Control Engineering
3A IA2R & 3A M3
Hugues Garnier
Floriane Collin

Midterm exam - October 3, 2022 - 1h00

Name firstname: ....

Diploma (IA2R/M3): ....

Instructions:

1. When necessary, you can answer in French or in English.
2. The only material you can consult is your personal A4 recto-verso piece of paper.
3. You may use a hand calculator with no communication capabilities.
4. Marking rule: for each question, +2 points for a correct answer and 0 point for a wrong or no answer
5. Good luck!

List 3 examples of feedback control systems (*1 point per correct answer*):
First example: ....
Second example: ....
Third example: ....

Multiple choice questions *(there is one correct answer from the choices only)*

Control engineering is applicable to which of the following engineering fields?

- ☐ Mechanical and aerospace engineering
- ☐ Chemical and biomedical engineering
- ☐ Electrical and civil engineering
- ☐ All of the previous answers

If the differential equation of a system is: $\dot{y}(t) + 0.1y(t) = 0.2u(t)$
then the steady-state gain of the system is:

- ☐ 0.2
- ☐ 1
- ☐ 0.1
- ☐ 2

If the transfer function of a first-order system is: $G(s) = \dfrac{10}{s+5}$
then the time-constant of the system is

- ☐ 5 seconds
- ☐ 1 second
- ☐ $\frac{1}{2}$ seconds
- ☐ $\frac{1}{5}$ seconds

If the transfer function of a second-order system is: $G(s) = \dfrac{1}{0.01s^2 + 0.2s + 1}$
then the damping ratio of the system is

- ☐ 0
- ☐ 1
- ☐ $\frac{1}{2}$
- ☐ 2

If the transfer function of a second-order system is: $G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{s}{(s+2)(s+4)}$
then its response to a unit step $u(t) = \Gamma(t)$ **is**

- ☐ $y(t) = 0.5(e^{2t} - e^{4t})\Gamma(t)$
- ☐ $y(t) = 0.5(e^{-2t} - e^{-4t})\Gamma(t)$
- ☐ $y(t) = (t + 0.5e^{-2t} - 0.5e^{-4t})\Gamma(t)$
- ☐ $y(t) = 0.5(1 - e^{-6t})\Gamma(t)$

Would you classify the step response from the previous question as

☐ undamped ☐ critically damped

☐ underdamped ☐ overdamped

Match each of the zero-pole plots labelled with a number (1 to 3) displayed in Figure 1.1

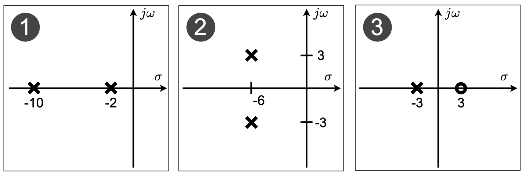

Figure 1.1: Zero-pole plot of 3 different linear systems

with one of the transfer functions $G_i(s)$ for $i = 1, \ldots, 6$ given below

$$G_1(s) = \frac{K(s-3)}{s+3}; \qquad G_2(s) = \frac{K}{s^2+12s+20}; \qquad G_3(s) = \frac{K}{(s+3)(s+6)}$$

$$G_4(s) = \frac{K}{(s+6)^2}; \qquad G_5(s) = \frac{K}{s^2+12s+45}; \qquad G_6(s) = \frac{K}{(s+6-3i)(s-6+3i)}$$

Your answer : ① **-** ② **-** ③ **-**

Match each of the four pole-zero diagrams labelled with a number (1 to 4) with one of the unit step responses labelled with a letter (A to D) displayed in Figure 1.2.

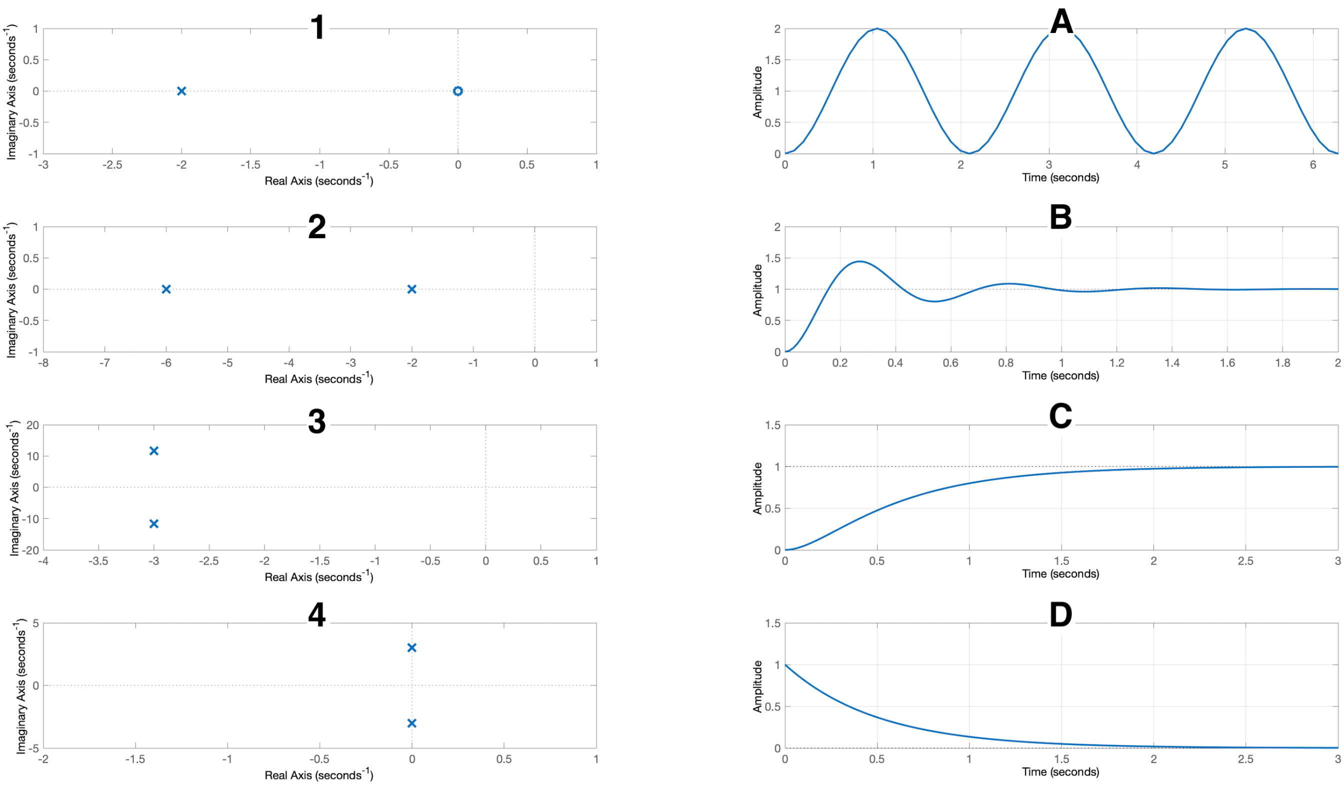

Figure 1.2: Pole-zero diagrams and step responses to be paired

Your answer :

- **1 -**
- **2 -**
- **3 -**
- **4 -**

From the pole-zero diagram shown in Figure 1.3, the system is

☐ stable ☐ marginally stable

☐ unstable ☐ possibly stable

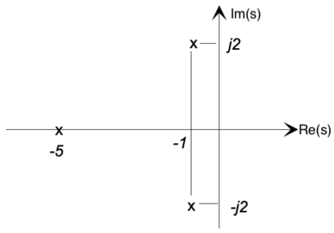

Figure 1.3: Pole-zero diagram.

Assuming the system has unit steady-state gain, from the pole-zero diagram shown in Figure 1.3, determine its transfer function: $G(s) =$

A system has the following characteristic equation: $s^3 + s^2 + 2s + 24 = 0$.
Select the number of roots in the right half of s**-plane that the system has:**

☐ zero ☐ one

☐ two ☐ four

The closed-loop block-diagram for a system is shown in Figure 1.4.

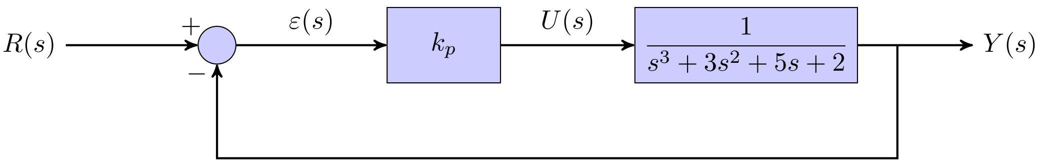

Figure 1.4: Closed-loop block-diagram of a feedback control system

The range of values for k_p **that ensures the stability of the closed-loop system is:**

☐ $0 < k_p < 2$ ☐ $k_p > 1$

☐ $k_p > 13$ ☐ $-2 < k_p < 13$

The block-diagram of a traditional closed-loop feedback system is shown in Figure 1.5 (*the correct answer gives 3 points*).

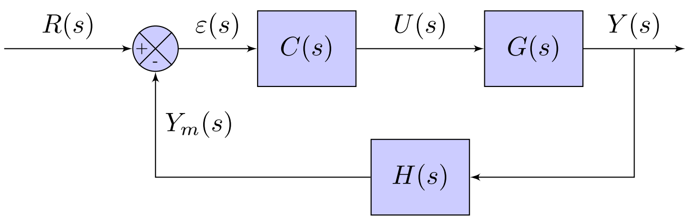

Figure 1.5: Block-diagram of a traditional closed-loop feedback system

Derive the closed-loop transfer function: $F_{CL}(s) = \dfrac{Y(s)}{R(s)}$. ***Do not only give the solution, explain how you obtain it!***

Some useful properties of the Laplace transform

$$L\left(\alpha x(t)+\beta y(t)\right)=\alpha X(s)+\beta Y(s)$$
$$L\left(\dot{x}(t)\right)=sX(s)-x(0)$$
$$\lim_{t\to+\infty} x(t)=\lim_{s\to 0} sX(s) \quad \textbf{if the limit exists}$$

Some Laplace transform pairs

Signal	Laplace transform
$\Gamma(t)$	$\frac{1}{s}$
$e^{-at}\Gamma(t)$	$\frac{1}{s+a}$
$t^n e^{-at}\Gamma(t)$	$\frac{n!}{(s+a)^{n+1}}$

First-order systems

$$G(s)=\frac{K}{1+Ts}$$

The 2 characteristic parameters of a first order system are:

- K: steady-state gain: $K=\lim_{s\to 0} G(s)$
- T: time-constant

Characteristic values of a first-order system step response

Rise-time at 63%	$T_m^{63\%}=T$
Rise-time at 95%	$T_m^{95\%}\approx 3T$
Settling-time at 5 %	$T_r^{5\%}\approx 3T$

Second-order systems

$$G(s)=\frac{K}{\frac{s^2}{\omega_0^2}+2\frac{z}{\omega_0}s+1}=\frac{K\omega_0^2}{s^2+2z\omega_0 s+\omega_0^2}$$

- K: steady-state gain
- z : damping ratio ($z>0$)
- ω_0 : undamped natural frequency

Characteristic values of a underdamped second-order system step response

Value of the first overshoot in %	$D_{1\%}=\frac{y(T_{D_1})-y(+\infty)}{y(+\infty)-y(0)}\times 100=e^{\frac{-\pi z}{\sqrt{1-z^2}}}\times 100$
Damping coefficient as a function of D_1 (not in %)	$z=\sqrt{\frac{(\ln(D_1))^2}{(\ln(D_1))^2+\pi^2}}$
Time-instant of the first overshoot	$T_{D_1}=\frac{\pi}{\omega_0\sqrt{1-z^2}}$
Value of the n^{th} overshoot in %	$D_{n\%}=-(-D_1)^n\times 100$
Time-instant of the n^{th} overshoot	$T_{D_n}=n\,T_{D_1}$