Midterm exam - October 11, 2021 - 1h00

Name firstname : ....

Diploma (IA2R/M3) : ....

Instructions :

1. When necessary, you can answer in French or in English.
2. The only material you can consult is your personal A4 recto-verso piece of paper.
3. You may use a hand calculator with no communication capabilities.
4. Marking rule : for each question, +2 points for a correct answer and 0 point for a wrong or no answer
5. Good luck !

Multiple choice questions *(there is one correct answer from the choices only)*

Which of the following statements is valid for a linear time-invariant dynamic system ?

- ☐ It is described by an ordinary differential equation with time-invariant coefficients
- ☐ Its response to a sinusoid of a given frequency is also a sinusoid of a different frequency
- ☐ It represents all dynamic behaviors of physical systems that exist on earth

Which statement best describes a transfer function ?

- ☐ It is a linear differential equation that describes the transient response of a system
- ☐ It is single mathematical function that relates the input and output Laplace transforms of a system
- ☐ It represents a dynamic system in state-space

Which of the following is a characteristic of a first order system ?

- ☐ Its step response has a non-zero slope at the origin
- ☐ It can be modelled using a first-order differential equation
- ☐ It does not exhibit oscillatory behavior when excited
- ☐ All of the above

If the differential equation of a system is : $\dot{y}(t) + 5y(t) = -10u(t)$
then the steady-state gain of the system is :

- ☐ 1
- ☐ -2
- ☐ 5
- ☐ -10

If the transfer function of a first-order system is : $G(s) = \dfrac{0.2}{s + 0.1}$
then the time-constant of the system is

- ☐ $\frac{1}{2}$ seconds
- ☐ $\frac{1}{10}$ seconds
- ☐ 10 seconds
- ☐ 2 seconds

If the transfer function of a second-order system is : $G(s) = \dfrac{2}{s^2 + s + 1}$
then the damping ratio of the system is

- ☐ 0
- ☐ 1
- ☐ $\frac{1}{2}$
- ☐ 2

If the transfer function of a second-order system is : $G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{2}{s(s+2)}$
then its response to a unit step $u(t) = \Gamma(t)$ **is**

- ☐ $y(t) = (t + 0.5e^{-2t})\Gamma(t)$
- ☐ $y(t) = (1 - e^{-2t})\Gamma(t)$
- ☐ $y(t) = (t + 0.5 + 0.5e^{-2t})\Gamma(t)$
- ☐ $y(t) = (t - 0.5 + 0.5e^{-2t})\Gamma(t)$

Match each of the four given transfer functions $G_i(s)$ for $i = 1, \ldots, 4$

$$G_1(s) = \frac{s+0.1}{s^2}; \qquad G_2(s) = \frac{1}{s^2+0.4s+1}; \qquad G_3(s) = \frac{1.3}{s^2+s+1}; \qquad G_4(s) = \frac{2}{s+2}$$

with one of the unit step responses labelled with a Roman number (I to VI) displayed in Figure 1.1.

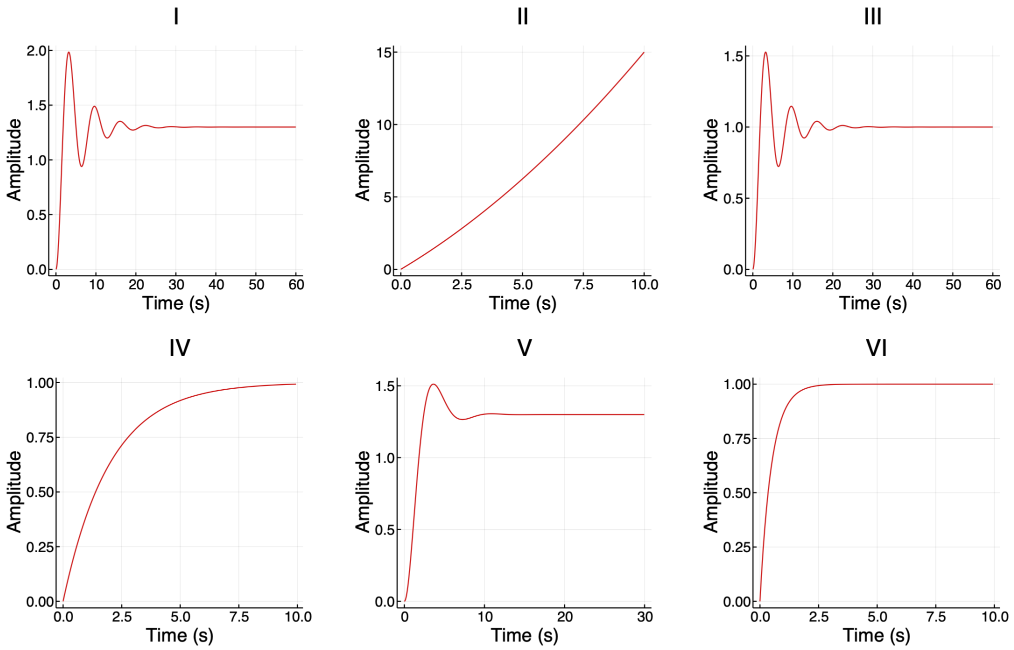

FIGURE 1.1: Step responses of 6 different linear systems

— $G_1(s)$ -
— $G_2(s)$ -
— $G_3(s)$ -
— $G_4(s)$ -

If for a closed-loop system, the Laplace transform of the error signal is : $\varepsilon(s) = \dfrac{8(s+3)}{s+10}$ **then the steady-state value of the error** $\varepsilon(t)$ **is**

☐ 3.6
☐ 2.4
☐ 1.8
☐ 0

From the pole-zero diagram shown in Figure 1.2, the system is

☐ stable
☐ unstable
☐ marginally stable
☐ possibly stable

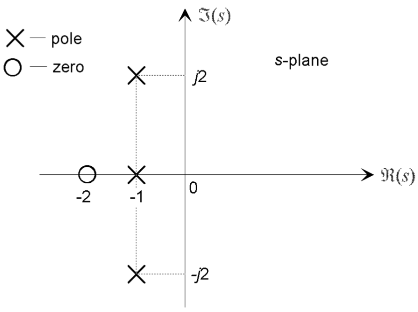

FIGURE 1.2: Pole-zero diagram.

Assuming the system has unit steady-state gain, determine from the pole-zero diagram shown in Figure 1.2, its transfer function is :

$G(s) =$

A system has the following characteristic equation : $s^4 + 2s^3 + 3s^2 + 10s + 8 = 0$**.**
Select the number of roots in the right half of s**-plane that the system has :**

- ☐ one
- ☐ two
- ☐ three
- ☐ four

The closed-loop block-diagram for a system is shown in Figure 1.3.

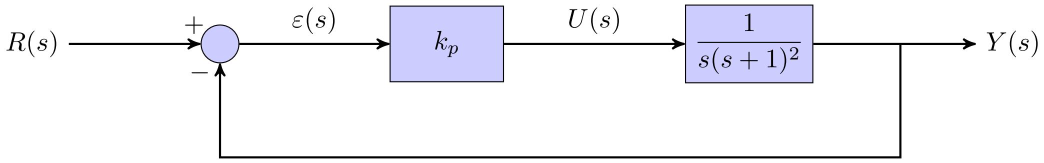

FIGURE 1.3: Closed-loop block-diagram of a feedback control system

The range of values for k_p **that ensures the stability of the closed-loop system is :**

- ☐ $0 < k_p < 2$
- ☐ $k_p > 1$
- ☐ $k_p > 2$
- ☐ $0 < k_p < 1$

For the range of values for k_p **that ensures the stability of the closed-loop system represented in Figure 1.3, the steady-state error to a unit step reference is :**

- ☐ 1
- ☐ k_p
- ☐ $+\infty$
- ☐ 0

The closed-loop block-diagram for a servo-control system is shown in Figure 1.4.

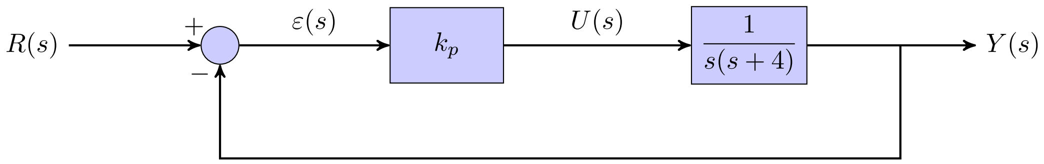

FIGURE 1.4: Closed-loop block-diagram of a servo-control system

The value for k_p **that makes the closed-loop step response to be critically overdamped is (*the correct answer gives 3 points*) :**

- ☐ $k_p = \frac{1}{4}$
- ☐ $k_p = 2$
- ☐ $k_p = 1$
- ☐ $k_p = 4$

The closed-loop block-diagram for a servo system with both feedback and feedforward control is shown in Figure 1.5 (*the correct answer gives 3 points*).

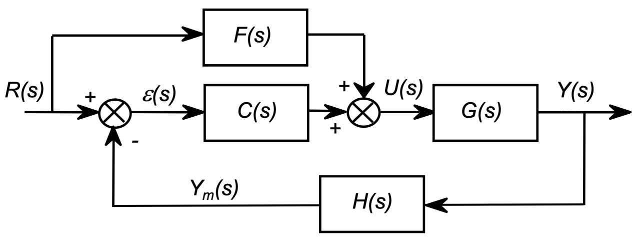

FIGURE 1.5: Closed-loop block-diagram of a servo system with both feedback and feedforward control

Determine the closed-loop transfer function : $F_{CL}(s) = \dfrac{Y(s)}{R(s)} =$

Some useful properties of the Laplace transform

$$L\left(\alpha x(t)+\beta y(t)\right) = \alpha X(s)+\beta Y(s)$$
$$L\left(\dot{x}(t)\right) = sX(s)-x(0)$$
$$\lim_{t\to+\infty} x(t) = \lim_{s\to 0} sX(s) \quad \textbf{if the limit exists}$$

Some Laplace transform pairs

Signal	Laplace transform
$\Gamma(t)$	$\frac{1}{s}$
$r(t) = t\Gamma(t)$	$\frac{1}{s^2}$
$e^{-at}\Gamma(t)$	$\frac{1}{s+a}$

First-order systems

$$G(s) = \frac{K}{1+Ts}$$

The 2 characteristic parameters of a first order system are :

— K : steady-state gain : $K = \lim_{s\to 0} G(s)$

— T : time-constant

Characteristic values of a first-order system step response

Rise-time at 63%	$T_m^{63\%} = T$
Rise-time at 95%	$T_m^{95\%} \approx 3T$
Settling-time at 5 %	$T_r^{5\%} \approx 3T$

Second-order systems

$$G(s) = \frac{K}{\frac{s^2}{\omega_0^2}+2\frac{z}{\omega_0}s+1} = \frac{K\omega_0^2}{s^2+2z\omega_0 s+\omega_0^2}$$

— K : steady-state gain

— z : damping ratio ($z > 0$)

— ω_0 : undamped natural frequency

Characteristic values of a underdamped second-order system step response

Value of the first overshoot in %	$D_{1\%} = \frac{y(T_{D_1})-y(+\infty)}{y(+\infty)-y(0)} \times 100 = e^{\frac{-\pi z}{\sqrt{1-z^2}}} \times 100$
Damping coefficient as a function of D_1 (not in %)	$z = \sqrt{\frac{(\ln(D_1))^2}{(\ln(D_1))^2+\pi^2}}$
Time-instant of the first overshoot	$T_{D_1} = \frac{\pi}{\omega_0\sqrt{1-z^2}}$
Value of the n^{th} overshoot in %	$D_{n\%} = -(-D_1)^n \times 100$
Time-instant of the n^{th} overshoot	$T_{D_n} = n\,T_{D_1}$