Sampling - Reconstruction

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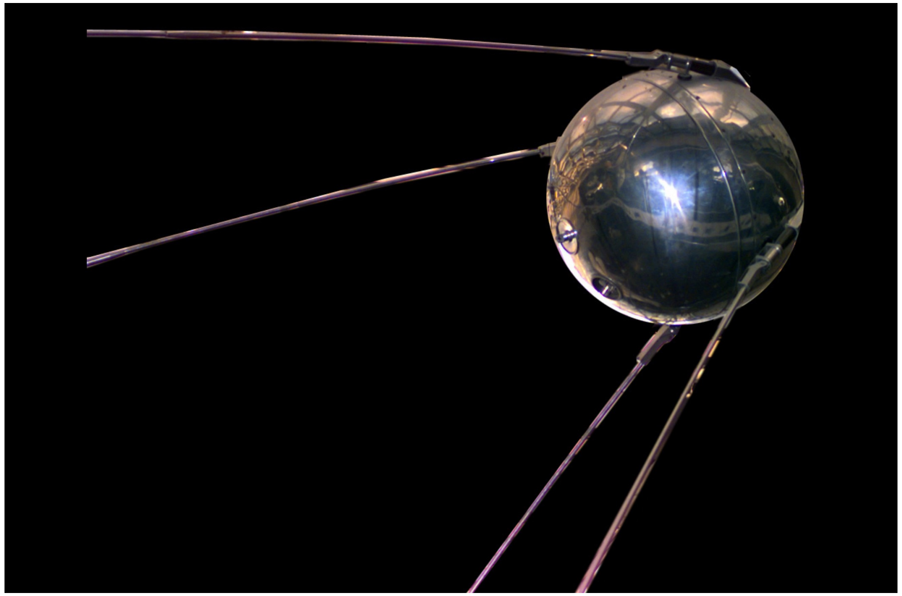
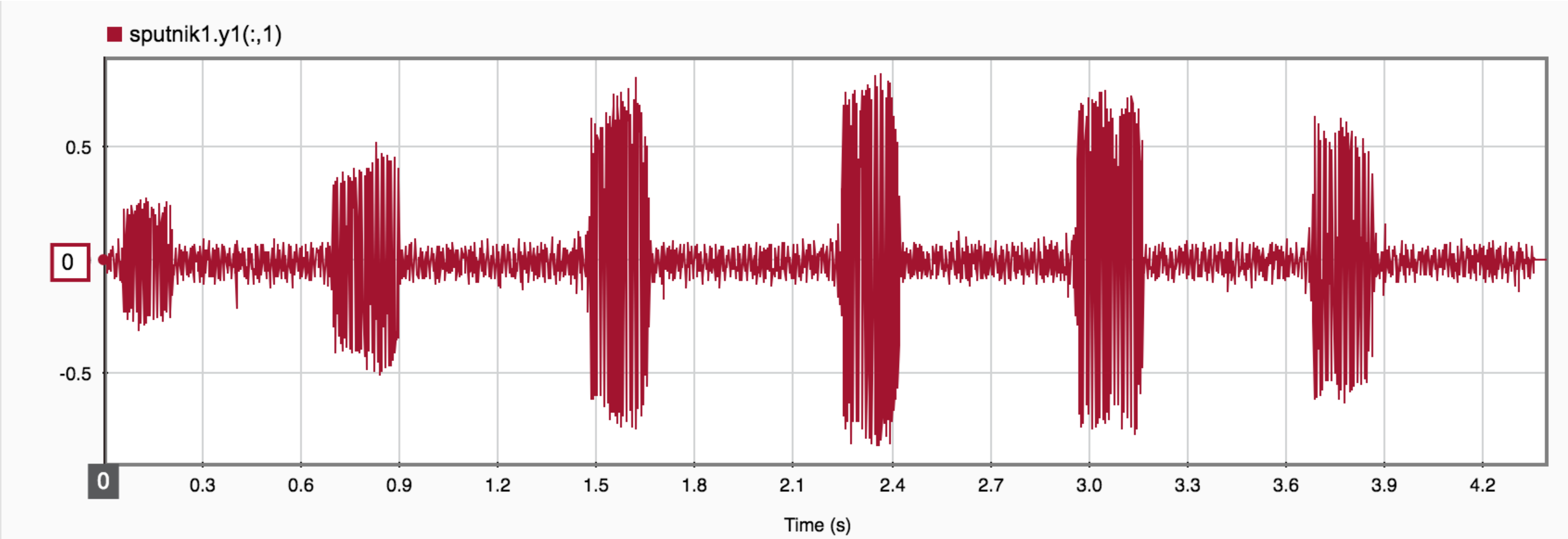
The Image of the day



- On october 4, 1957, Spoutnik-1, first artificial satellite *(spherical shape of 83 cm diameter equipped with 4 antennas) is sent in Earth orbit by the Russians*
- In 1958, the NASA is created !

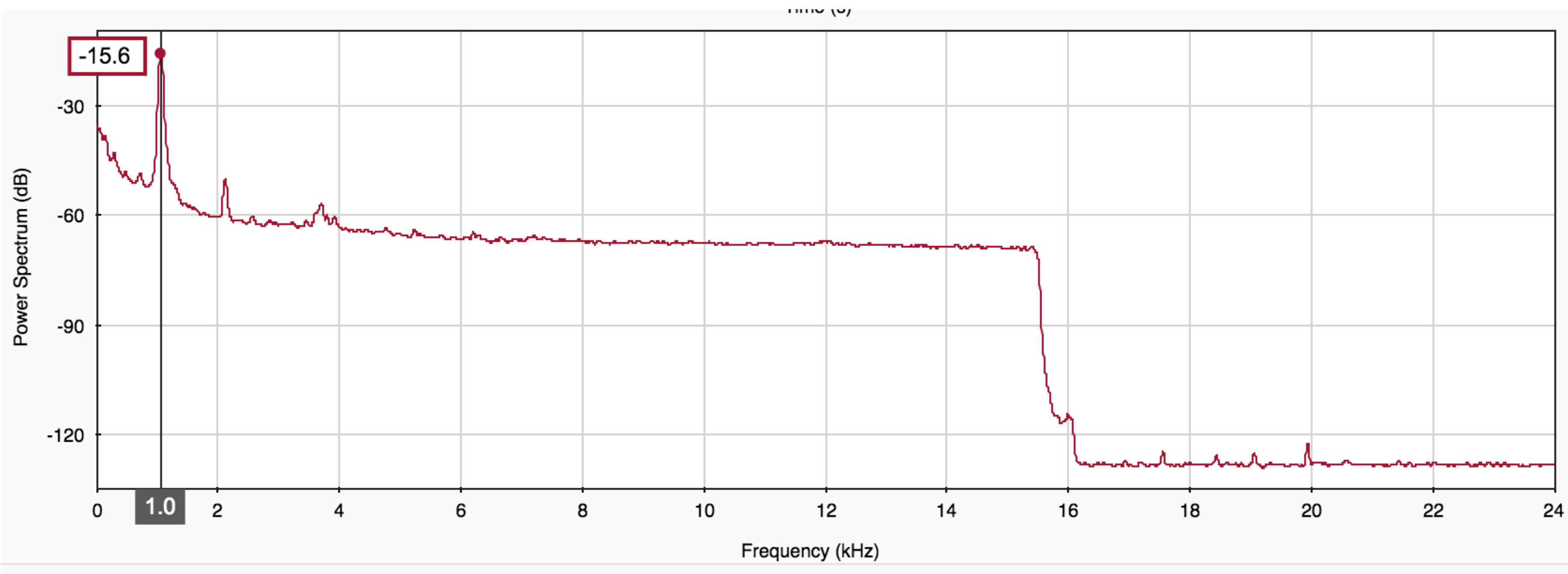
Signal of the day
The Spoutnik« Beep-beep » sound

- Although invisible from Earth, Sputnik-1 can be heard!
- Its "beep-beep", broadcast by two transmitters in the 20 and 40 MHz bands, was picked up by amateur radio operators around the world

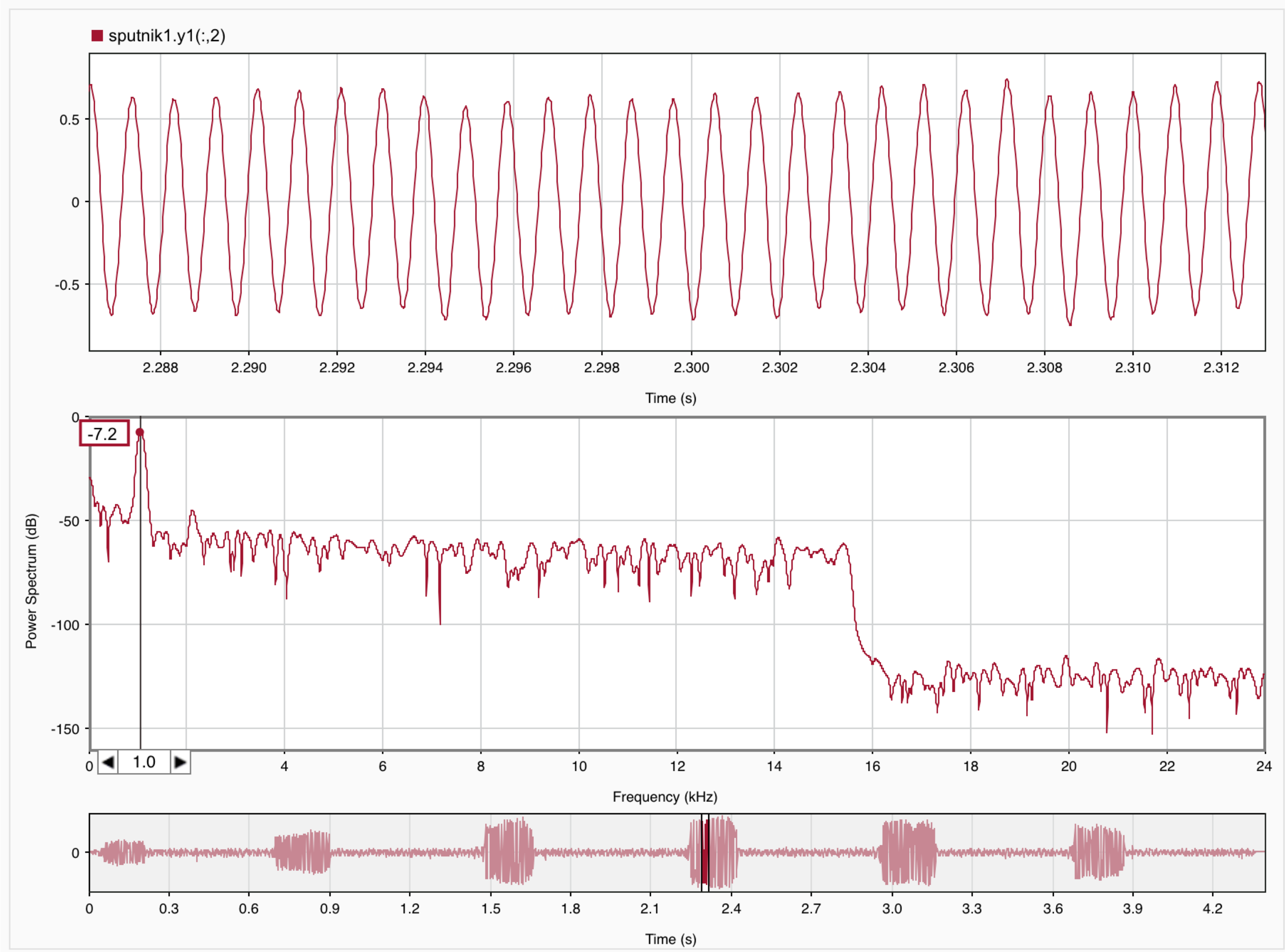


- ***Source***: https://soundcloud.com/nasa/sputnik-beep

Spectrum of Sputnik signal



Spectrum of the « beep » broadcast by Sputnik



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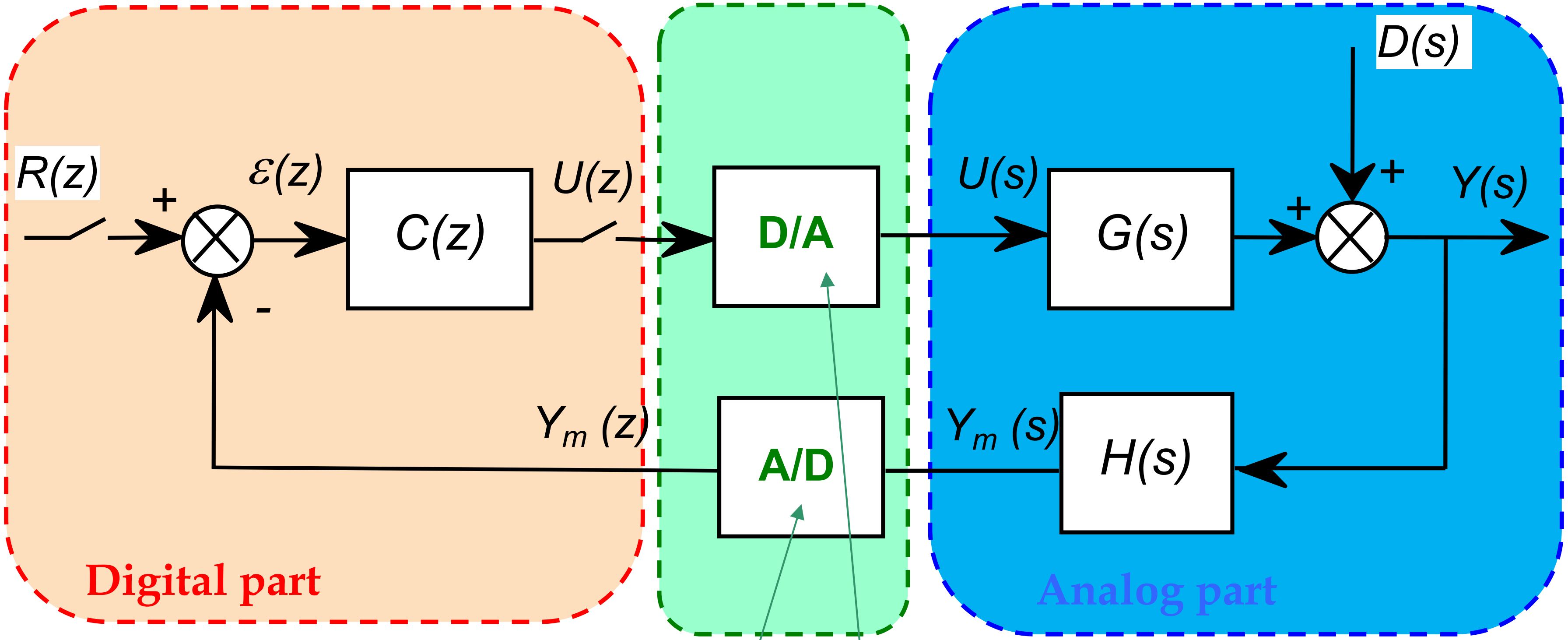
Digital control block-diagram



- Need for blocks to make analog and digital parts interact:

A/D & D/A

Sampling - Reconstruction

- A/D: Analog to Digital converter *to read the analog signals into the micro-controller*

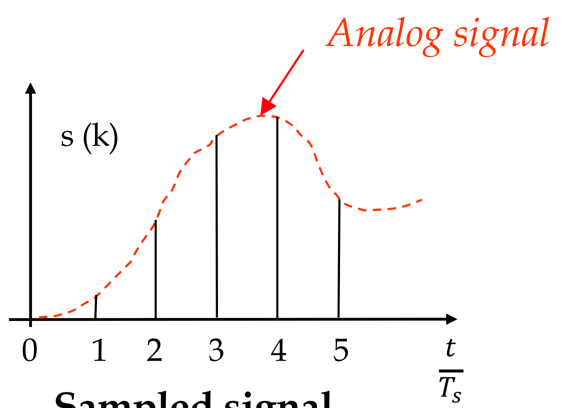
The process is called **SAMPLING**

- D/A: Digital to Analog converter *to take the desired control signals out of the micro-controlled and present them in a form whereby they can be applied to the physical process*

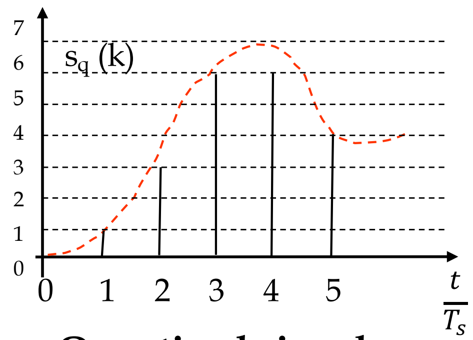
The process of signal **RECONSTRUCTION**

Converting an analog signal into a digital signal: A/D

- A/D conversion is characterized by two **discretizations**
  - The first concerns **TIME**, and is known as ***sampling***: this involves taking samples of the analog signal at regularly spaced instants
  - The second concerns **AMPLITUDE** and is called ***quantization***: it consists in coding the signal amplitude on a finite number of binary elements



Sampled signal



Quantized signal

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Choices to be made when digitizing an analog signal

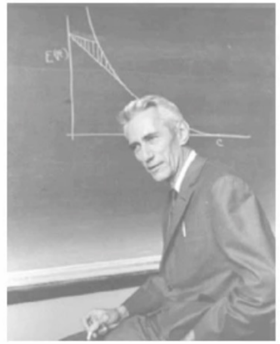
- Discretization accuracy via the choice of the sampling frequency (f_s)
  - f_s must be sufficiently high so as not to loose information about the signal
  - However, the higher f_s, the shorter the time available for numerical calculations and the greater the number of samples to be processed

How can we choose the sampling frequency f_s ?

$$f_s = \frac{1}{T_s}$$ where T_s is the sampling period

Sampling theorem (*Shannon 1949*)

Harry Nyquist
1889-1976



Claude Shannon
1916-2001

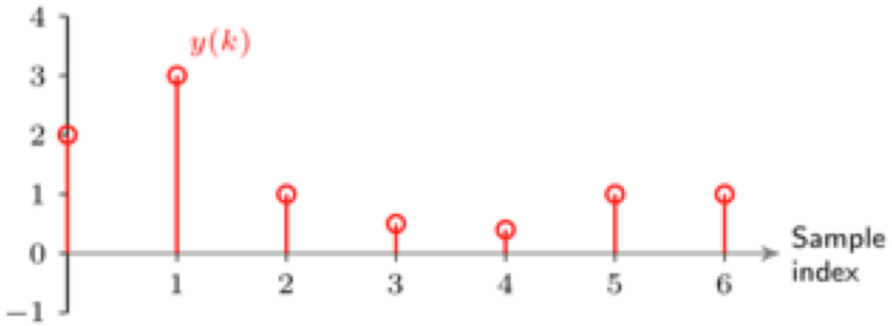
A band-limited signal x(t) in the frequency range [-f_{max} ; +f_{max}] can be reconstructed exactly from its samples

if $f_s > 2 f_{max}$

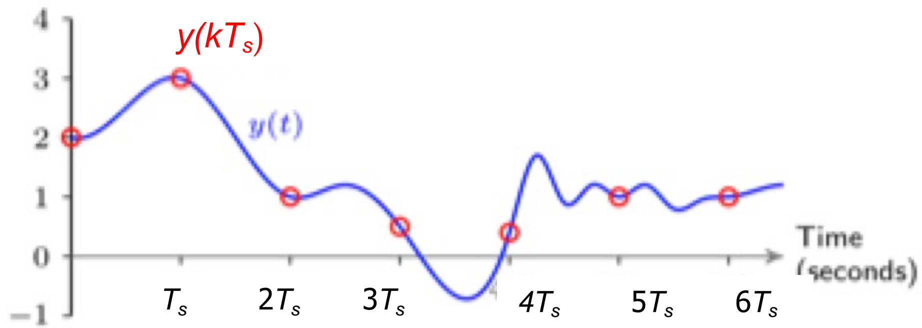
The limiting frequency $\frac{f_s}{2}$ is called the ***Nyquist frequency***

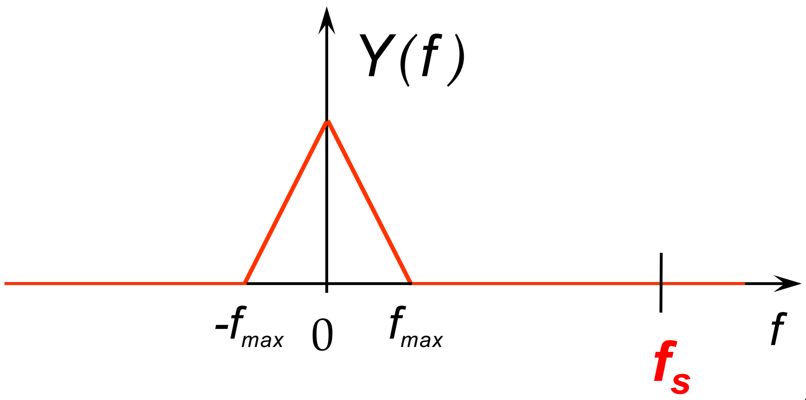
Shannon's theorem - Interpretation

- Samples *y(k)* are available. How can we deduce *y(t)?*



- If $f_s > 2\, f_{max}$ then *y(t)* can be perfectly reconstructed from $y(kT_s)$





Spectrum of *y(t)*

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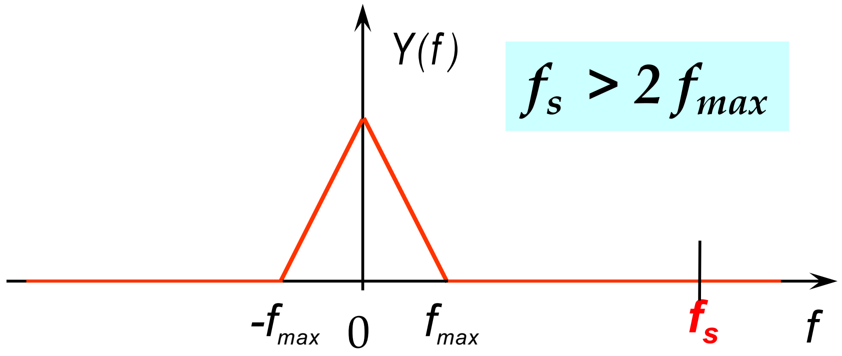
Shannon's theorem in practice

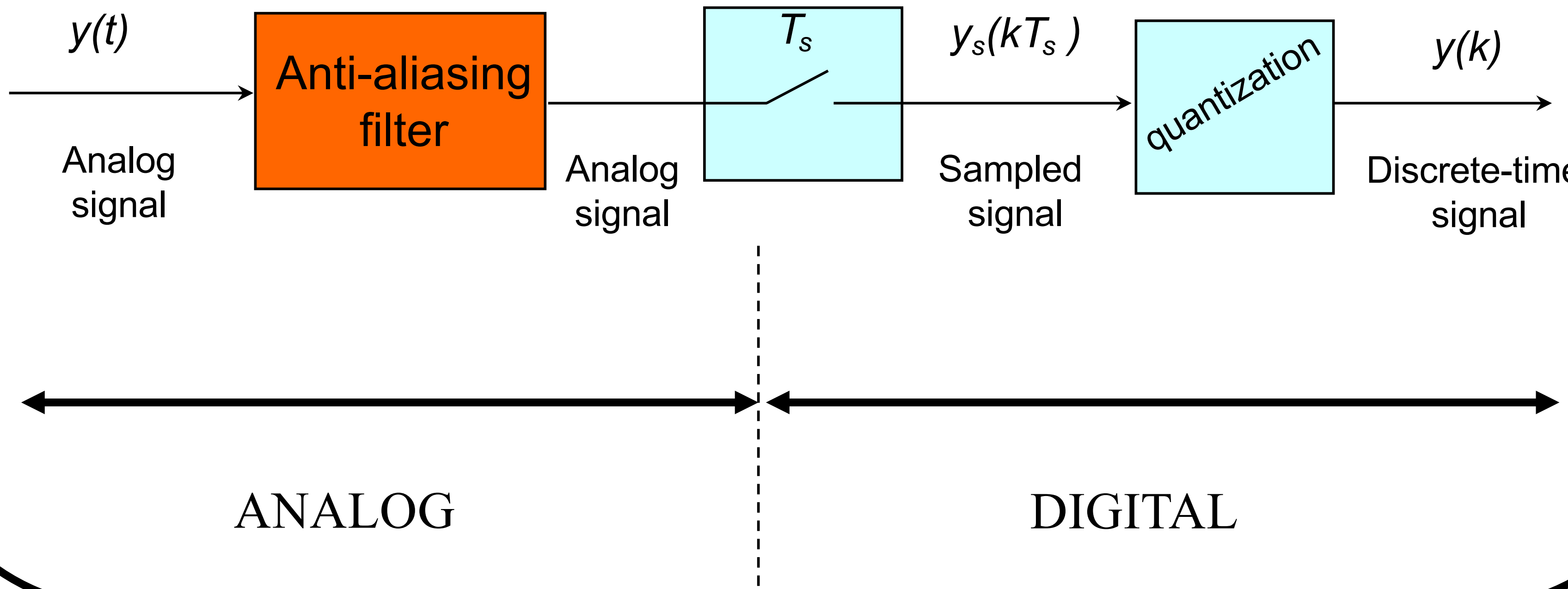


- Shannon's theorem only gives a **lower bound** on the sampling frequency that must not be exceeded
  - In practice, we need to choose a **much higher** sampling frequency
- The frequency f_{max} is rarely known precisely
  - It is necessary to filter the analog signal with a low-pass analog filter. Such a filter is called an ***anti-aliasing filter***
- For digital control, the choice of the sampling period is a much more complex problem
  - It depends on the characteristics of the desired closed-loop response and therefore it depends on the desired performance

Practical chain for analog-to-digital conversion (A/D)

- In practice:
  - It is essential to precede the sampling operation with a **low-pass filter** called an ***anti-aliasing filter***, with a cut-off frequency f_c slightly lower than the Nyquist frequency $f_c \approx f_s \,/\, 2$

- The practical chain for converting an analog signal into a digital signal therefore consists of the following elements



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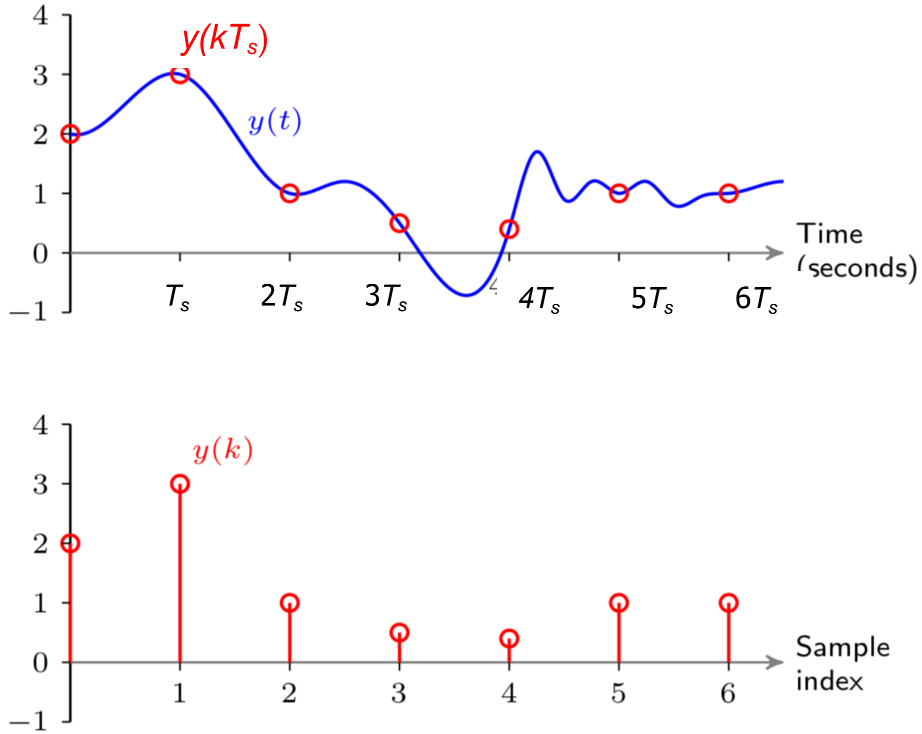
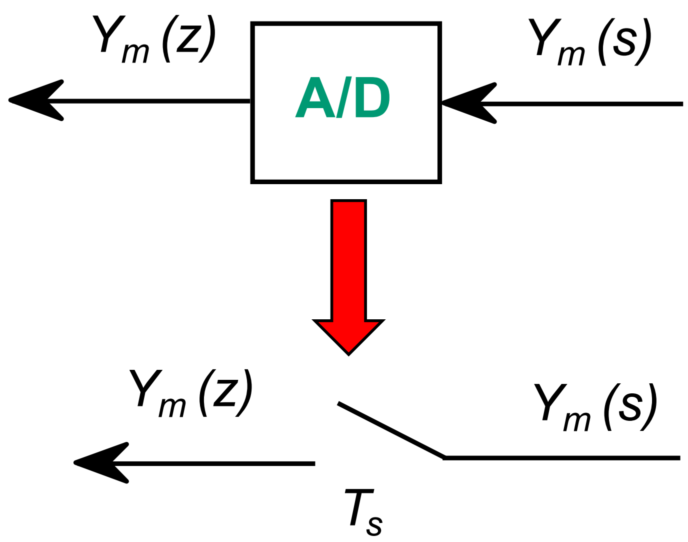
Analog-to-digital operation (A/D)
Simplified representation

- The usual representation of the A/D operation consists in representing **only the sampler block** *(not the anti-aliasing filter and quantization block)*

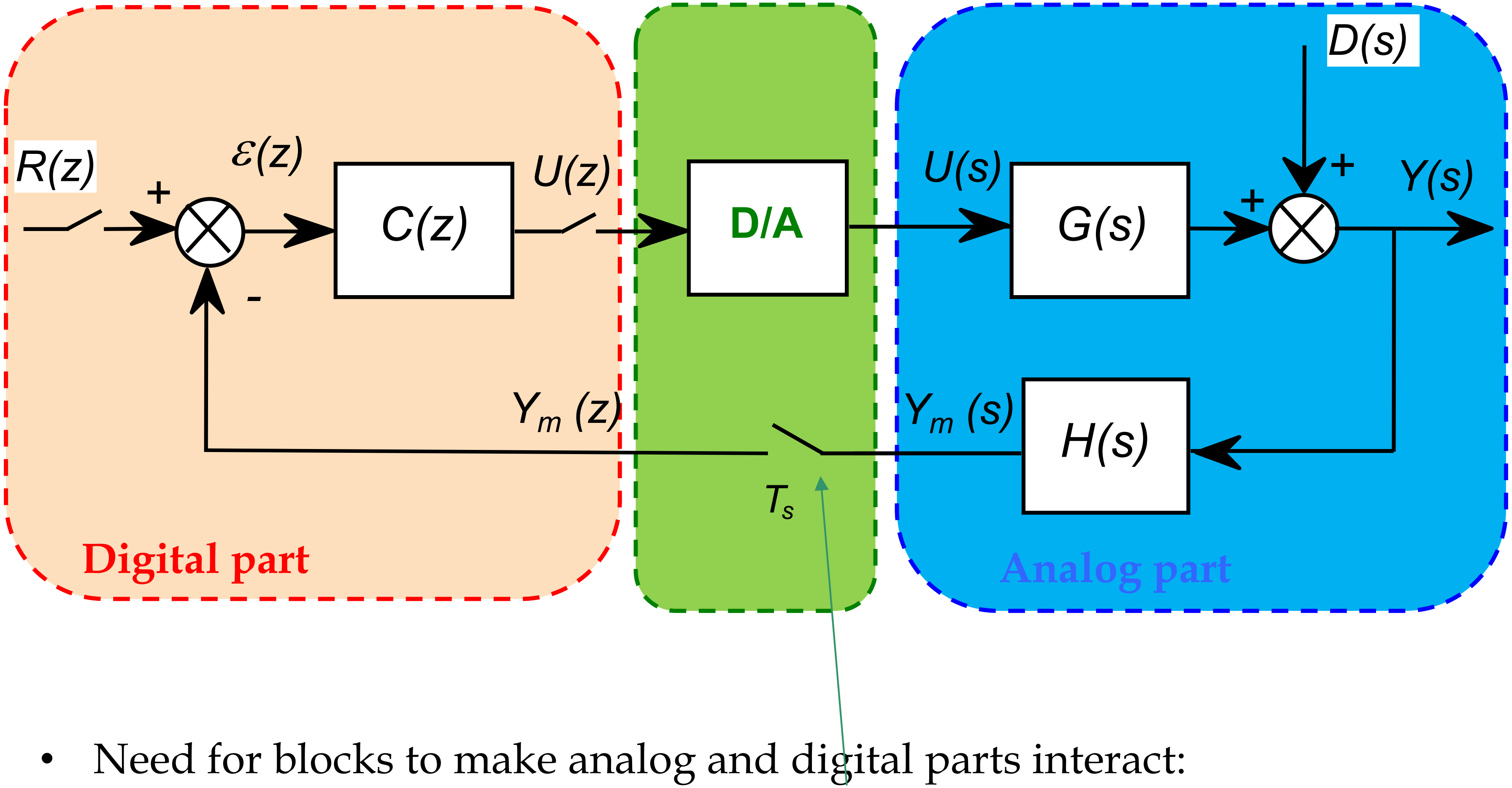




Some standard values for the sampling period

	T_s **in seconds**
• Servo and regulation	
– Position	0,001 à 0,1
– Speed	0,001 à 0,1
– Flow	1 à 3
– Level	5 à 10
– Pressure	1 à 5
– Temperature	10 à 45
• Systèmes industriels	
– Distillation columns	10 à 180
– Ciment mills	20 à 45
– Dryers	20 à 45

Digital control block-diagram

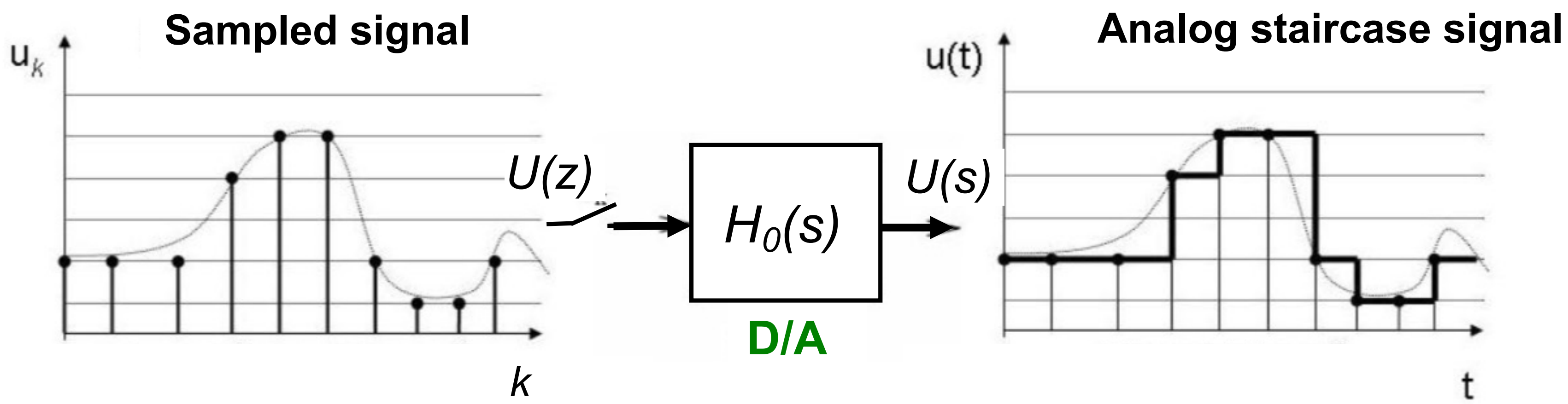


- Need for blocks to make analog and digital parts interact:

D/A & A/D

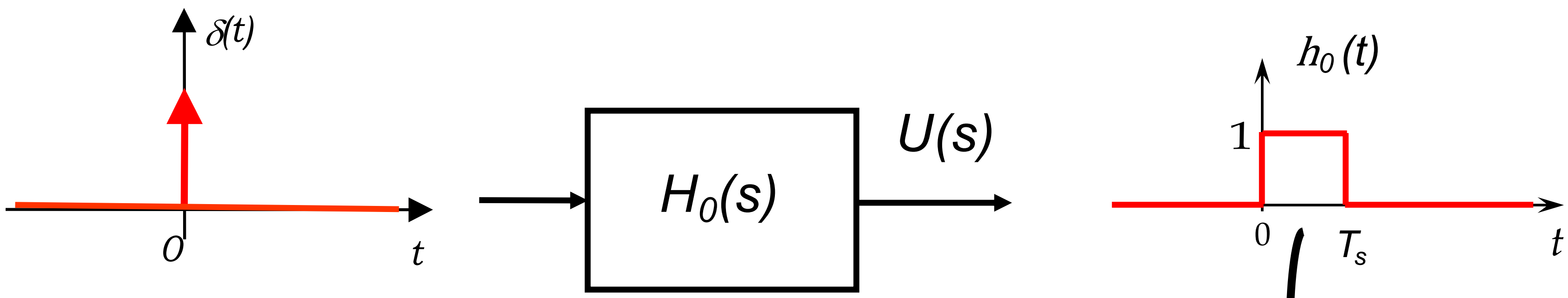
Digital-to-analog conversion (D/A)
Practical reconstruction

- The most common D/A operation consists in producing a continuous-time control signal *u(t)* from the sampled values *u(k)* by keeping *u(k)* constant throughout the sampling period via a **zero-order hold (*ZOH*)**



Laplace transfer function of a zero-order hold

- **<u>Reminder</u>: transfer function = $\mathcal{L}$(impulse response)**

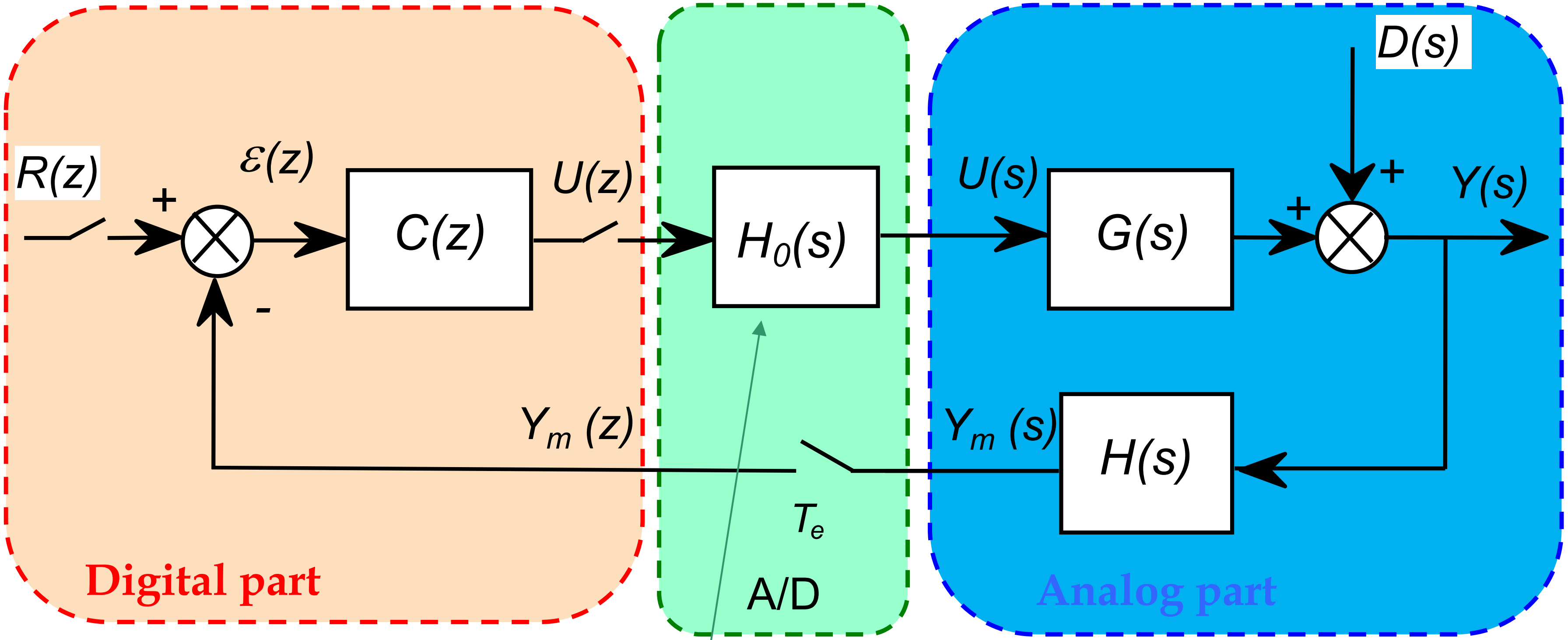


$$h_0(t) = \Gamma(t) - \Gamma(t - T_s)$$

$$\mathcal{L}(h_0(t)) = H_0(s) = \frac{1}{s} - \frac{e^{-T_s s}}{s}$$

$$H_0(s) = \frac{1 - e^{-T_s s}}{s}$$

Digital control block-diagram



- Need for blocks to make analog and digital parts interact:

D/A & A/D

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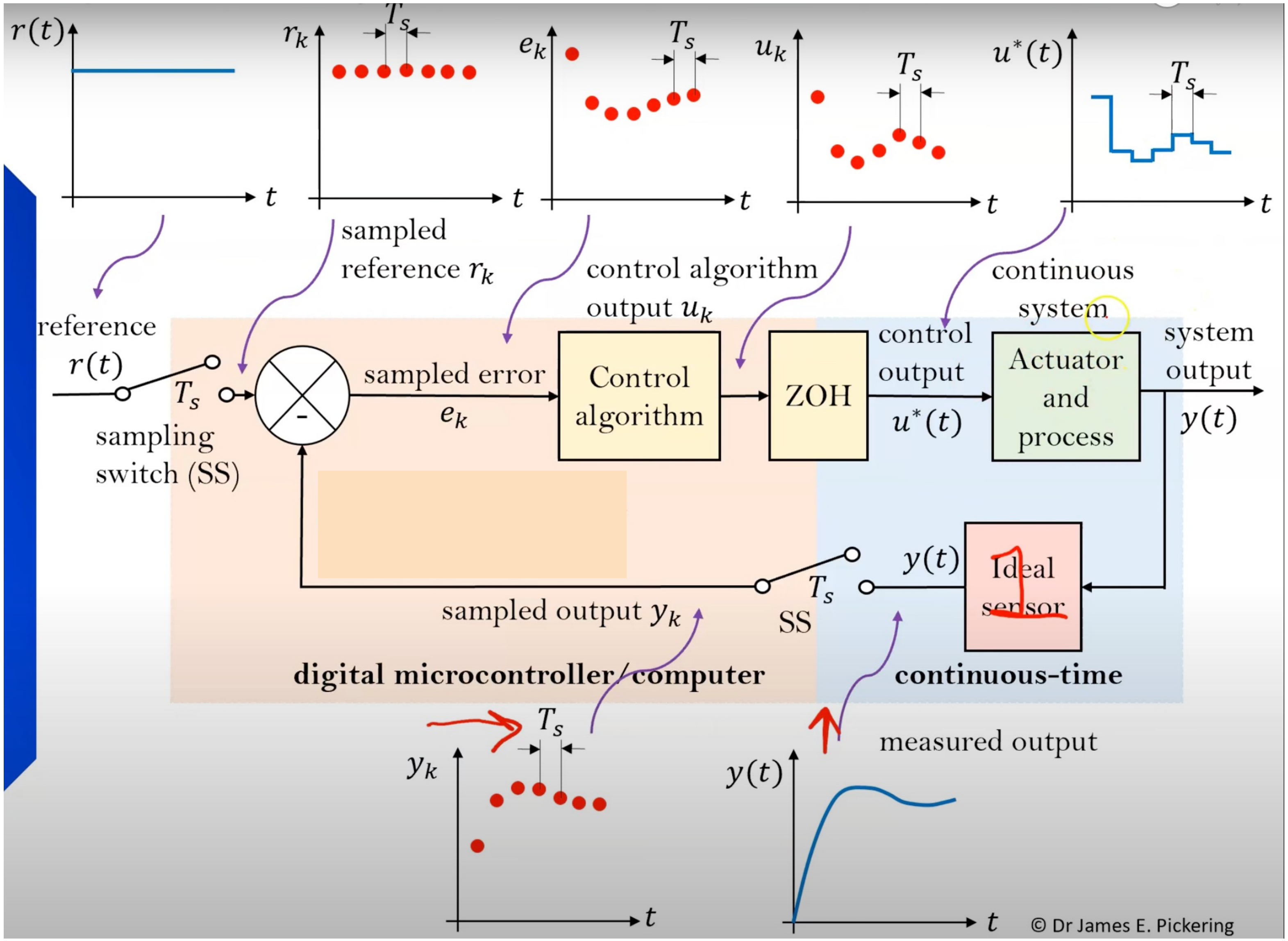
Signals in a digital control scheme



From James Pickering - University of Aston –
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