Fault detection and isolation with robust principal component analysis

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CRAN

Context

Principal Component Analysis

Determination of the redundancy equations

Context

Principal Component Analysis

Determination of the redundancy equations

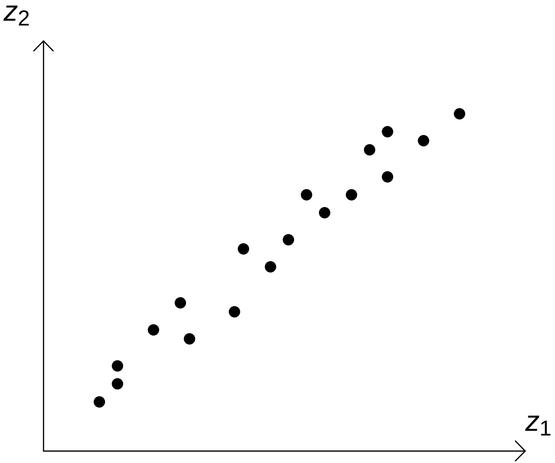
Data

PCA need fault-free data

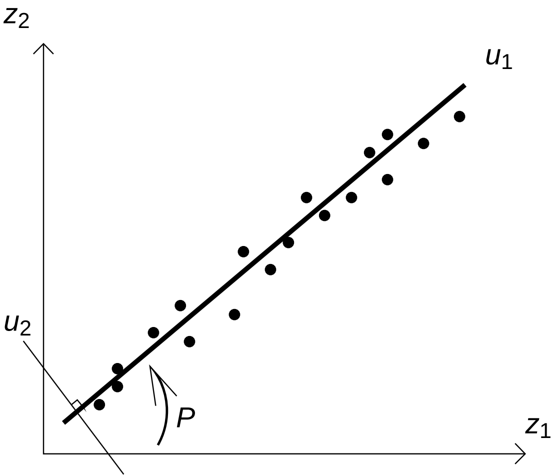
Real case ->Data Validity ??

Outline

Principle of the Principal component analysis



Principle of the Principal component analysis



Principle of the Principal component analysis

- Data matrix $X \in \Re^{N \times n}$ in a normal process operation

PCA

Maximization of the variance projections $T = X\,P$

- $T \in \Re^{N \times n}$: principal component matrix
- $P \in \Re^{n \times n}$: projection matrix

Decomposition in eigenvalues/eigenvectors of the covariance matrix

$$\Sigma = \frac{1}{N-1} X^T X = P \Lambda P^T \qquad \text{with} \qquad P P^T = P^T P = I_n$$

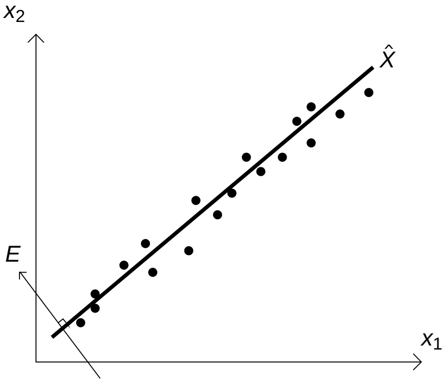
$$\Sigma = \left[\begin{array}{cc} P_\ell & P_{n-\ell} \end{array}\right] \left[\begin{array}{cc} \Lambda_\ell & 0 \\ 0 & \Lambda_{n-\ell} \end{array}\right] \left[\begin{array}{c} P_\ell^T \\ P_{n-\ell}^T \end{array}\right]$$

Principle of the Principal component analysis

Decomposition

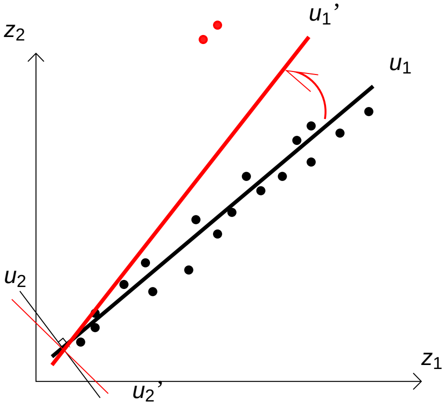
Principal part : $\hat{X} = X P_\ell P_\ell^T = X C_\ell$

Residual part : $E = X - \hat{X} = X (I - C_\ell)$



PCA weakness

- Sensitive to outliers



Robust covariance matrix with respect to outliers

$\longrightarrow$ Outliers detection and isolation

Robust principal component analysis

Robust covariance matrix

$$V = \frac{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} w_{i,j}(x_i - x_j)(x_i - x_j)^T}{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} w_{i,j}}$$

where the weights $w_{i,j}$ themselves are defined by:

$$w_{i,j} = \exp\left(-\frac{\beta}{2}(x_i - x_j)^T \Sigma^{-1}(x_i - x_j)\right)$$

with β turning parameter

Reconstruction principal

Estimate r variables using the $n-r$ remaining variables and the model

$$\hat{x}_R = [I - \Xi_R(\tilde{\Xi}_R^T \tilde{\Xi}_R)^{-1} \tilde{\Xi}_R^T)]\, x$$

$$\tilde{\Xi}_R = (I - C_\ell)\, \Xi_R$$

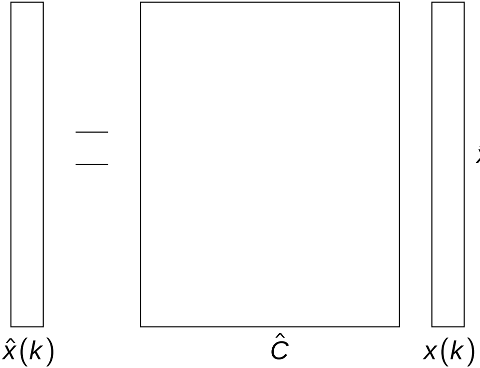
with Ξ_R : reconstruction directions matrix

To reconstruct 2 variables ($R = 2, 4$) among 5 variables

$$\Xi_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T$$

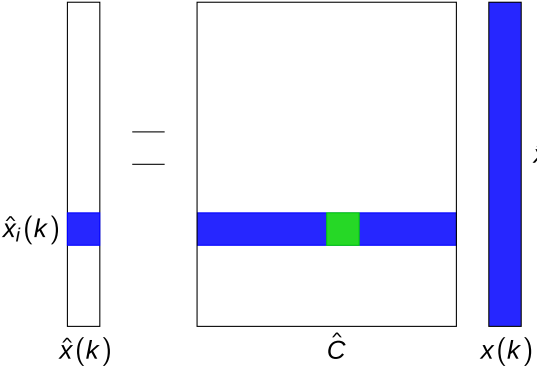
Reconstruction condition : $(\tilde{\Xi}_R^T \tilde{\Xi}_R)^{-1}$

Reconstruction of one variable



$$\hat{x}_i(k) = \frac{\begin{pmatrix} c_{-i} & 0 & c_{+i} \end{pmatrix}}{1 - c_{ii}} x(k)$$

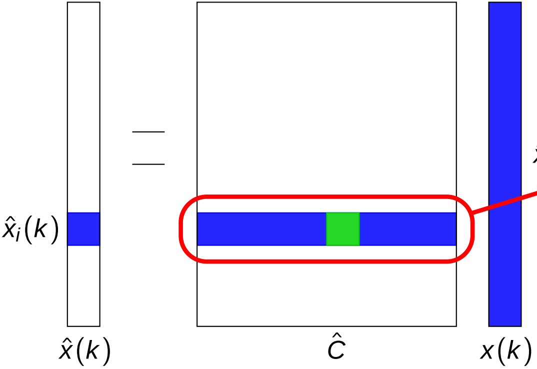
Reconstruction of one variable



$$\hat{x}_i(k) = \frac{\begin{pmatrix} c_{-i} & 0 & c_{+i} \end{pmatrix}}{1 - c_{ii}} x(k)$$

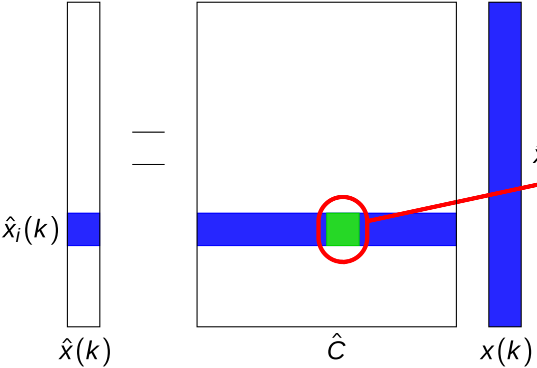
Residual generation

Reconstruction of one variable



$$\hat{x}_i(k) = \frac{\begin{pmatrix} c_{-i} & 0 & c_{+i} \end{pmatrix}}{1 - c_{ii}} x(k)$$

Reconstruction of one variable



$$\hat{x}_i(k) = \frac{\begin{pmatrix} c_{-i} & 0 & c_{+i} \end{pmatrix}}{1 - c_{ii}} x(k)$$

Residual generation

x_R projection in the residual space

$$\tilde{x}_R = P_R^{(\ell)}\, x, \qquad P_R^{(\ell)} = (I - C_\ell)\, [I - \Xi_R(\tilde{\Xi}_R^T \tilde{\Xi}_R)^{-1} \tilde{\Xi}_R^T)]$$

Proprieties : $P_R^{(\ell)}\, \Xi_R = 0, \qquad \Xi_R^T\, P_R^{(\ell)} = 0$

Illustration :

$$x = x^* + \varepsilon + \Xi_F\, d$$

$$\tilde{x}_R = P_R^{(\ell)}(x^* + \varepsilon + \Xi_F\, d) = P_R^{(\ell)}(\varepsilon + \Xi_F\, d)$$

its expected value is: $\mathcal{E}(\tilde{x}_R) = P_R^{(\ell)}\, \Xi_F\, d$

- $\Xi_F = \Xi_R$: $P_R^{(\ell)}\, \Xi_R = 0$ et $\mathcal{E}(\tilde{x}_R) = 0$
- $\Xi_F \neq \Xi_R$: $P_R^{(\ell)}\, \Xi_R \neq 0$ et $\mathcal{E}(\tilde{x}_R) \neq 0$

Residual generation

Fault isolation using the vector $\tilde{x}_R$

Global indicator computed for each observation

$$\Delta_R = \| \tilde{x}_R \|^2_{V_R^{-1}} < \delta$$

with V_R the covariance matrix

Numerical example: multi-fault case

One considers:

- $X = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \end{bmatrix}$
- $N = 108$ observations

$$x_{i,1} = v_i^2 + \sin(0.1i), \quad v_i \sim \mathcal{N}(0,1)$$
$$x_{i,2} = 2\sin(i/6)\cos(i/4)\exp(-i/N)$$
$$x_{i,3} = \log(x_{i,2}^2)$$
$$x_{i,4} = x_{i,1} + x_{i,2}$$
$$x_{i,5} = x_{i,1} - x_{i,2}$$
$$x_{i,6} = 2x_{i,1} + x_{i,2}$$
$$x_{i,7} = x_{i,1} + x_{i,3}$$
$$x_{i,8} \sim \mathcal{N}(0,1)$$

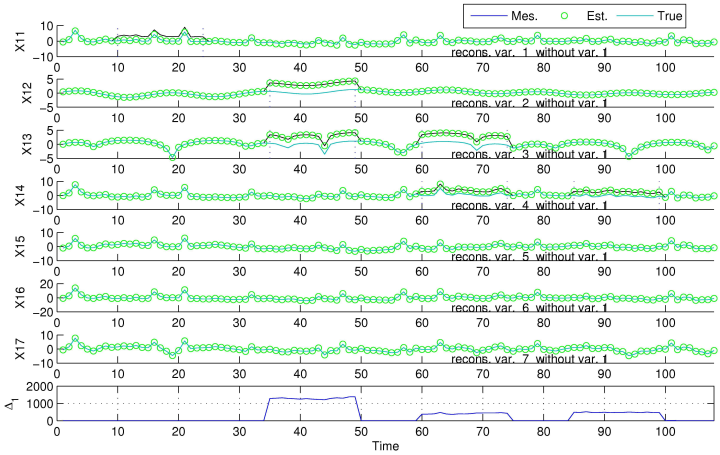
	I_1 {10 : 24}	I_2 {35 : 49}	I_3 {60 : 74}	I_4 {85 : 99}
x_1	×	0	0	0
x_2	0	×	0	0
x_3	0	×	×	0
x_4	0	0	×	×

Numerical example: multi-fault case

Sensitivity of the global indicator Δ_R with respect to fault δ

	δ_1	δ_2	δ_3	δ_4	δ_{12}	δ_{13}	δ_{14}	δ_{23}	δ_{24}	δ_{34}
Δ_1	0	×	×	×	×	×	×	×	×	×
Δ_2	×	0	×	×	×	×	×	×	×	×
Δ_3	×	×	0	×	×	×	×	×	×	×
Δ_4	×	×	×	0	×	×	×	×	×	×
Δ_5	×	×	×	×	×	×	×	×	×	×
Δ_6	×	×	×	×	×	×	×	×	×	×
Δ_{12}	0	0	×	×	0	×	×	×	×	×
Δ_{13}	0	×	0	×	×	0	×	×	×	×
Δ_{14}	0	×	×	0	×	×	0	×	×	×
Δ_{15}	0	×	×	×	×	×	×	×	×	×
Δ_{16}	0	×	×	×	×	×	×	×	×	×
Δ_{23}	×	0	0	×	×	×	×	0	×	×

Variables reconstruction without using variable 1



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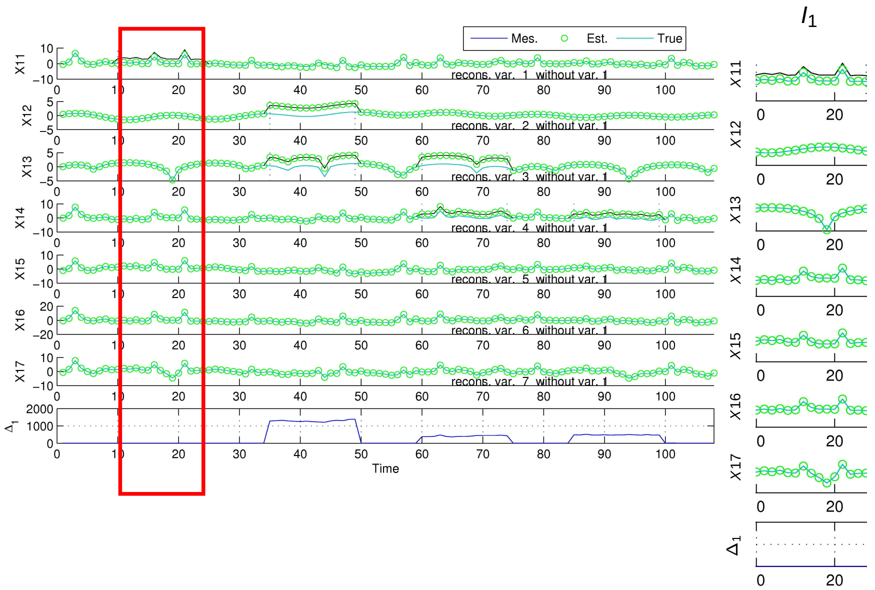

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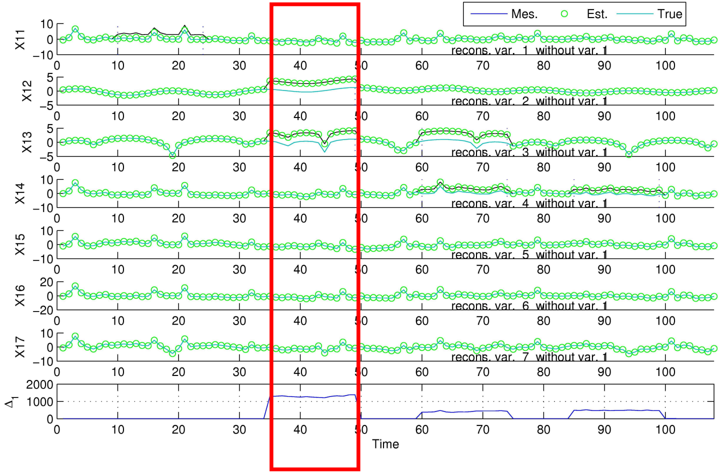

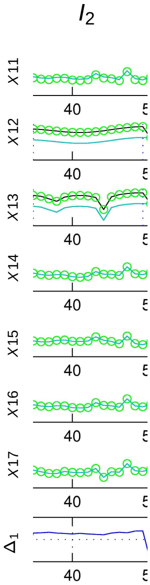

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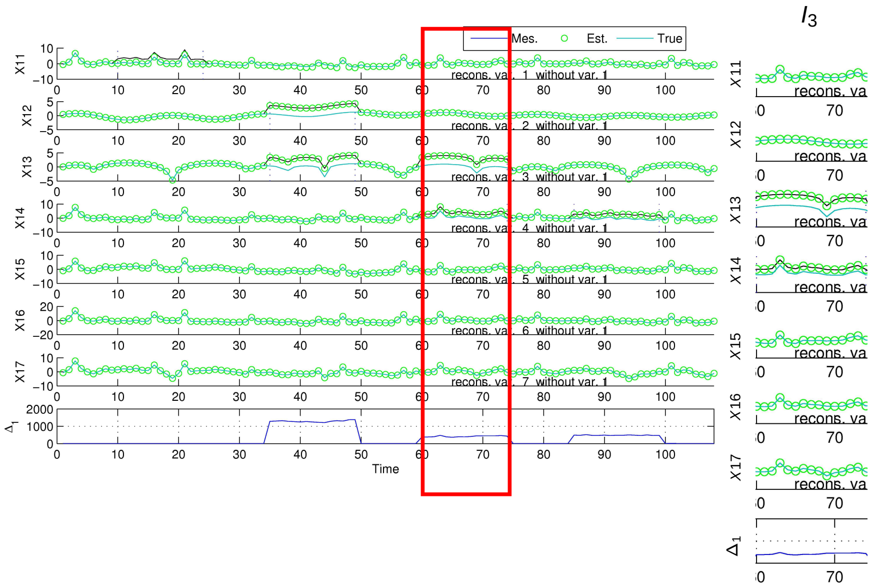

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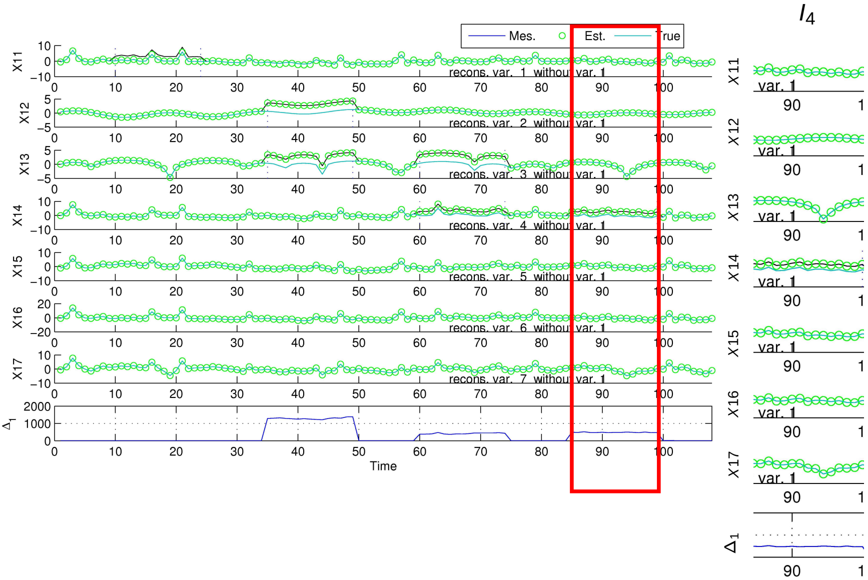

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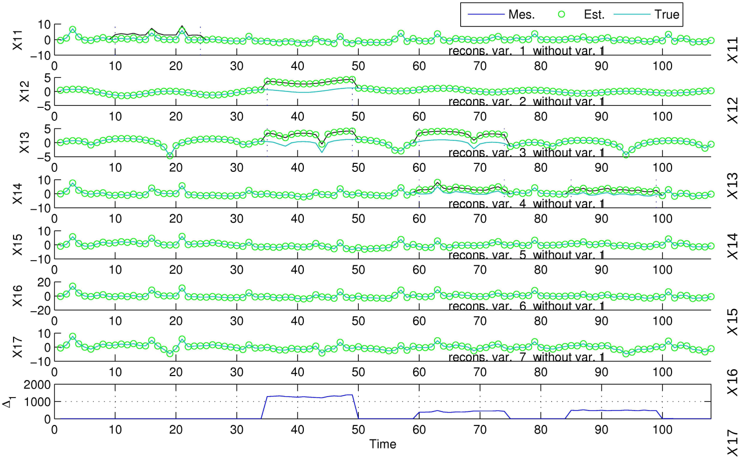

	I_1	I_2	I_3	I_4
Δ_1	0	$\times$	$\times$	$\times$

$\Rightarrow$ in the interval I_1, x_1 is faulty

Numerical example: multi-fault case

	I_1	I_2	I_3	I_4
Δ_1	0	$\times$	$\times$	$\times$
Δ_{23}	$\times$	0	$\times$	$\times$
Δ_{24}	$\times$	$\times$	$\times$	0
Δ_{34}	$\times$	$\times$	0	0

Fault signatures

Numerical example: multi-fault case

	I_1	I_2	I_3	I_4
Δ_1	0	$\times$	$\times$	$\times$
Δ_{23}	$\times$	0	$\times$	$\times$
Δ_{24}	$\times$	$\times$	$\times$	0
Δ_{34}	$\times$	$\times$	0	0

Fault signatures

- in the interval I_1, x_1 is faulty
- in the interval I_2, x_2 and x_3 are faulty
- in the interval I_3, x_3 and x_4 are faulty
- in the interval I_4, x_4 is faulty

Conclusion & prospects

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  –> directly applicable on data containing potential faults

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Prospects

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Prospects

- Reduction of the computational load
  –> reduction of the reconstruction and projection number