Polytech Nancy - Université de Lorraine 3A IA2R - 3A M3

Control engineering

Final exam

November 7, 2022 - 1h30

Name & firstname: ....H G....................

Diploma (IA2R/M3/Erasmus): ....

Ex 1 - 1	Ex 4 _ 16
Ex 2 - 3	Ex 5 - 2
Ex 3 - 1	Total 23

Instructions:

1. Do not forget to write your name above and include this sheet with your copy.
2. You can answer either in French or in English but do not mix both languages.
3. The only material you can consult is your personal A4 piece of paper.
4. You may use a hand calculator with no communication capabilities.
5. The exercises must be solved in the given order.
6. Good luck!

Exercise 1 - Temperature control of the TCLab (≈ 5 mn)

Three different feedback control strategies have been implemented on the TCLab kit that you have used in Lab 1.

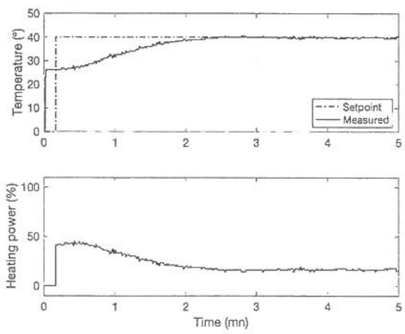


Figure 1: Control A

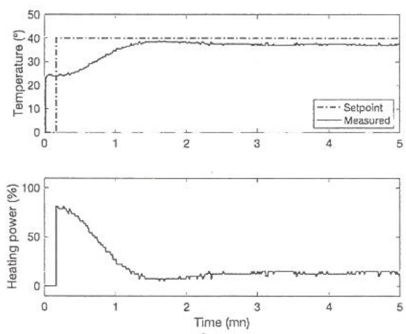


Figure 2: Control B

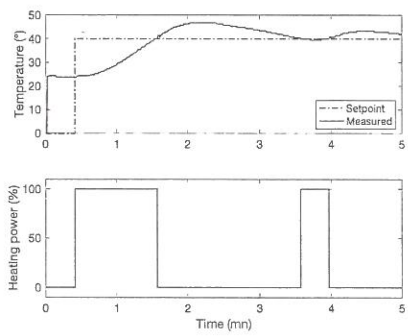


Figure 3: Control C

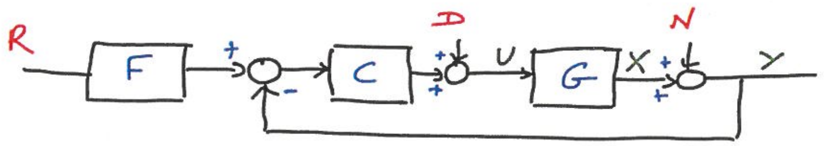
Match each one of the feedback control strategies below to one of the step responses (Control + letter). Justify your answer in each case.

1. On-off control - ..Control C..............................
2. P control - ..Control B.................................
3. PI control - ..Control A................................

1

Exercise 2 - The Gang of Six (≈ 15mn)

The general block diagram of a control system is displayed in Figure 4. For most control designs, as explains by Brian Douglas in his video that was watched during Lab 2, six transfer functions appear to be important leading to the so-called **Gang of Six**.
Complete the missing transfer functions in the 3 empty boxes in Figure 4. Do not give the details of your calculations but the final results only.



Reference Disturbance Noise

State $X = \frac{FCG}{1+CG} R + \frac{G}{1+CG} D \boxed{-\frac{CG}{1+CG}} N$

3

Output $Y = \boxed{\frac{FCG}{1+CG}} R + \frac{G}{1+CG} D + \frac{1}{1+CG} N$

Command $U = \frac{FC}{1+CG} R \boxed{+\frac{\text{[illegible]}}{1+CG}} D - \frac{C}{1+CG} N$

Figure 4: The **Gang of Six** important transfer functions in closed-loop control.

Exercise 3 - Closed-loop block-diagram for the 3pi+ line follower robot (≈ 5mn)

The block diagram of the closed-loop control for the 3pi+ robot that was used in Lab 3 is displayed in Figure 5. Complete the block diagram to link the speed difference command and the nominal speed to the left and right wheel speeds.

1

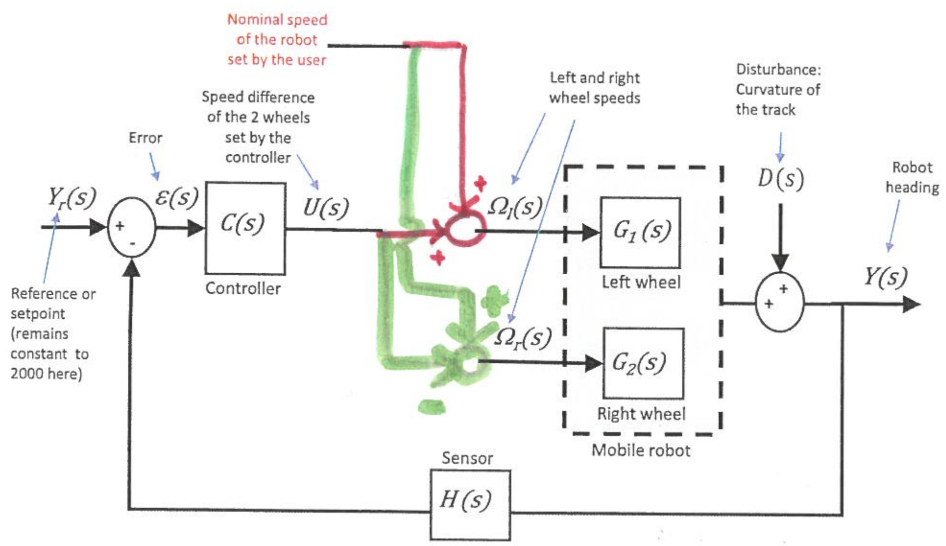


Figure 5: Block diagram of the closed-loop control for the 3pi+ robot.

Exo 4 - Automatic cruise control

1 - Modelling

$$m\dot{v}(t) + b\,v(t) = F(t) - mg\,\theta(t) \quad (1)$$

$m = 1000\,kg$

$b = 100\,Ns/m$

$g = 10\,m/s^2$

1 - a) output $y(t) = v(t)$: speed of the car (m/s)

input $u(t) = F(t)$: force generated (in N) at the road/tire interface

disturbance $d(t) = \theta(t)$: angle of the road with the horizontal axis (in rad)

1 - b) By applying the Laplace transform to (1)

$$m\,s\,Y(s) + b\,Y(s) = F(s) - mg\,\Theta(s)$$

$$(ms + b)\,Y(s) = F(s) - mg\,\Theta(s)$$

$$Y(s) = \frac{1}{ms+b}F(s) - \frac{mg}{ms+b}\Theta(s)$$

$$Y(s) = \frac{\frac{1}{b}}{1 + \frac{m}{b}s}F(s) + \frac{-\frac{mg}{b}}{1 + \frac{m}{b}s}\Theta(s)$$

$$Y(s) = G(s)\,F(s) + G_D\,\Theta(s)$$

with $G(s) = \dfrac{K}{1+Ts} = \dfrac{\frac{1}{b}}{1+\frac{m}{b}s}$

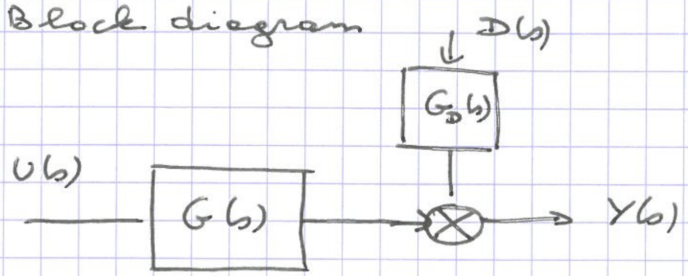
$G_D(s) = \dfrac{-\frac{mg}{b}}{1+\frac{m}{b}s} = \dfrac{K_D}{1+Ts}$

$$\boxed{\begin{aligned} K &= \frac{1}{b} = 0{,}01 \\ T &= \frac{m}{b} = 10\,s \\ K_D &= -\frac{mg}{b} = -100 \end{aligned}}$$

1 - c) poles of $G(s)$ and $G_D(s)$

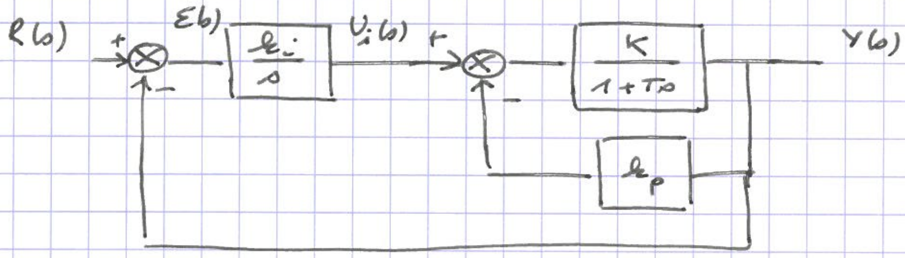
$\boxed{p_1 = -\frac{1}{T} = -\frac{b}{m} = -0{,}1 < 0}$ $\Rightarrow$ $G(s)$ and $G_D(s)$ are stable

1- d) Block diagram



2 - PI feedback control.

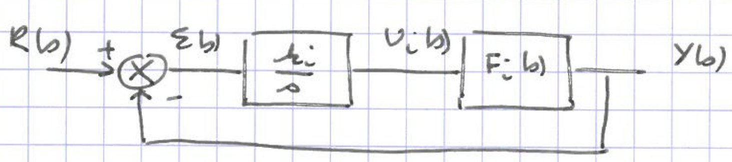
$\theta(t) = d(t) = 0$.



2.a)
$$Y(s) = \frac{\frac{K}{1+Ts}}{1 + \frac{K k_p}{1+Ts}} U_i(s)$$

$$F_i(s) = \frac{Y(s)}{U_i(s)} = \frac{K}{Ts + 1 + K k_p}$$

2.b)



2.c)
$$F_{CL}(s) = \frac{Y(s)}{R(s)} = \frac{k_i K}{Ts^2 + (1 + k_p K)s + k_i K}$$

2.d) Routh-Hurwitz criterion

$$T > 0 \Rightarrow \left| \begin{array}{l} 1 + k_p K > 0 \\ k_i K > 0 \end{array} \right. \qquad \left| \begin{array}{l} k_p > -\frac{1}{K} \\ k_i > 0 \quad \text{since } K > 0, \end{array} \right.$$

to ensure the stability of the closed-loop

2.e) $r(t) = 25\,\Gamma(t)$ $\qquad R(s) = \frac{25}{s}$

$$\lim_{t \to +\infty} \varepsilon(t) = \lim_{s \to 0} s\,E(s)$$

$$= \lim_{s \to 0} s\left[R(s) - Y(s)\right]$$

$$= \lim_{s \to 0} s\,R(s)\left[1 - \frac{Y(s)}{R(s)}\right]$$

$$= \lim_{s \to 0} s\,R(s)\left[1 - F_{CL}(s)\right]$$

$$= \lim_{s \to 0} s \times \frac{25}{s}\left[1 - F_{CL}(s)\right]$$

$$= 1 - F_{CL}(0)$$

$$= 1 - 1$$

$$\boxed{\lim_{t \to +\infty} \varepsilon(t) = 0}$$

In servo mode, the car will be able to track the reference

2.f)

$$D_{1\%} = 4,3\,\% \iff \xi = \sqrt{\frac{(\ln D_1)^2}{\pi^2 + (\ln D_1)^2}} = 0,707.$$

$$T_r^{5\%} = 40\,s \implies \omega_0 = \frac{3}{\xi\, T_r^{5\%}} = \frac{3}{40 \times 0,707}$$

$$F_{CL}(s) = \frac{1}{\frac{s^2}{\omega_0^2} + \frac{2\xi}{\omega_0}s + 1} = \frac{1}{\frac{T}{k_i K}s^2 + \frac{1 + k_p K}{k_i K}s + 1}$$

$$\left\{\begin{array}{ll}
\xi = 0,707 & \\
\omega_0 = \frac{3}{40\xi} \iff \omega_0 \xi = \frac{3}{40} & \\
\frac{1}{\omega_0^2} = \frac{T}{k_i K} & \iff k_i = \frac{T\omega_0^2}{K} = \frac{T}{K\xi^2} \times \left(\frac{3}{40}\right)^2 = \boxed{5,625} \\
\frac{2\xi}{\omega_0} = \frac{1 + k_p K}{k_i K} & 1 + k_p K = \frac{2\xi k_i K}{\omega_0}
\end{array}\right.$$

$$1 + k_p K = \frac{2\cancel{K}}{\omega_0} \times \cancel{\xi} \times \frac{T}{\cancel{K}\cancel{\xi}} \times \left(\frac{3}{40}\right)^2$$

$$k_p = \left[\frac{2}{\omega_0} \times T \times \left(\frac{3}{40}\right)^2 - 1\right] \times \frac{1}{K}$$

$$k_p = \left[2 \times \frac{\cancel{40}}{\cancel{3}} 3 \times T \times \left(\frac{\cancel{3}}{\cancel{40}}\right)^{\cancel{2}} - 1\right] \times \frac{1}{K}$$

$$k_p = \left[\frac{3}{20}\xi \times T - 1\right] \times \frac{1}{K}.$$

$\boxed{k_p \simeq 6,\text{[illegible]}5}$

2.g) $T_{D_1} = \dfrac{\pi}{\omega_0\sqrt{1-\xi^2}} = \dfrac{\cancel{40}\,\pi}{\cancel{8}\sqrt{1-0{,}707^2}} = \cancel{177{,}4}\,s \simeq \cancel{3}\,mn.$

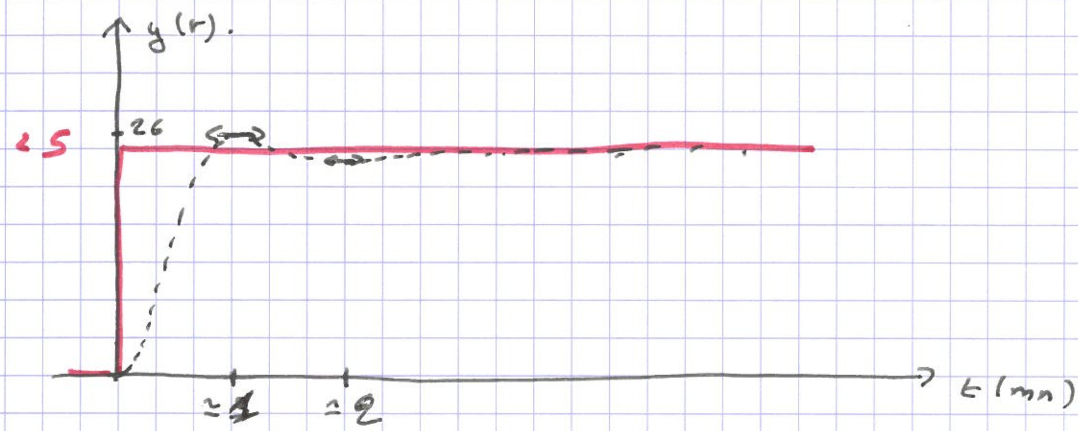
$0{,}075$ $\qquad 59{,}22\,s \simeq 1\,mn.$

$$y(T_{D_1}) = \left(1 + \frac{4{,}3}{100}\right) \times y(+\infty)$$

$$= 1{,}043 \times 25 = 26{,}075\ m/s.$$

$$y(T_{D_1}) = 1{,}043 \times 90 = 93{,}87\ km/h.$$

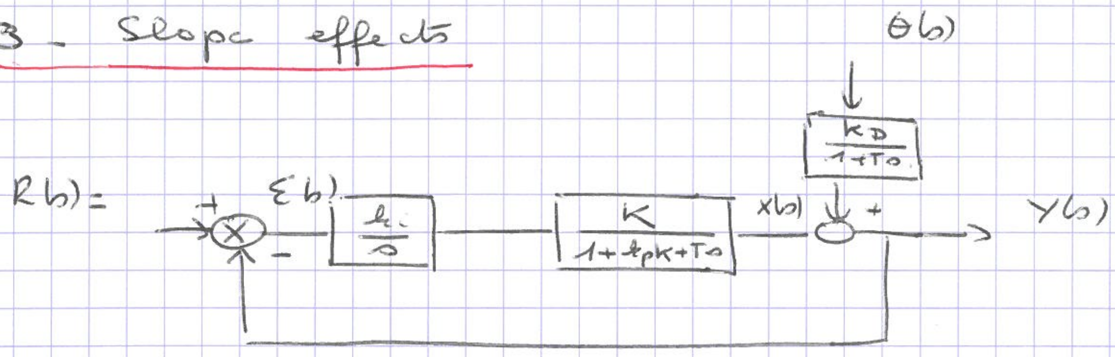
2.h)



2.i) Yes, the performance requirements in servo mode are satisfied : - there is no steady-state error

- $D_{1\%} = 4{,}3\%$
- $T_r^{5\%} = 40\,s$.

3 - Slope effects



3 - a) $\theta(t) = \theta_0 \, \Gamma(t-t_0)$

$$\theta(s) = \frac{\theta_0}{s} \times e^{-t_0 s}.$$

$$\lim_{t\to+\infty} \varepsilon(t) = \lim_{s\to 0} s\,\varepsilon(s) = \lim_{s\to 0} -\left[\frac{k_D}{1+Ts}\theta(s) + \right.$$

$$\lim_{t\to+\infty} \varepsilon(t) = \lim_{s\to 0} -s\,Y(s)$$

$$Y(s) = \frac{k_D}{1+Ts}\theta(s) + X(s)$$

$$X(s) = -\frac{k_i K}{s(1+k_pK+Ts)} Y(s).$$

$$Y(s)\left[1 + \frac{k_i K}{Ts^2 + (1+k_pK)s}\right] = \frac{k_D}{1+Ts}\theta(s)$$

$$\boxed{Y(s) = \frac{k_D}{1+Ts} \times \frac{s(Ts+1+k_pK)}{Ts^2+(1+k_pK)s+k_iK} \times \theta(s).}$$

$$\theta(s) = \frac{\theta_0}{s} \times e^{-t_0 s}.$$

$$\lim_{t\to+\infty} \varepsilon(t) = \lim -s\left[\frac{k_D}{1+Ts} \times \cancel{s}\frac{(Ts+1+k_pK)}{Ts^2+(1+k_pK)s+k_iK} \times \frac{\theta_0}{\cancel{s}} e^{-t_0 s}\right.$$

$$\lim_{t\to+\infty} \varepsilon(t) = 0.$$

If $\theta(t) = \theta_0\,\Gamma(t-t_0)$, the steady-state speed of the car will be maintained.

3. b) $\theta(t) = \theta_0\, r(t - t_0)$

$$\Theta(s) = \frac{\Theta_0}{s^2} e^{-t_0 s}.$$

$$\lim_{t \to +\infty} \varepsilon(t) = \lim_{s \to 0} s\, Y(s)$$

$$= \lim -\cancel{s}\left[\frac{K_D}{1+Ts} \times \cancel{s}\,\frac{(Ts + 1 + k_p K)}{Ts^2 + (1 + k_p K)s + k_i K} \times \frac{\Theta_0 e^{-t_0 s}}{\cancel{s^2}}\right]$$

$$= -\frac{K_D(1 + k_p K)\,\Theta_0}{k_i K} \neq 0.$$

If $\theta(t) = \Theta_0\, r(t - t_0)$, the steady-state speed of the car will not be maintained.

3. c) The performance requirements in regulation mode will not be totally satisfied with this PI structure.

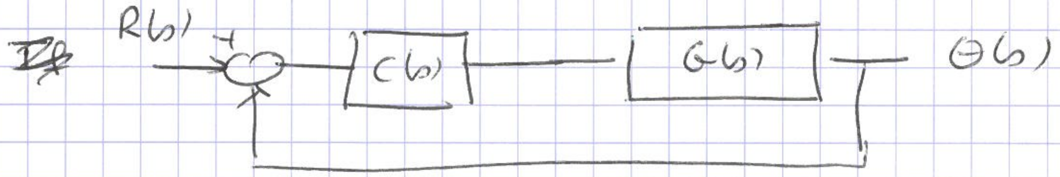
Exercise 5. Feedback control of an inverted pendulum on a cart.

$$\ddot{\theta}(t) - \frac{g}{l}\,\theta(t) = -\frac{1}{l}\,\ddot{x}(t)$$

$$s^2\,\Theta(s) - \frac{g}{l}\,\Theta(s) = -\frac{1}{l}\,s^2 X(s)$$

$$\Theta(s) = \frac{-\frac{1}{l}s^2}{s^2 - \frac{g}{l}}\,X(s).$$

$$\boxed{G(s) = \frac{-s^2/l}{s^2 - g/l}} = -\frac{s^2}{l s^2 - g}.$$



If $C(s) = k_p$ $\qquad F_{CL}(s) = \dfrac{k_p s^2}{(k_p - l)s^2 + g}$

↳ not stable $\forall k_p$ inappropriate

If $C(s) = k_p + k_d s$

$$F_{CL}(s) = \frac{(k_p + k_d)s^2}{k_d s^3 + (k_p - l)s^2 + g}.$$

↳ no term in s → not stable $\forall k_p$ and k_d

⇒ inappropriate

If $\boxed{C(s) = k_p + \frac{k_i}{s}}$

$$F_{CL}(s) = \frac{(k_p s + k_i)s}{(k_p - l)s^2 + k_i s + g}.$$

stable if $k_i > 0$, $k_p > l$.

⇒ only option to stabilise the pendulum.