Nonlinear Joint State-Parameter Observer for VAV Damper position Estimation*

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***Abstract*— Variable Air Volume (VAV) based Heating Ventilation and Air Conditioning (HVAC) systems are common in large non-residential buildings. The dynamic model of a VAV system along with the Air Handling Unit (AHU) and the zones has a nonlinear characteristic. In this paper, a nonlinear model based joint state and parameter observer is proposed to estimate the VAV damper position in such systems. First, a Takagi-Sugeno (T-S) equivalent model for the AHU-VAV-Zone model is obtained using sector nonlinearity approach. The damper position estimation problem is then posed as a time varying parameter estimation problem. A procedure based on existing literature results on T-S joint state and parameter estimation is implemented. Simulation results show the effectiveness of this approach. A bank of observers based approach is then described that can help in detecting and isolating VAV damper faults in the system using the state and parameter estimates.**

I. INTRODUCTION

Building energy optimization is increasingly becoming an important problem in the overall context of energy footprint of a country. The large non-residential buildings form a key part of this challenge. Variable Air Volume (VAV) based systems supplied by an Air Handling Unit (AHU) are popular in such buildings to maintain a constant temperature in the occupant zones by varying the supplied air volume. The dampers which enable this action are prone to getting stuck and that could lead to both reduction in comfort of the occupants as well as loss of energy.

Model-based approaches are popular in fault diagnosis of systems. The AHU-VAV-Zones combination has a nonlinear characteristic, specifically, a bilinear structure. Takagi-Sugeno (T-S) models are suitable for such *mildly* nonlinear systems. Sector nonlinearity based derivation of T-S models [1] can represent nonlinear characteristics exactly, if the range of values over which the states and inputs of the systems vary is within a known sector [2].

An unknown input observer for polynomial type inputs is designed in [3], for detection of fouling in heat exchangers. The authors assume a specific symmetry in the physical characteristics (the temperature of inlet of hot fluid is equal to that of outlet of cold fluid and vice versa). In [4], a T-S model based polynomial fuzzy observer is designed for the same problem of fouling detection in heat exchangers. This approach requires the unknown parameter is either constant or has very slow dynamics. In a damper position estimation scenario such limitations would not hold.

To mitigate this problem, an approach that follows the results in [5] is proposed. In that paper, the authors rewrite the system matrices with time varying parameters into two terms, one containing the terms dependent on the unknown parameter and another not dependent on it. The only assumption required is that the parameters vary within a known range of values. In the problem considered in this paper, the damper position estimation, this assumption is reasonable. The same sector nonlinearity approach is used to represent the unknown time varying parameter and estimated.

The contributions of this paper and the organization is summarized as follows:

- Derive a system model for the AHU-VAV-Zones combination with a focus on estimation scenario. (Sec. II).
- Derive the T-S models for the AHU-VAV-Zones model using sector nonlinearity approach. Extend this T-S model to that of a T-S model with an unknown time varying parameter (Sec. III).
- Propose an implementation tailored version of the results in literature for a joint state and parameter estimation (Sec IV).
- Propose systematic algorithmic steps to implement joint state and parameter estimation (Sec IV).
- Illustrate how to connect the joint state and parameter estimation results with fault detection in VAV dampers using bank of observers (Sec. V).

II. SYSTEM MODEL

A. Models from first principles

A schematic of the system considered is given in Fig. 1.

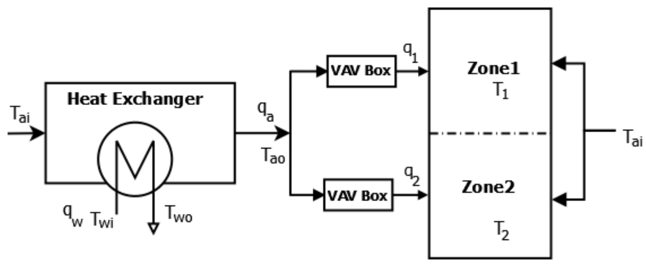

Fig. 1. Schematic of the system under consideration

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The following are the points to be noted for the model:

- Only two zones are considered for the system, though this could be easily extended to any number of zones supplied by a single AHU.
- The VAV boxes have an internal control loop that adjusts the damper position based on the deviation of zone temperature from the set point.
- The two zone temperatures are measured along with the air mass flow rate q_a.
- The mass flow rate of water (q_w) is the manipulated variable. The water temperature is considered constant and known.
- Further the overall heat transfer coefficient of the heat exchanger, $U_A(t)$, is assumed constant, since fouling takes place over a very long period of time.
- The heat exchanger is assumed extract air from the environment, which is also the same ambient temperature outside the walls of the zone.

In the AHU, the dynamics of heat exchanger are modeled using the energy balance approach, inspired by that in [3]:

$$\begin{aligned}\frac{dT_{ao}(t)}{dt} &= \frac{q_a(t)}{M_a}(T_{ai}(t) - T_{ao}(t)) + \frac{U_A}{2C_{pa}M_a}\Delta T(t) \\ \frac{dT_{wo}(t)}{dt} &= \frac{q_w(t)}{M_w}(T_{wi}(t) - T_{wo}(t)) - \frac{U_A}{2C_{pw}M_w}\Delta T(t) \quad (1)\end{aligned}$$

where,

T_{ao}, T_{ai}	: output/input air temperature (K)
T_{wo}, T_{wi}	: output/input water temperature (K)
$q_a,\ q_w$	: air and water mass flow rates (g/s)
$C_{pa},\ C_{pw}$	: specific heat capacities (J/g.K)
$M_w,\ M_a$	: mass of the water with metal and air (kg)
U_A	: overall heat transfer coefficient (J/K)
ΔT	: $T_{wo} + T_{wi} - T_{ao} - T_{ai}$ (approximation)

The zone models were derived based on an energy balance approach and inspired by the thermal modeling (see for e.g., [6], [7]) and given by:

$$\begin{aligned}C_1\frac{dT_1(t)}{dt} &= q_1(t)C_{pa}(T_{ao}(t) - T_1(t)) + K_{12}(T_2(t) - T_1(t)) \\ &\quad + K_{1amb}(T_{ai}(t) - T_1(t)) + K_{d_2}d_2(t) \quad (2) \\ C_2\frac{dT_2(t)}{dt} &= q_2(t)C_{pa}(T_{ao}(t) - T_2(t)) + K_{21}(T_1(t) - T_2(t)) \\ &\quad + K_{2amb}(T_{ai}(t) - T_2(t)) + K_{d_3}d_3(t) \quad (3)\end{aligned}$$

where, q_1, q_2 are the air mass flow rate into each zones. The known, measured or forecast disturbances like occupancy, solar radiation are combined and represented as d_2 and d_3. C_1, C_2 are the corresponding zone capacitance. The gains $K_{12},\ K_{21},\ K_{1amb},\ K_{2amb}$ correspond to the heat transfer coefficient offered by a wall or a window that remains between the zone temperature and the other zone and the zone and the external environment respectively. K_{d2} and K_{d3} represent the general resistance offered for the exchange of thermal energy between objects in the room with the air. The values are computed following the approach in [6].

The modeling of the VAV boxes and other components on the air flow rate path is simplified by the following assumptions:

- The control loop in VAV terminal boxes have a negligible time constant compared to that of the AHU and the zones and the actuator actions are instantaneous.
- Let β_1 and β_2 be ratios of the air flow rate into each zone. The air mass balance gives,

$$\begin{aligned}q_1(t) = \beta_1 q_a(t), \qquad q_2(t) = \beta_2 q_a(t) \\ \beta_1 + \beta_2 = 1 \quad (4)\end{aligned}$$

- The fan dynamics are ignored, while the fan speed and hence $q_a(t)$, is measured.

With these assumptions in place, the overall model of the AHU-VAV-Zone combination could be given as (time is dropped for simplicity),

$$\dot{x}_1 = \alpha_1 q_a(d_1 - x_1) + \alpha_{2au}(T_{wi} + x_2 - d_1 - x_1) \quad (5)$$

$$\dot{x}_2 = \alpha_3 u(T_{wi} - x_2) - \alpha_{2wu}(T_{wi} + x_2 - d_1 - x_1) \quad (6)$$

$$\begin{aligned}\dot{x}_3 &= \alpha_4\beta_1 q_a(x_1 - x_3) + \alpha_5(x_4 - x_3) \\ &\quad + \alpha_6(d_1 - x_3) + \alpha_7 d_2 \quad (7)\end{aligned}$$

$$\begin{aligned}\dot{x}_4 &= \alpha_8(1 - \beta_1)q_a(x_1 - x_4) + \alpha_9(x_3 - x_4) \\ &\quad + \alpha_{10}(d_1 - x_4) + \alpha_{11}d_3 \quad (8)\end{aligned}$$

where the state and the input variables are defined as,

$$\begin{aligned}x_1 \triangleq T_{ao},\ x_2 \triangleq T_{wo},\ x_3 \triangleq T_1,\ x_4 \triangleq T_2 \\ u \triangleq q_w,\ d_1 \triangleq T_{ai}\end{aligned}$$

and the constants α_is as,

$$\begin{aligned}&\alpha_1 \triangleq \frac{1}{M_a}, \alpha_3 \triangleq \frac{1}{M_w}, \alpha_{2au} \triangleq \frac{U_A}{2C_{pa}M_a}, \alpha_{2wu} \triangleq \frac{U_A}{2C_{pw}M_w} \\ &\alpha_4 \triangleq \frac{C_{pa}}{C_1},\ \alpha_5 \triangleq \frac{K_{12}}{C_1},\ \alpha_6 \triangleq \frac{K_{1amb}}{C_1},\ \alpha_7 \triangleq \frac{K_{d1}}{C_1}, \\ &\alpha_8 \triangleq \frac{C_{pa}}{C_2},\ \alpha_9 \triangleq \frac{K_{21}}{C_2},\ \alpha_{10} \triangleq \frac{K_{2amb}}{C_2},\ \alpha_{11} \triangleq \frac{K_{d3}}{C_2}\end{aligned}$$

III. Takagi-Sugeno Models

A. *Takagi-Sugeno equivalent model*

A Takagi-Sugeno model is a polytopic model represented by [2],

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{2^p}\mu_i(z(t))(A_i x(t) + B_i u(t)) \\ y(t) &= \sum_{i=1}^{2^p}\mu_i(z(t))(C_i x(t) + D_i u(t)) \quad (9)\end{aligned}$$

where $z(t)$ refers to the p premise variable(s), which can be function of states and/or inputs, varying over a certain known sector and leads to 2^p submodels. The weighting functions (μ_i) absorb the nonlinearity in the model and they also satisfy the convex sum property,

$$\sum_{i=1}^{2^p}\mu_i(z(t)) = 1, \quad 0 \le \mu_i(z(t)) \le 1, \forall t, \forall i \in \{1, 2, ...2^p\}$$

To obtain the T-S equivalent model of the AHU-VAV-Zones system in the polytopic form, the equations (5)-(8) are written in state space form as,

$$\begin{aligned}\dot{x}(t) &= A(t)x(t) + B_u(t)u(t) + B_d(t)d(t) + B_T T_{wi} \\ y(t) &= Cx(t) + H\nu(t) \quad (10)\end{aligned}$$

where, the premise variables to be considered for this problem are $z_1(t) \triangleq q_a(t)$ and $z_2(t) \triangleq x_2(t)$. Here, the time dependence (t) of the matrices is implicit as they depend on variables that time-varying. These premise variables are assumed to be within a sector, $z_j \in [z_j^{min}\ z_j^{max}]$. $\nu(t)$ is the measurement noise and H its distribution matrix. Given that the AHU water output temperature is not measured, this is an unmeasured premise variable. The matrices in (10) are given by,

$$A(t) = [A_1(t)\ A_2(t)\ A_3(t)]$$

where,

$$A_1(t) = \begin{bmatrix} -\alpha_1 z_1(t) - \alpha_{2au} & \alpha_{2au} \\ \alpha_{2wu} & -\alpha_{2wu} \\ \alpha_4\beta_1 z_1(t) & 0 \\ \alpha_8(1-\beta_1)z_1(t) & 0 \end{bmatrix}$$
$$A_2(t) = \begin{bmatrix} 0 \\ 0 \\ -\alpha_4\beta_1 z_1(t) - \alpha_5 - \alpha_6 \\ \alpha_9 \end{bmatrix}$$
$$A_3(t) = \begin{bmatrix} 0 \\ 0 \\ \alpha_5 \\ -\alpha_8(1-\beta_1)z_1(t) - \alpha_9 - \alpha_{10} \end{bmatrix} \tag{11}$$
$$B_u(t) = \begin{bmatrix} 0 \\ \alpha_3(T_{wi} - z_2(t)) \\ 0 \\ 0 \end{bmatrix},\ B_T = \begin{bmatrix} \alpha_{2au} \\ -\alpha_{2wu} \\ 0 \\ 0 \end{bmatrix}$$
$$B_d(t) = \begin{bmatrix} \alpha_1 z_1(t) - \alpha_{2au} & 0 & 0 \\ \alpha_{2wu} & 0 & 0 \\ \alpha_6 & \alpha_7 & 0 \\ \alpha_{10} & 0 & \alpha_{11} \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

To obtain the model of the form (9), the matrices' entries of $z_1(t)$ and $z_2(t)$ are replaced with the corresponding sector extremum values corresponding to the submodel i, i.e., z_j^{min} or z_j^{max}. The membership functions are obtained by,

$$\mu_i^1(z_j(t)) = \frac{z_j(t) - z_j^{min}}{z_j^{max} - z_j^{min}},\ \mu_i^2(z_j(t)) = \frac{z_j^{max} - z_j(t)}{z_j^{max} - z_j^{min}}$$

and the weighting functions are obtained through the product of these membership functions for the corresponding extremum values. More details on this could be obtained from, say [2].

B. Models for T-S parameter estimation

The T-S model obtained from (12) can be sufficient if the parameter β_1 is known. However, there are a couple of points to note about its characteristic:

- The β_1 parameter is unknown and time varying because the VAV damper positions determine the actual flow rates into each zone. This action is dependent on the local temperature controller.
- When there is a fault in the VAV damper position, say a damper stuck fault, the variation or lack of thereof in the estimated parameter β_1 can be used to detect, isolate and estimate the fault.

In such a scenario, the estimation of $\beta_1(t)$ is necessary. This leads to an observer design requirement which can do simultaneous state and parameter estimation. This problem can be mapped to the results in [5], where a T-S system of type (9), with time varying $A_i(t), B_i(t)$ depending upon n_θ time varying parameters, is rewritten as,

$$\dot{x}(t) = \sum_{i=1}^{2^p}\sum_{j=1}^{2^{n_\theta}} \mu_i^z(z(t))\mu_j^\theta(\theta(t))(A_{ij}x(t) + B_{ij}u(t))$$
$$y(t) = Cx(t) + H\nu(t) \tag{13}$$

with

$$A_{ij} = \breve{A}_i + \sum_{k=1}^{n_\theta} \theta_k^m \bar{A}_k$$
$$B_{ij} = \breve{B}_i + \sum_{k=1}^{n_\theta} \theta_k^m \bar{B}_k \tag{14}$$

where $\theta_k \in [\theta_k^1, \theta_k^2]$ and the weighting functions μ_j^θ are obtained using the sector extremum values of θ following the same process of obtaining the weighting functions for the T-S models. The index j in (13) is a combination of the two indices $m = 1, 2$ and k in (14).

For the application problem scenario, there is one unknown parameter $\beta_1 \in [\beta_1^1,\ \beta_1^2]$ which affects only the state matrix. Hence, the corresponding representation for (14) would be, $A_{ij} = \breve{A}_i + \beta_1^j \bar{A}_i$, where $j = 1, 2$ and β_1^j corresponds to either the minimum of maximum value of the parameter. The system matrices corresponding to (13) are given by,

$$\breve{A}_i = \begin{bmatrix} \breve{A}_i^1 & \breve{A}_i^2 & \breve{A}_i^3 \end{bmatrix}$$

where,

$$\breve{A}_i^1 = \begin{bmatrix} -\alpha_1 z_1^i - \alpha_{2au} & \alpha_{2au} \\ \alpha_{2wu} & -\alpha_{2wu} \\ 0 & 0 \\ \alpha_8 z_1^i & 0 \end{bmatrix}$$
$$\breve{A}_i^2 = \begin{bmatrix} 0 \\ 0 \\ -\alpha_5 - \alpha_6 \\ \alpha_9 \end{bmatrix}, \breve{A}_i^3 = \begin{bmatrix} 0 \\ 0 \\ \alpha_5 \\ -\alpha_8 z_1^i - \alpha_9 - \alpha_{10} \end{bmatrix}$$

$$\bar{A}_i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \alpha_4 z_1^i & 0 & -\alpha_4 z_1^i & 0 \\ -\alpha_8 z_1^i & 0 & 0 & \alpha_8 z_1^i \end{bmatrix}$$
$$\breve{B}_{ui} = \begin{bmatrix} 0 \\ \alpha_3(T_{wi} - z_2^i) \\ 0 \\ 0 \end{bmatrix}, \breve{B}_{di} = \begin{bmatrix} \alpha_1 z_1^i - \alpha_{2au} & 0 & 0 \\ \alpha_{2wu} & 0 & 0 \\ \alpha_6 & \alpha_7 & 0 \\ \alpha_{10} & 0 & \alpha_{11} \end{bmatrix}$$

and $\bar{B}_{ui} = 0_{4\times 1}$, $\bar{B}_{di} = 0_{4\times 3}$ with B_T and C remaining unchanged from (12). z_1^i and z_2^i refer to the values of the premise variables corresponding to the ith submodel.

IV. VAV DAMPER POSITION ESTIMATION

A. T-S joint state and parameter estimation

In this section, a modified version of the T-S joint state and parameter estimation result is derived. Only an outline of the proof is given as it closely follows the details in [5]. For a system defined by (13)-(14), a joint state and parameter observer with a first order structure is envisaged,

$$\begin{aligned}
\dot{\hat{x}}(t) &= \sum_{i=1}^{2^p}\sum_{j=1}^{2^{n_\theta}} \mu_i^z(z)\mu_j^\theta(\hat{\theta})[A_{ij}\hat{x}(t) + B_{ij}u(t) \\
&\qquad + L_{ij}(y(t) - \hat{y}(t))] \\
\dot{\hat{\theta}}(t) &= \sum_{i=1}^{2^p}\sum_{j=1}^{2^{n_\theta}} \mu_i^z(z)\mu_j^\theta(\hat{\theta})[K_{ij}(y(t) - \hat{y}(t)) - \eta\hat{\theta}(t)] \\
\hat{y}(t) &= C\hat{x}(t)
\end{aligned} \tag{15}$$

where L_{ij} and K_{ij} are the gains to be computed to obtain a good estimate. The gain matrix η is fixed a priori as discussed later. To obtain the error dynamics, the system model in (13) is converted to an uncertain-like representation (see for e.g., [8]), so that both system and observer depend on the same weighting functions. This gives an error dynamics of the form,

$$\dot{e}_a(t) = \sum_{i=1}^{2^p}\sum_{j=1}^{2^{n_\theta}} \mu_i^z(z)\mu_j^\theta(\hat{\theta})[\Phi_{ij}e_a(t) + \Psi_{ij}(t)\tilde{u}(t)] \tag{16}$$

where $e_a(t) \triangleq [e_x(t) \;\; e_\theta(t)]^T$ combines the error dynamics of both the states and the parameter, with $\tilde{u} \triangleq \begin{bmatrix} x(t) & \theta(t) & \dot{\theta}(t) & u(t) & \nu(t)\end{bmatrix}^T$, and,

$$\begin{aligned}
\Phi_{ij} &= \begin{bmatrix} A_{ij} - L_{ij}C & 0 \\ -K_{ij}C & -\eta \end{bmatrix} \\
\Psi_{ij}(t) &= \begin{bmatrix} \Delta A(t) & 0 & 0 & \Delta B(t) & -L_{ij}H \\ 0 & \eta & I & 0 & -K_{ij}H \end{bmatrix}
\end{aligned} \tag{17}$$

The uncertain terms $\Delta A(t)$ and $\Delta B(t)$ are functions of the difference between the known and estimated weighting functions $(\mu_j(\theta) - \mu_j(\hat{\theta}))$. The problem is now reduced to finding observer gains such that the error will decay and the effect from the external inputs $\tilde{u}$ is minimized with guaranteed bounds. Bounded Real Lemma (BRL) [9] is used to obtain the sufficient conditions for (16) to be stable and bounded. Considering a quadratic Lyapunov function, $V(t) = e^T(t)Pe(t)$, $P = P^T$, the following inequality needs to hold,

$$\dot{V}(t) + e^T(t)e(t) - \tilde{u}^T(t)\Gamma_2\tilde{u}(t) < 0 \tag{18}$$

where Γ_2 corresponds to the matrix with block entries Γ_2^k and k refers to the corresponding component in $\tilde{u}$. This leads to,

$$\sum_{i=1}^{2^p}\sum_{j=1}^{2^{n_\theta}} \mu_i(z)\mu_j(\hat{\theta}) \begin{bmatrix} e_a(t) \\ \tilde{u}(t)\end{bmatrix}^T \begin{bmatrix} \Phi_i^T P + P\Phi_i + I & P\Psi_i(t) \\ \Psi_i^T(t)P & -\Gamma_2 \end{bmatrix} \begin{bmatrix} e_a(t) \\ \tilde{u}(t)\end{bmatrix}$$

However, this application of BRL requires the error dynamics to have constant matrices. The uncertain terms in $\Psi_{ij}(t)$ in (17) depend on weighting functions which are time varying but follow the convex sum property. Hence well known matrix theory results could be applied to obtain a bound for them.

Further, the application of the BRL leads to bilinear terms involving the positive definite matrix P and the observer gains. This problem is solved in two steps. First, the matrix is restricted to have a diagonal structure, $P = \text{diag}(P_0, P_1)$, then the bilinear terms are replaced with a new variable, such that $R_{ij} = P_0L_{ij}$ and $F_{ij} = P_1K_{ij}$. This leads to a Linear Matrix Inequality (LMI) condition which is summarized in the following theorem.

Theorem 1: There exists a robust state and parameter observer of type (15) for the T-S time varying parameter system (13)-(14) with a bounded gain of $\Gamma = [\Gamma_x \; \Gamma_\theta \; \Gamma_2^{\dot{\theta}} \; \Gamma_2^u \; \Gamma_2^\nu]^T$ from $\tilde{u}(t)$ to $e_a(t)$, if there exists $P_0 = P_0^T > 0$, $P_1 = P_1^T > 0$, $\lambda_1, \lambda_2 > 0$, F_{ij}, R_{ij} such that (for $i = 1,..,2^p$ and $j = 1,..,2^{n_\theta}$),

$$\begin{bmatrix}
T_{11} & T_{12} & 0 & 0 & 0 & 0 & -R_{ij}I_\nu & P_0\mathcal{A} & P_0\mathcal{B} \\
* & T_{22} & 0 & \eta P_1 & P_1 & 0 & -F_{ij}I_\nu & 0 & 0 \\
* & * & T_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\Gamma_2^\theta & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\Gamma_2^{\dot{\theta}} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & T_{55} & 0 & 0 & 0 \\
* & * & * & * & * & * & -\Gamma_2^\nu & 0 & 0 \\
* & * & * & * & * & * & * & -\lambda_1 I & 0 \\
* & * & * & * & * & * & * & * & -\lambda_2 I
\end{bmatrix} < 0$$

where, $T_{11} = P_0A_{ij} + A_{ij}^TP_0 - R_{ij}C - C^TR_{ij}^T + I_{n_x}$, $T_{12} = -C^TF_{ij}^T$, $T_{22} = -2\eta P_1 + 1$, $T_{33} = -\Gamma_2^x + \lambda_1E_A^TE_A$ and $T_{55} = -\Gamma_2^u + \lambda_2E_B^TE_B$. The entry '$*$' refers to the transpose of the corresponding symmetric element in the matrix. The matrices $\mathcal{A}$ and $\mathcal{B}$ are given by,

$$\mathcal{A} \triangleq \begin{bmatrix} A_{11} & ... & A_{2^p2^{n_\theta}}\end{bmatrix}, \; \mathcal{B} \triangleq \begin{bmatrix} B_{11} & ... & B_{2^p2^{n_\theta}}\end{bmatrix}$$

The observer gains are given by:

$$L_{ij} = P_0^{-1}R_{ij} \qquad \text{and} \qquad K_{ij} = P_1^{-1}F_{ij} \tag{19}$$

for $i = 1,..,2^p$ and $j = 1,..,2^{n_\theta}$.

The proof of this theorem follows that in [5].

B. Algorithm for VAV damper position estimation

The implementation of the theorem in the previous section requires a number of numerical aspects to be taken care of. This would be in the form of additional constraints that can ensure that the obtained observer gains are appropriate for the application problem. In this section, the experiences of the implementation for this AHU-VAV-Zones model is summarized in the form of algorithmic steps.

- Choose η in (17). The gain matrix is chosen such that its eigen values are comparable to the eigenvalues of the matrix Φ_{ij} in (17). This may need to go through iterations as the eigenvalues of Φ_{ij} are not known beforehand. Simplest way is choosing a diagonal matrix with appropriate eigen values.
- Choose Γ_2^k values in (18). This is primarily to reduce the number of variables for which the LMI is solved.
- Enforce $P_0 > P_{0init}$ and $P_1 > P_{1init}$ for sufficiently large P_{0init} and P_{1init}. This would ensure that P_0^{-1} and

P_1^{-1} are not close to singular and make the computation of the observer gains K_{ij} and L_{ij} unreliable.

- To ensure that there is a balance between the gains K_{ij} and η, an additional LMI constraint is considered as,

$$F_{ij} > \rho P_1 \eta \tag{20}$$

where ρ is chosen such that the observer first order characteristic would not let the estimation vanish to zero due to the effect of η. ρ shall be a diagonal matrix of different values depending upon the amplitude levels of each unknown parameter θ.

C. Simulation results

To illustrate the methodology, a simulation of the AHU-VAV-Zones system combination was executed in MATLAB. The yalmip [10] LMI parser was used along with the lmilab solver in the LMI toolbox for solving the LMI feasibility problem. The system model parameters were derived to be reasonably representative and not accurate. The heat exchanger component was designed based on steady state analysis of the model after fixing some of the variables. The zone models were obtained considering zones of dimensions 3×5×4 and 3×4×4 (m). The sector maximum and minimum vales of premise variables and the unknown time varying parameters are given in the Table I(a). It is to be noted that the parameter β_1 was scaled by 100 to allow for a reasonably close scale for the parameters and states of the observer. The simulation parameters that were chosen for the LMI feasibility problem are given in the Table I(b). The results are shown in Figures 2 and 3. The inputs used to generate these results are shown in Fig 4. The error statistics of the observer estimates are summarized in Table II. It is clear that the main error deviation on the estimated parameter is at the initial periods of the simulation. It was observed that by dropping the first few estimates, the standard deviation came down by 5%. The other significant behaviour is the noisy estimate.

TABLE I
SIMULATION AND MODEL PARAMETERS

(a) Sector Min and Max Values

Parameter	Min	Max
z_1	0.16 kg/s	1.6 kg/s
z_2	293 K	368 K
β_1	0	100

(b) Simulation parameters

Parameters	Values
η	10^{-4}
ρ	10^5
Γ_2^θ	0.1
Γ_2^θ	0.1
Γ_2^x	$0.1 I_4$
Γ_2^u	0.1
Γ_2^v	0.1

TABLE II
SIMULATION RESULTS: $\beta_1(t)$ ESTIMATION

Error	Mean (%)	Standard Deviation (%)
$\lvert e_{x_1}\rvert$	0.04	0.4
$\lvert e_{x_2}\rvert$	0.07	0.55
$\lvert e_{x_3}\rvert$	0.03	0.3
$\lvert e_{x_4}\rvert$	0.03	0.3
$\lvert e_\theta\rvert$	10.9	18.34

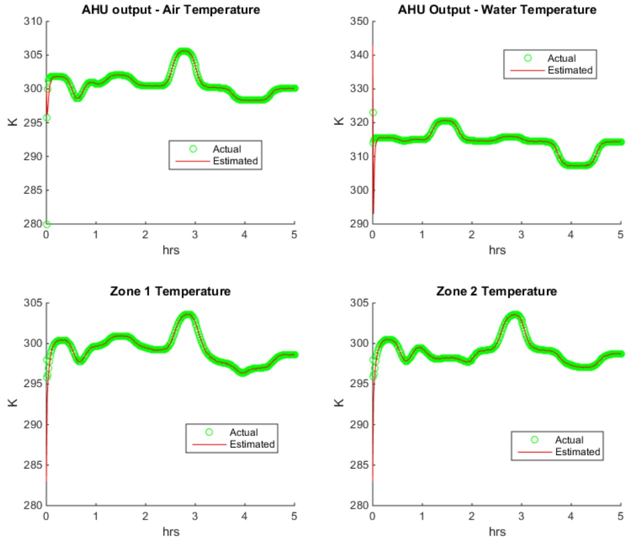

Fig. 2. Estimated and actual states

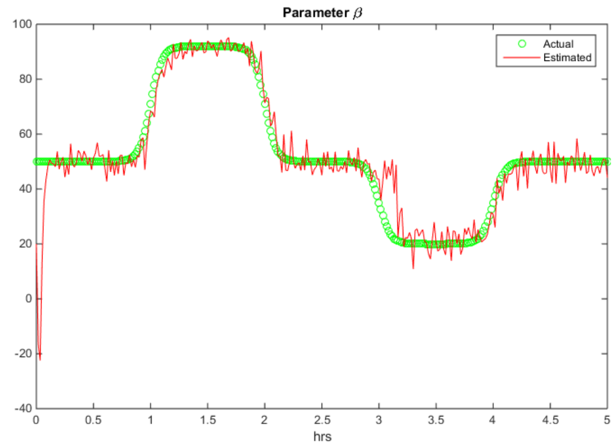

Fig. 3. Estimated and actual parameter β

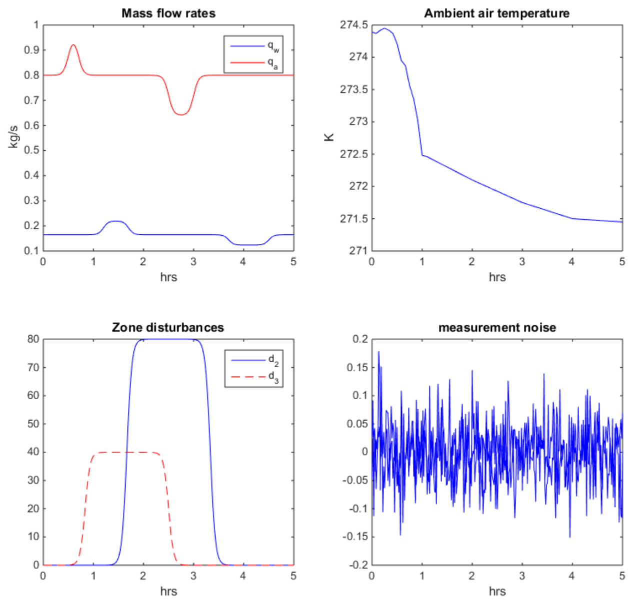

Fig. 4. Various inputs used for the simulation

While this could be partially mitigated through filtering the estimate, a more inclusive approach by embedding this into the observer would be interesting.

V. FAULT DIAGNOSIS USING BANK OF OBSERVERS

This section outlines a strategy to use the estimated β_1 parameter to detect and isolate a VAV damper stuck fault. The proposed strategy is illustrated in Fig. 5. Given the relationship between β_1 and β_2 in (4), the two observers, 'Obs1' and 'Obs2' provide a redundant estimate. Both these observers have the same structure, except for the system matrix changes to reflect the parameter being estimated. However, this redundancy is useful in a fault detection scenario. To analyze the feasibility of β_1 (or β_2) estimation, consider the equations (7)-(8), from the observer's perspective, with $y_1 = x_3$ and $y_2 = x_4$, summarized as,

$$\dot{y}_1 = f_1(\hat{\beta}_1, q_a, \hat{x}_1, y_1, y_2, d_1, d_2) \tag{21}$$

$$\dot{y}_2 = f_2(\hat{\beta}_1, q_a, \hat{x}_1, y_1, y_2, d_1, d_3) \tag{22}$$

which are two nonlinear equations with two unknowns in $\hat{\beta}_1$ and $\hat{x}_1$. Theoretically, this allows for determining the two unknowns when there is no noise and the simulation results agree with this. Hence this estimation could be used to generate residuals for detecting and isolating damper faults.

To generate the residuals using $\hat{\beta}_i$ $(i = 1, 2)$, computation of these parameters from the actual measurements and the knowledge of control loop configuration is required. In a typical VAV set up, the temperature control would have two control loops. The outer loop would use the difference in the measured temperature (T_i) and the set point (T_i^{SP}) to generate an air flow rate set point for the inner control loop. This controller would then compute a damper position and drive the actuator. If the control loop configuration is known, an estimate for the damper position and hence the parameter β_i could be obtained. This is indicated in the figure as β_i^{SP}. The residuals hence generated could detect as well as isolate the fault to a particular zone.

The illustration of this strategy would involve inclusion of control loops in the model and would be dealt with in another paper. More details on T-S model based strategies for residual generation, fault diagnosis etc., could be referred to in [11].

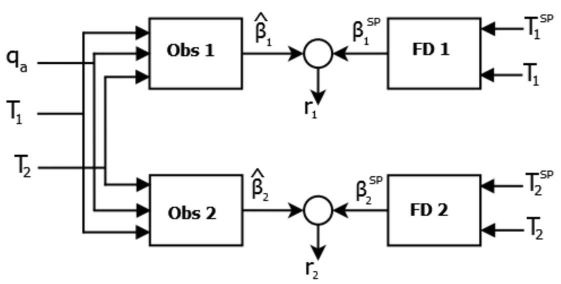

Fig. 5. Bank of Observers for residue generation

VI. CONCLUDING REMARKS

The paper illustrated a methodology to implement a parameter estimation strategy for a nonlinear system of T-S type. The application of this strategy was shown through simulation of a AHU-VAV-Zones model. A proposal on how to use this estimation for fault detection purposes was given. Based on the results and discussion, an obvious extension is to illustrate a complete fault detection and isolation for the discussed problem. Further, the following could be explored:

- An adaptive K_{ij} value can allow for an optimal transient and steady state response for parameter estimation.
- VAV dampers with only ON/OFF or a limited number of positions will mean $\beta_1(t)$ takes discrete values and correspondingly need a hybrid T-S observer.
- Noise reduction through pre-filtering the measurement data or by adding specific LMI constraints that could limit the rate of change of estimated parameter.

REFERENCES

[1] Hiroshi Ohtake, Kazuo Tanaka, and Hua O Wang. Fuzzy modeling via sector nonlinearity concept. *Integrated Computer-Aided Engineering*, 10(4):333–341, 2003.

[2] Kazuo Tanaka and Hua O Wang. *Fuzzy control systems design and analysis: a linear matrix inequality approach*. John Wiley & Sons, 2004.

[3] Sabrina Delrot, Thierry Marie Guerra, Michel Dambrine, and François Delmotte. Fouling detection in a heat exchanger by observer of Takagi–Sugeno type for systems with unknown polynomial inputs. *Engineering Applications of Artificial Intelligence*, 25(8):1558–1566, 2012.

[4] François Delmotte, Michel Dambrine, Sabrina Delrot, and Sylvain Lalot. Fouling detection in a heat exchanger: A polynomial fuzzy observer approach. *Control Engineering Practice*, 21(10):1386–1395, 2013.

[5] Souad Bezzaoucha, Benoît Marx, Didier Maquin, and José Ragot. State and parameter estimation for nonlinear systems: a Takagi-Sugeno approach. In *American Control Conference, ACC 2013, Washington, DC, USA, June 17-19*, 2013.

[6] Bourhan Tashtoush, Mohammed Molhim, and Mohammed Al-Rousan. Dynamic model of an hvac system for control analysis. *Energy*, 30(10):1729–1745, 2005.

[7] Frauke Oldewurtel, Alessandra Parisio, Colin N Jones, Dimitrios Gyalistras, Markus Gwerder, Vanessa Stauch, Beat Lehmann, and Manfred Morari. Use of model predictive control and weather forecasts for energy efficient building climate control. *Energy and Buildings*, 45:15–27, 2012.

[8] Dalil Ichalal, Benoît Marx, José Ragot, and Didier Maquin. An approach for the state estimation of takagi-sugeno models and application to sensor fault diagnosis. In *the Joint 48th Conference on Decision and Control and 28th Chinese Control Conference, Shangai, China*, December 15-18, 2009.

[9] Stephen P Boyd, Laurent El Ghaoui, Eric Feron, and Venkataramanan Balakrishnan. *Linear matrix inequalities in system and control theory*, volume 15. SIAM, 1994.

[10] Johan Löfberg. Yalmip: A toolbox for modeling and optimization in matlab. In *International Symposium on Computer Aided Control Systems design, Taipei, Taiwan,*, September 2-4, 2004.

[11] Dalil Ichalal, Benoît Marx, José Ragot, and Didier Maquin. Fault detection, isolation and estimation for Takagi–Sugeno nonlinear systems. *Journal of the Franklin Institute*, 351(7):3651–3676, 2014.