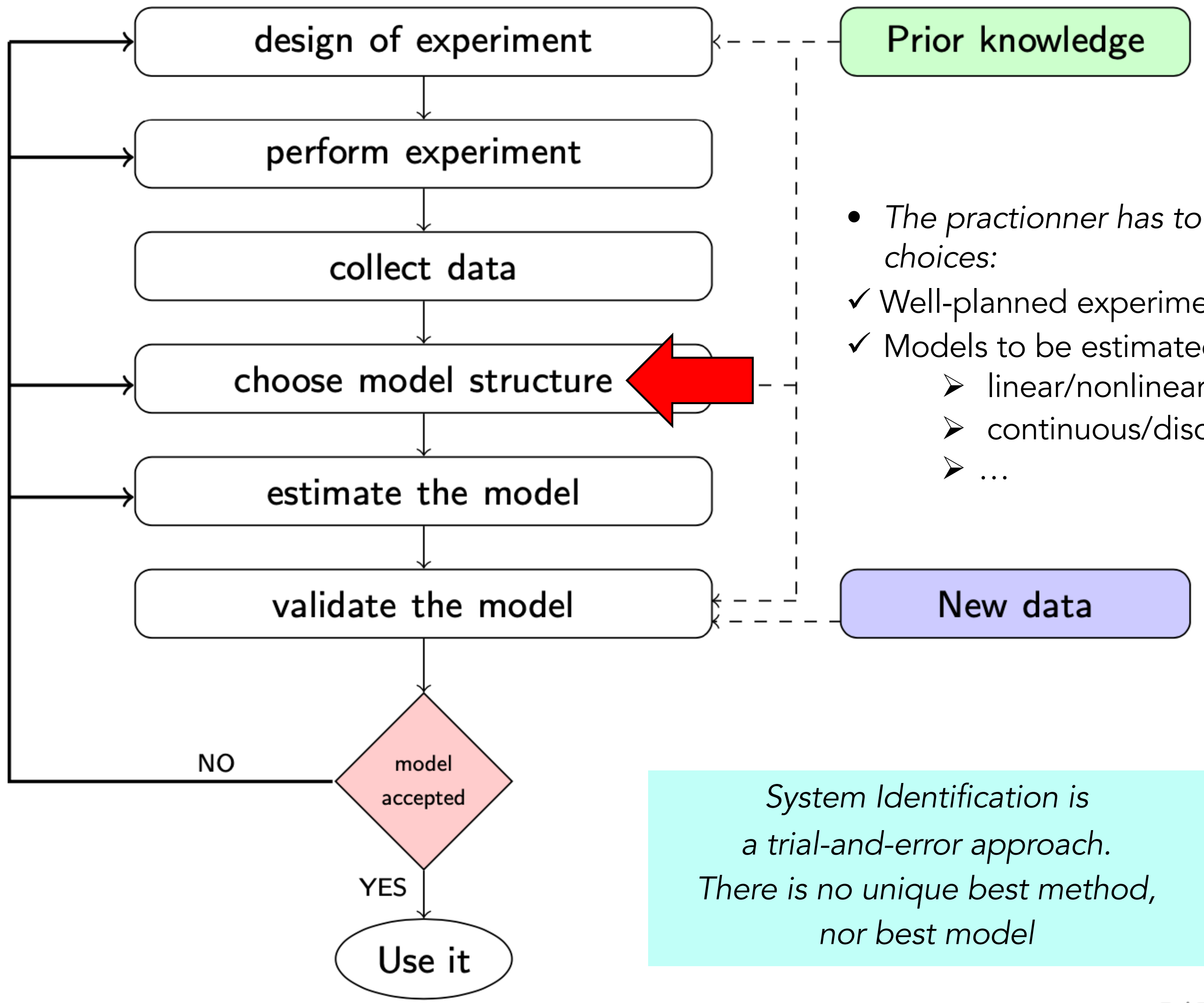
The iterative system identification workflow



- *The practionner has to make many choices:*

✓ Well-planned experiment

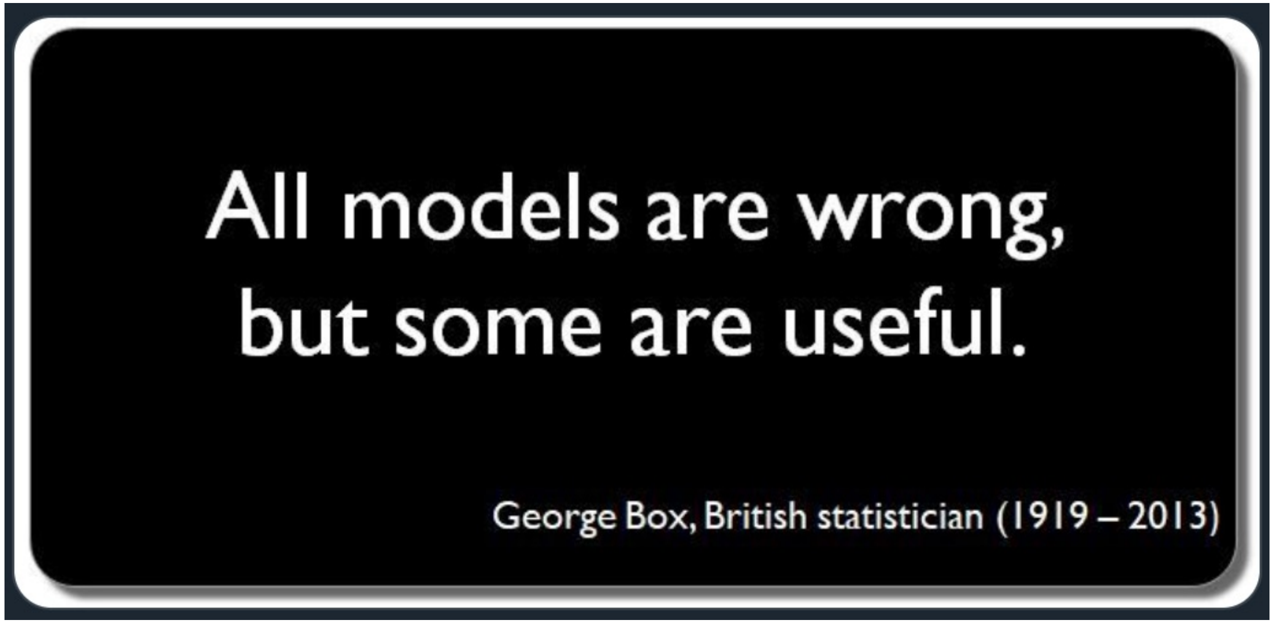
✓ Models to be estimated:

  - linear/nonlinear
  - continuous/discrete-time
  - …

System Identification is a trial-and-error approach. There is no unique best method, nor best model

Do not forget this quote from

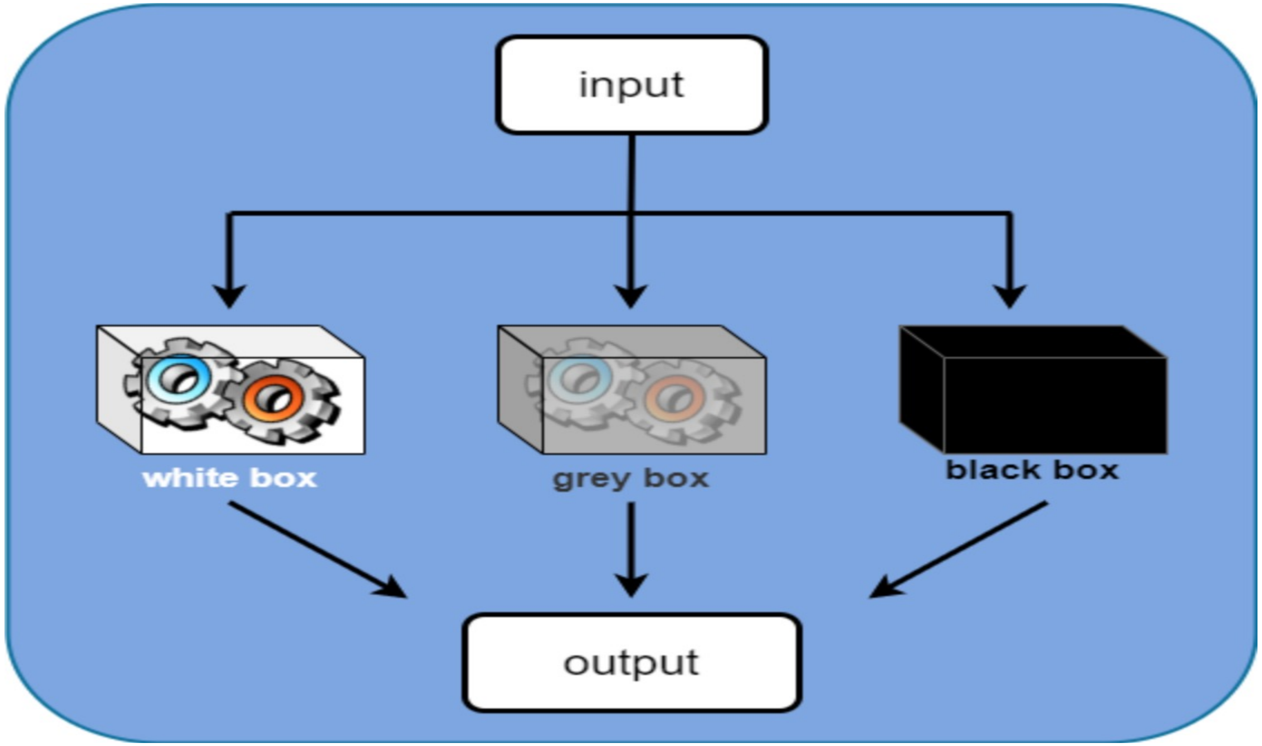
George E. P. Box



Choice of the model structure

- Must be guided by the application-specific goals
  - Do we want to use the model for
    - Simulation ?
    - Control design ?
    - Predictive maintenance ?
    - … ?
- Different types of models:
  - *White-box*: nonlinear model structure derived from first principles
    - Both model structure and parameters assumed perfectly known
  - *Gray-box*: nonlinear model structure derived from first principles
    - Model parameters estimated from measured data
  - *Black-box*: choice between linear or nonlinear model structure
    - Both model structure and parameters estimated from measured data

A palette of color for the models (structure + parameters)



White-box model
- Structure built from Physics
- Parameters a priori known

Gray-box model
- Structure built from Physics
- Some parameters are estimated from data

Black-box model
- Structure built mainly from data
- Parameters estimated from data

Gray-box non-linear models

When a linear model is inadequate, we might consider first principles and specify a nonlinear gray-box model

- *Main advantages*:
  - models in ***continuous-time*** derived from physical principles
  - parameters have a direct physical interpretation

- *Main shortcomings*:
  - some parameters might not be identifiable
  - cannot be built in the case of complex systems where the physical principles are not well established or too involved

Black-box non-linear models

When the physical principles are not well established, we might consider a black-box nonlinear modelling approach

- *Main advantages*:
  - offer potentially unlimited modeling flexibility
  - have the ability to fit any data-generating system

- *Main shortcomings*:
  - offers little or no insight into how the system is actually working
  - parameters have no direct physical interpretation
  - risk of overfitting—a failure to capture the true governing mechanisms of the system

Black-box linear models

- Linear models
  - offer a strong baseline
  - are easy to fit
  - provide excellent interpretability

- The black-box aspect offers modeling flexibility

- While few systems are truly linear, many systems are described well locally by a linear model
  - A system actively regulated to stay around an operating point is, for instance, often well described by a linear model
  - Linear models further facilitate easy control design thanks to the very well-developed theory for linear control system analysis and design

Model identification of linear time-invariant systems

- For ***control design and analysis***, Linear Time-Invariant (LTI) models have been hugely important, mainly motivated by
  - their simplicity
  - their performance and robustness properties are well understood

- Classical SYSID methods of linear models
  - share these properties in many regards
  - have a relatively low computational complexity
  - have strong systems-theoretical background, with well developed concepts such as stability, identifiability, input design, informative data selection

- We will focus on data-driven *black-box model* methods for identifying *linear time-invariant dynamical systems*

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There is a wealth of books on System Identification !!!

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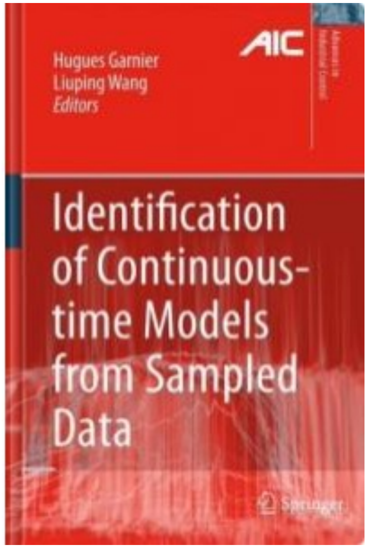
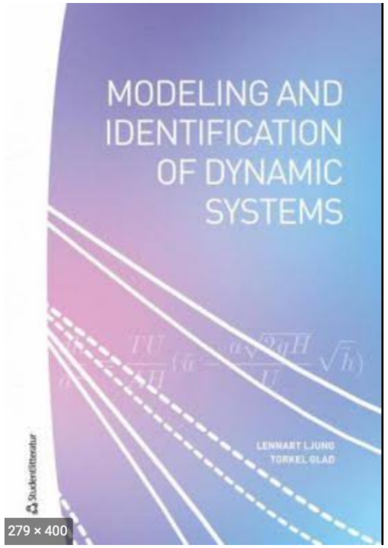
Course website & recommended textbooks

- Website of the course
  - w3.cran.univ-lorraine.fr/hugues.garnier/?q=content/teaching

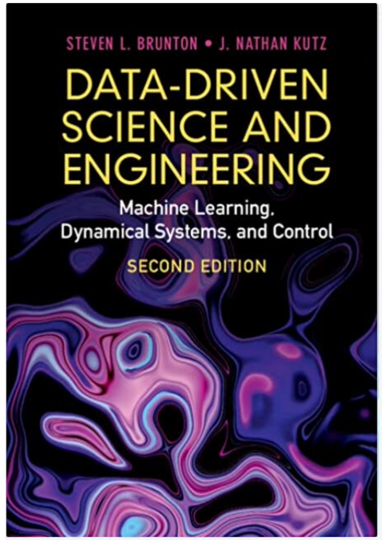
- Recommended textbooks



H. Garnier & L. Wang
2008



L. Ljung & T. Glad
2016
New version in 2021



S. Brunton & N. Kutz
2022

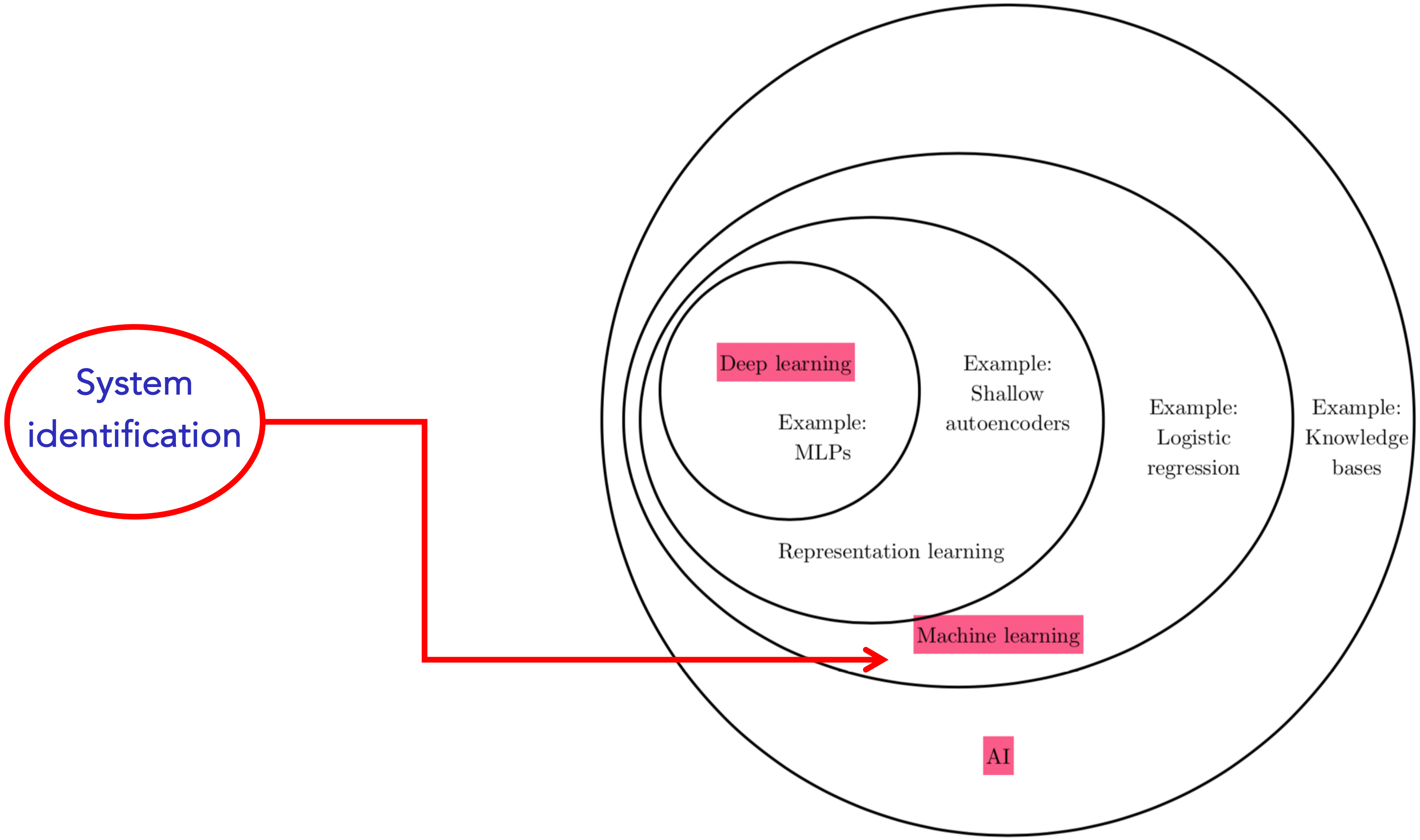
System identification is part of Data science and is connected to Machine learning

- From Wikipedia
  - Data science *is an interdisciplinary field that uses scientific methods, processes, algorithms and systems to extract knowledge and insights from data in various forms*
  - Machine learning *is a field of study in artificial intelligence concerned with the development and study of statistical algorithms that can learn from data and generalize to unseen data*
  - System identification *uses statistical methods to build mathematical models of dynamical systems from measured data*

- System identification
  - Well-established research area within Automatic Control
  - The term *System identification* first introduced by Lofti Zadeh in 1957 !
  - The first IFAC Symposium on System Identification was organized in Prague in 1967. This is now the longest running IFAC symposium series; the next edition will be held in Lyon in July 2027

System identification as Machine Learning

- System Identification may be considered as *an early form of machine learning*, where a dynamical system is characterized through training data



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System identification is connected to Machine learning (ML)

Vocabulary used SYSID versus ML

- Estimate = train
- Identify = learn
- Validate = generalize
- Model structure = network topology (architecture)
- Estimation data = training set
- Validation data = generalization data
- Overfitting = overtraining
- Output = target

The cost of the modelling phase in a control-design project

- The modelling phase of an unknown system can be quite time-consuming and is often a significant part of a control-design project
  - *Normally modelling costs account for over 75% of the expenditures !*

- This is true in particular for Physics-based modelling

- Convenient alternative: data-driven modeling via system identification

Software tools for data-driven system identification

- System identification is typically an iterative procedure, where the insights and judgements of the user can be mingled with:
  - extensive data handling
  - sophisticated parameter optimization algorithms
  - practical considerations and user experience

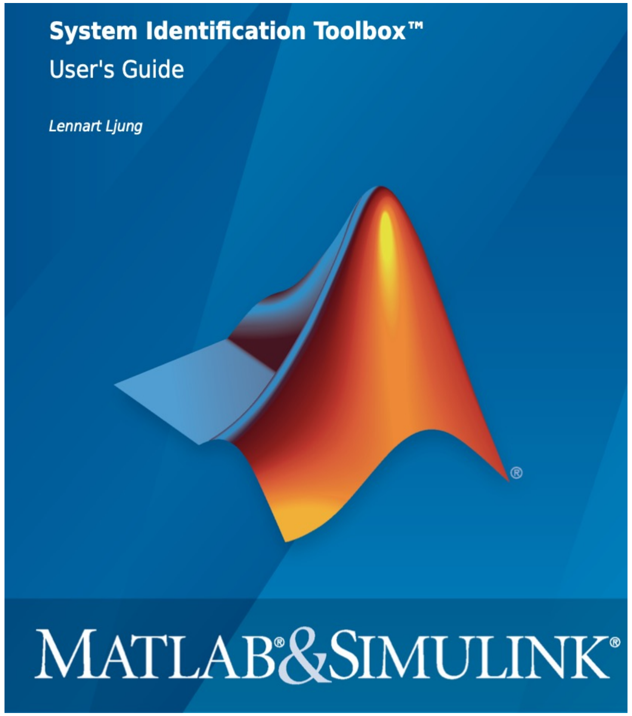
- To make the application of the system identification procedure successful:
  - it is ***necessary to exploit some user-friendly software tools***

Watch the recent video Modeling-dynamic-systems-with-Matlab-and-Simulink

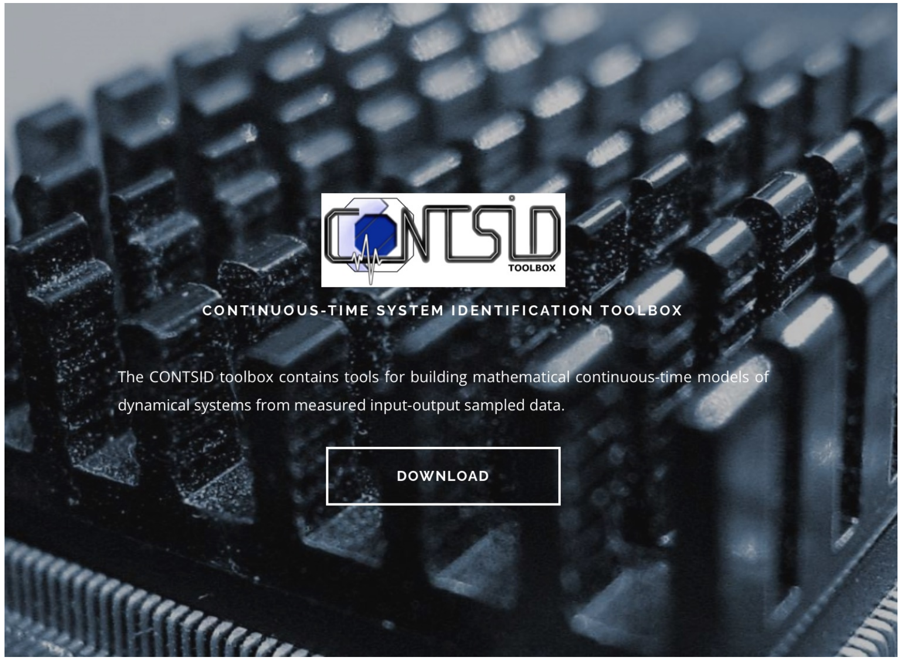
https://fr.mathworks.com/videos/modeling-dynamic-systems-with-matlab-and-simulink-1730977830582.html

Software requirements for the course

We will make use of the Matlab SID Toolbox and the CONTSID Toolbox



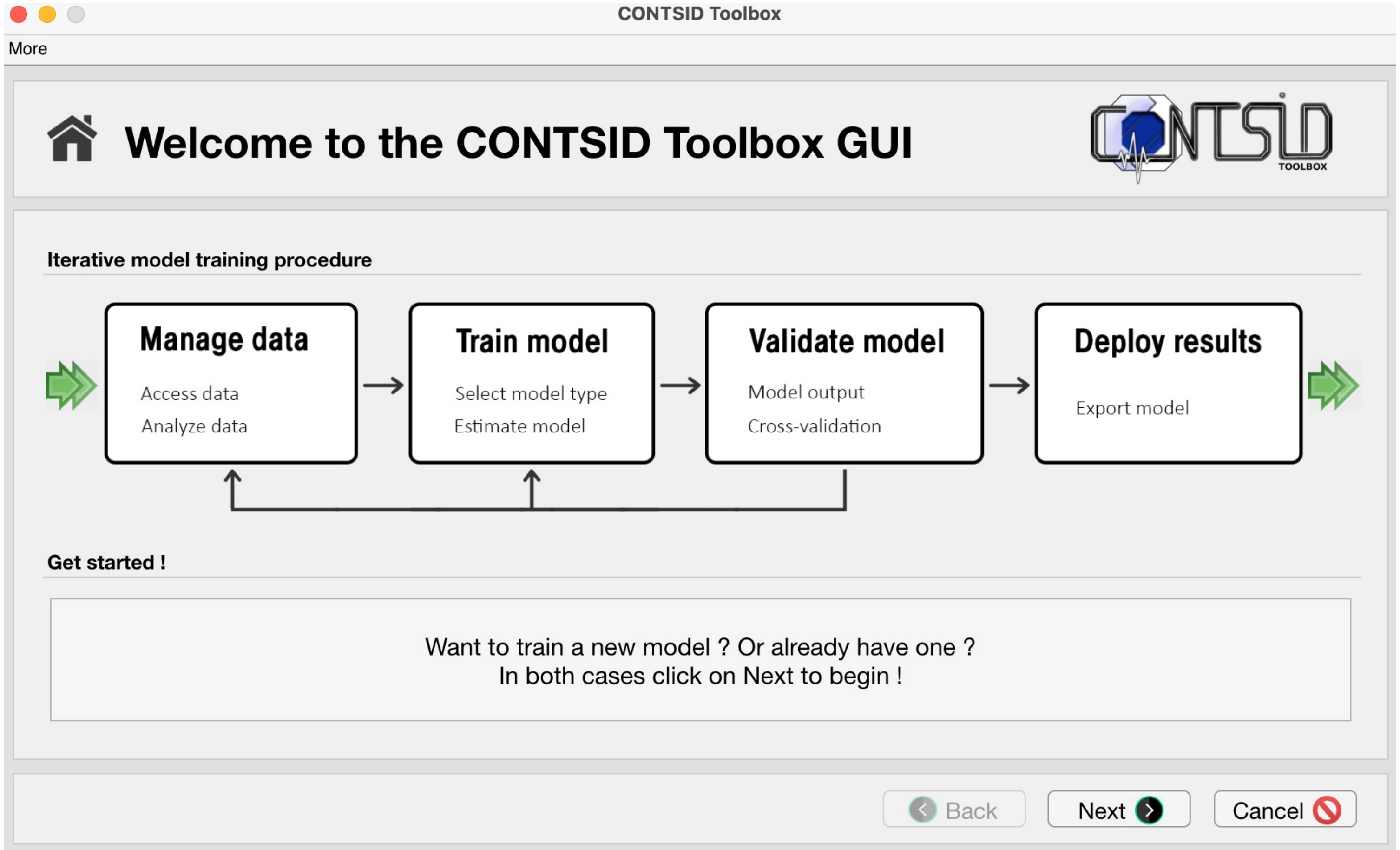
The principal architect of the toolbox is Prof. Lennart Ljung, a recognized leader in the field of system identification



Developed by the CRAN team (Hugues Garnier and his co-workers) at Polytech Nancy

Software requirements for the course

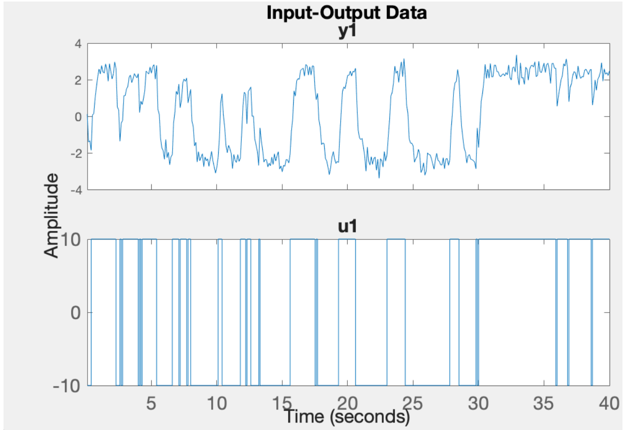
We will also make use of the recent *CONTSID* App developed by Hugo Muller, former student of the IA2R diploma



A typical session with Matlab

```
% Load the data
load dcmotor;
```

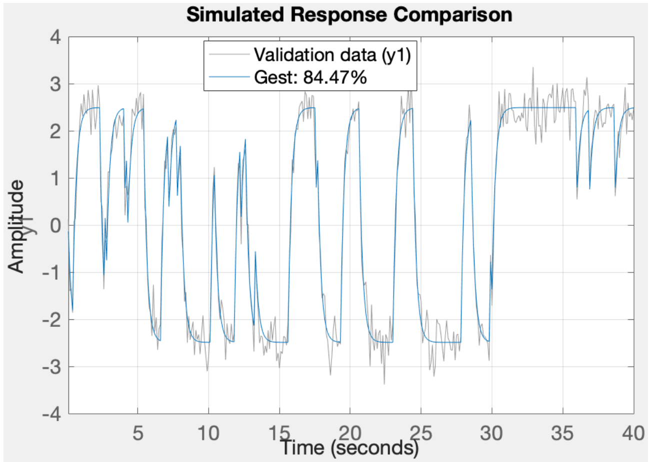
```
% Plot and examine the data
data=iddata(y,u,Ts);
idplot(data);
```



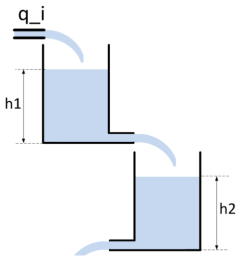

```
% Choose a model structure and estimate the parameters
% Let us test a 1st-order transfer function with no time-delay
Gest = tfsrivc(data,1,0,'TdMax',0);
present(Gest)
Gest =
  From input "u1" to output "y1":
   1.037 (+/- 0.02287)
  ---------------------
  s + 4.161 (+/- 0.0994)
```

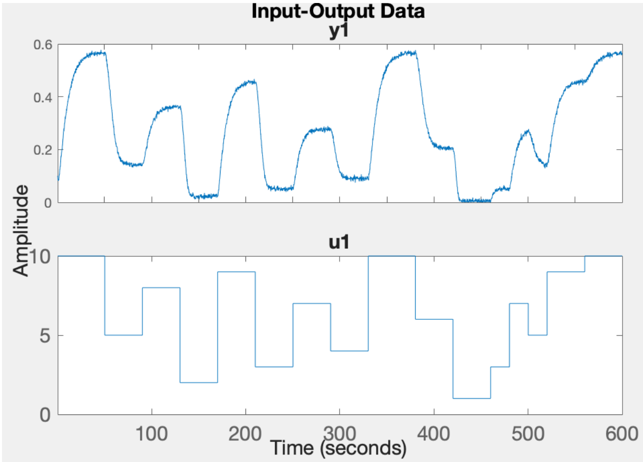
Is-it always that simple ?

```
% Validate the model
compare(data, Gest);
```



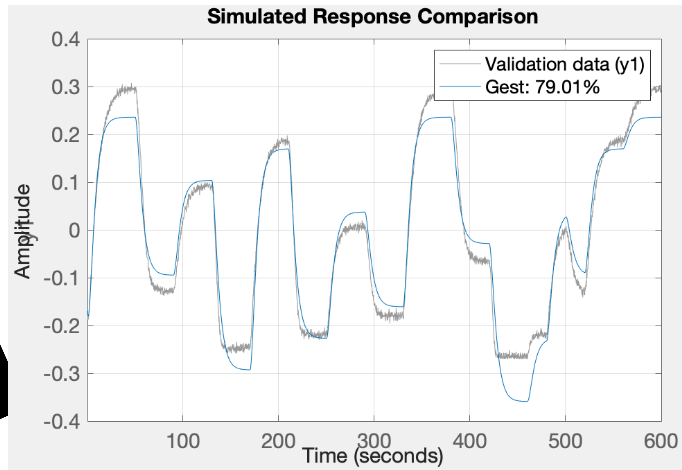
Another typical session with Matlab






```
% Load the data
load twotankdata;

% Plot and examine the data
data0=iddata(y,u,0.2);idplot(data0);

% Suppress the mean of both input and output
Data=detrend(data0);

% Choose a model structure and estimate the parameters
% Let us test a 2nd-order transfer function with no time-delay
Gest = tfsrivc(data,2,0,'TdMax',0);
present(Gest)
Gest =
  From input "u1" to output "y1":
              0.001657 (+/- 5.6e-05)
  ----------------------------------------------
  s^2 + 0.21 (+/- 0.0083) s + 0.036 (+/- 0.0012)

% Validate the model
compare(data, Gest);
```

Not too bad !
But nonlinear effects
not well captured
by a linear model



Key takeaways from the course

- Introduction to classical linear model identification methods along with implementation on real-life data examples. Objectives are to:
  - Know traditional methods and be able to choose adequate methods for different types of systems
  - Understand how different models and methods can be used for control design
  - Know how to evaluate and compare the performance of parameter estimation methods

System identification

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Data-driven model learning of dynamical systems

is a discipline

that requires practical skills, practice and experience

Identification of dynamical systems
Course outline

*Before exploring recent machine learning and deep learning methods, it is a good idea to ensure you have tried classical data-driven **linear model identification** methods*

These methods are performing extremely well on a wide range of problems

- *in particular when the number of data is relatively limited (in contrast to deep learning methods)*

Course outline

I. Review of linear regression for static and dynamical systems

II. Parameter estimation of continuous-time transfer function models

III. The practical side of system identification

IV. Mini-project