Takagi-Sugeno model based Nonlinear Parameter Estimation in Air Handling Units ⋆

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Abstract: Air Handling Units (AHU) are responsible for efficiently transferring the energy produced for heating or cooling to the occupant area in the building. This energy transfer dynamics is modeled as a bilinear system. Such a structure poses problems to develop observers either with asymptotically vanishing error or with guaranteed error bounds. Takagi-Sugeno based polytopic models allow modeling of such nonlinear systems as well as to extend the results from linear system theory. Parameter estimation of such models with guaranteed error bounds is presented in Bezzaoucha et al. (2013). The present work applies this nonlinear parameter estimation technique on the energy balance model of a heat exchanger, the key component in an AHU. Three parameter estimation scenarios are proposed and extension to the theoretical results in the reference from an implementation point of view are given. Simulation results are provided to show the feasibility of the approach.

Keywords: Heat Exchanger, Takagi-Sugeno Modeling, Parameter Estimation, Fault Detection and Diagnosis, Bank of Observers

1. INTRODUCTION

Building energy management is important in the effort to reduce the overall energy footprint of a country. Faults in the building HVAC (Heating, Ventilation and Air Conditioning) systems can lead to significant effect on the energy consumed as well as the occupant comfort. AHUs are responsible for the transfer of heating or cooling produced at the source to the rooms and zones that require conditioning. A number of components in the building energy system can be modeled through the energy balance equations similar to that of heat exchanger. For example, the return air heat recovery or even the occupant area temperature change could be modeled using a similar approach.

Different process and equipment faults (including actuators, sensors) are responsible for reduction in the efficiency of the AHU. Classical faults are heat exchanger fouling, hot water supply valve stuck, supply fan fault, to name a few. A majority of the works in the fault detection and diagnosis (FDD) in HVAC systems focuses on the use of models learned from the measurement data (See Katipamula and Brambley (2005) and references thereof). One of the main reasons for this is the bilinear nature of the dynamics. This makes the overall AHU model complex and difficult to develop model based observers with guaranteed error bounds.

Takagi-Sugeno (T-S) method based modeling offers an alternative to the nonlinear representation. For systems with known differential equations, the sector nonlinearity approach (Ohtake et al. (2003)) based T-S method allows for an exact representation of the nonlinear characteristics, when they are bounded. Given that physical systems have natural bounds, this is an attractive approach to model these systems.

In Delrot et al. (2012), a T-S observer was designed considering unknown inputs of polynomial type. The authors introduce parameters that are a function of the heat transfer coefficient and estimate the variation in these parameters around the nominal value. However, there is an inherent assumption that both the hot and cold fluids have to be the same or almost same characteristics. This is typically not the case in HVAC systems. In Delmotte et al. (2013), a polynomial fuzzy observer is proposed for the uncertain case. The model includes a parameter that can indicate fouling, as a state of the system without dynamics. This assumption works well in case of constant or slowly time-varying nature of parameter, like fouling. However such guarantees are not applicable in the applications considered in this paper. This motivated the use of parameter estimation methods which are not restricted by this constraint.

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The paper is organized as follows: Sec. 2 proposes the parameter estimation scenarios to be considered and derives the T-S model system matrices for each scenario. The parameter estimation results from Bezzaoucha et al. (2013) are summarized in Sec. 3 and its customizations and extensions for the problems scenarios are discussed. The Sec. 4 illustrates the results and discusses the inferences. The concluding remarks and future works are summarized in Sec. 5.

2. SYSTEM MODELING

The heat exchanger model from the energy balance adapted from Delrot et al. (2012) is given by the nonlinear differential equations:

$$\frac{dT_{ao}(t)}{dt} = \frac{q_a(t)}{M_a}(T_{ai}(t) - T_{ao}(t)) + \frac{U_A(t)}{2C_{pa}M_a}\Delta T(t) \quad (1)$$

$$\frac{dT_{wo}(t)}{dt} = \frac{q_w(t)}{M_w}(T_{wi}(t) - T_{wo}(t)) - \frac{U_A(t)}{2C_{pw}M_w}\Delta T(t) \quad (2)$$

where,

T_{ao}, T_{ai} : Output/input air temperature (K)
T_{wo}, T_{wi} : Output/input water temperature (K)
$q_a,\ q_w$: Air and water mass flow rates (g/s)
$C_{pa},\ C_{pw}$: Specific heat capacities (J/g.K)
U_A : Overall heat transfer coefficient (J/K)

The following are assumed for the model:

- The mass flow rates of air and water do no undergo any change inside the heat exchanger.
- The temperature variation is considered in a lumped form at the end of the heat exchanger.
- There is no heat loss in the transfer through the metal conductor.
- For the factor ΔT, as discussed in Delrot et al. (2012), this paper adopts the value as $T_{wo} + T_{wi} - T_{ao} - T_{ai}$. This factor is considered a better approximation for ΔT over others.
- $U_A(t)$, the heat transfer coefficient is a time varying factor. However, it changes very slowly (over weeks) and hence could be considered constant for problems where time duration is small.

By making the following definitions for the states and inputs of the system: $x_1(t) \triangleq T_{ao}(t),\ x_2(t) \triangleq T_{wo}(t),\ d(t) \triangleq T_{ai}(t),\ \ u_1(t) \triangleq T_{wi}(t)$, and defining the constants as $\alpha_1 \triangleq \frac{1}{M_a},\ \alpha_{2a} \triangleq \frac{1}{2C_{pa}M_a},\ \alpha_{2w} \triangleq \frac{1}{2C_{pw}M_w}$, the model could be represented as,

$$\begin{aligned}\dot{x}_1(t) &= \alpha_1 q_a(t)(d(t) - x_1(t)) \\ &\quad + \alpha_{2a}U_A(t)(x_2(t) + u_1(t) - x_1(t) - d(t)) \quad (3)\\ \dot{x}_2(t) &= \alpha_3 q_w(t)(u_1(t) - x_2(t)) \\ &\quad - \alpha_{2a}U_A(t)(x_2(t) + u_1(t) - x_1(t) - d(t)) \quad (4)\end{aligned}$$

This can further be represented in a matrix form as,

$$\dot{x}(t) = A(x,u)x(t) + B(x,u)u(t) \quad (5)$$

where,

$$A(x,u) = \begin{bmatrix} -\alpha_1 q_a(t) - \alpha_{2a}U_A(t) & \alpha_{2a}U_A(t) \\ \alpha_{2w}U_A(t) & -\alpha_3 q_w(t) - \alpha_{2w}U_A(t)\end{bmatrix}$$

$$B(x,u) = \begin{bmatrix} \alpha_1 q_a(t) - \alpha_{2a}U_A(t) & \alpha_{2a}U_A(t) \\ \alpha_{2w}U_A(t) & \alpha_3 q_w(t) - \alpha_{2w}U_A(t)\end{bmatrix}$$

The input air temperature is considered as a measured disturbance since the external weather is not controlled, but measured.

2.1 Takagi-Sugeno Models

A Takagi-Sugeno model is a polytopic model represented by (Tanaka and Wang (2004)),

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{2^p} \mu_i(z(t))(A_i x(t) + B_i u(t)) \\ y(t) &= \sum_{i=1}^{2^p} \mu_i(z(t))(C_i x(t) + D_i u(t)) \quad (6)\end{aligned}$$

where $z(t)$ refers to the p premise variable(s), which can be a function of states and/or inputs, varying over a certain known sector and leads to 2^p submodels. The weighting functions (μ_is) absorb the nonlinearity in the model and they also satisfy the convex sum property,

$$\sum_{i=1}^{r} \mu_i(z(t)) = 1 \ \text{ and } \ 0 \le \mu_i(z(t)) \le 1, \forall t, \forall i \in \{1, 2, ...r\}$$

The sector nonlinearity approach allows to define the weighting functions such that the resultant T-S model exactly represents the original nonlinear behaviour within the sector. For a given premise variable z_1, enclosed within the sector of $[z_1^1, z_1^2]$, the membership functions are given by,

$$\tilde{\mu}_1^1(z) = \frac{z(t) - z_1^1}{z_1^2 - z_1^1}, \text{ and } \tilde{\mu}_1^2(z) = \frac{z_1^2 - z(t)}{z_1^2 - z_1^1} \quad (7)$$

The weighting functions $\mu_i(z)$ are then obtained by normalizing the products of the membership functions of individual premise variables. For instance, for a system with two premise variables, $\mu_1 = \frac{\tilde{\mu}_1^1\tilde{\mu}_2^1}{\sum_{ij}\tilde{\mu}_1^i\tilde{\mu}_2^j}$, ..., $\mu_4 = \frac{\tilde{\mu}_1^2\tilde{\mu}_2^2}{\sum_{ij}\tilde{\mu}_1^i\tilde{\mu}_2^j}$. Any nonlinear system could be written in a matrix form similar to that in (5). The premise variables hence chosen would replace the corresponding functions of x and u in the matrices. By replacing these premise variables with their appropriate sector extremum values, the matrices A_i, B_i, C_i, D_i in (6) are obtained.

2.2 T-S Models for Parameter Estimation

The idea of representing T-S models dependent on time varying parameters is given in Bezzaoucha et al. (2013). For a T-S system of the type in (6), consider the system matrices to be time varying $A_i(t), B_i(t)$ depending upon n_θ time varying parameters. Considering constant output equation but affected by a measurement noise $\nu(t)$, this system is rewritten by the authors as,

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{2^p}\sum_{j=1}^{2^{n_\theta}} \mu_i^z(z(t))\mu_j^\theta(\theta(t))(A_{ij}x(t) + B_{ij}u(t)) \\ y(t) &= Cx(t) + \nu(t) \quad (8)\end{aligned}$$

with,

$$\begin{aligned}A_{ij} &= \breve{A}_i + \sum_{j=1}^{n_\theta} \theta_j^k \bar{A}_j \\ B_{ij} &= \breve{B}_i + \sum_{j=1}^{n_\theta} \theta_j^k \bar{B}_j \quad (9)\end{aligned}$$

where $\theta_j \in [\theta_j^1, \theta_j^2]$ and the weighting functions μ_j^θ are using these bounds as given in (7). For more details regarding the model, refer to Bezzaoucha et al. (2013).

2.3 T-S Models for Heat Exchanger

To represent the heat exchanger model, the premise variables considered are the mass flow rates $z(t) = [q_a(t)\ q_w(t)]$ and would replace the mass flow rates in the matrices in (5). Thus the time varying system matrices are given by,

$$\begin{aligned} A_i(t) &= \begin{bmatrix} -\alpha_1 z_1^i - \alpha_{2a} U_A(t) & \alpha_{2a} U_A(t) \\ \alpha_{2w} U_A(t) & -\alpha_3 z_2^i - \alpha_{2w} U_A(t) \end{bmatrix} \\ B_i(t) &= \begin{bmatrix} \alpha_1 z_1^i - \alpha_{2a} U_A(t) & \alpha_{2a} U_A(t) \\ \alpha_{2w} U_A(t) & \alpha_3 z_2^i - \alpha_{2w} U_A(t) \end{bmatrix} \\ C &= [1\ 0] \\ D &= 0 \end{aligned} \tag{10}$$

for $i = 1, 2, 3, 4$. The superscript i on the premise variable refers to the corresponding maximum or minimum value of the premise variable of the ith submodel. As is evident, the model assumes the temperature of output air as measured. Given this system model, three parameter estimation problem scenarios in the AHU are described and the corresponding system matrices are given below. The subscript j for matrices $\bar{A}$ and $\bar{B}$ in (9) are omitted as $n_\theta = 1$ in all three cases.

Heat Transfer Coefficient Estimation The heat exchanger surfaces suffer deposition of unwanted materials over time. This phenomenon, known as fouling, leads to reduction of effectiveness of the heat exchanger. Due to this deposition, the heat transfer coefficient $U_A(t)$ changes over a long period of time. This can be analyzed by constructing a parameter estimator for $\theta(t) \triangleq U_A(t)$. With the premise variables as $z_1(t) \triangleq q_a(t)$ and $z_2(t) \triangleq q_w(t)$, the system matrices in (8) and (9) for this problem can be given by,

$$\begin{aligned} \breve{A}_i &= \begin{bmatrix} -\alpha_1 z_1^i & 0 \\ 0 & -\alpha_3 z_2^i \end{bmatrix} \quad \bar{A} = \begin{bmatrix} -\alpha_{2a} & \alpha_{2a} \\ \alpha_{2w} & -\alpha_{2w} \end{bmatrix} \\ \breve{B}_i &= \begin{bmatrix} \alpha_1 z_1^i & 0 \\ 0 & \alpha_3 z_2^i \end{bmatrix} \quad \bar{B} = \begin{bmatrix} -\alpha_{2a} & \alpha_{2a} \\ \alpha_{2w} & -\alpha_{2w} \end{bmatrix} \end{aligned} \tag{11}$$

with $i = 1, 2, 3, 4$.

Water Mass Flow Rate Estimation The mass flow rate of the water is one of the control inputs used in the industry to regulate and maintain the output air temperature of the heat exchanger. The water mass flow rate could be affected by a number of reasons: a stuck or malfunctioning valve, a malfunctioning pump, etc. Since the time scale over which this occurs is considerably small, the heat transfer coefficient is considered constant: $\alpha_{2wu} \triangleq \alpha_{2w} U_A$ and $\alpha_{2au} \triangleq \alpha_{2a} U_A$. With $z(t) \triangleq q_a(t)$ and $\theta(t) \triangleq q_w(t)$, the system matrices for the parameter estimation become,

$$\begin{aligned} \breve{A}_i &= \begin{bmatrix} -\alpha_1 z^i - \alpha_{2au} & \alpha_{2au} \\ \alpha_{2wu} & -\alpha_{2wu} \end{bmatrix} \quad \bar{A} = \begin{bmatrix} 0 & 0 \\ 0 & -\alpha_3 \end{bmatrix} \\ \breve{B}_i &= \begin{bmatrix} \alpha_1 z^i - \alpha_{2au} & \alpha_{2au} \\ \alpha_{2wu} & -\alpha_{2wu} \end{bmatrix} \quad \bar{B} = \begin{bmatrix} 0 & 0 \\ 0 & \alpha_3 \end{bmatrix} \end{aligned} \tag{12}$$

with $i = 1, 2$.

Air Mass Flow Rate Estimation The mass flow rate of air is maintained by the fan speed as well as the pressure balance in the air network. An estimation of the air mass flow rate as a parameter can potentially point out to malfunctioning of the fan or problems in the duct. With $z(t) \triangleq q_w(t)$ and $\theta(t) \triangleq q_a(t)$, the system matrices for the parameter estimation become,

$$\begin{aligned} \breve{A}_i &= \begin{bmatrix} -\alpha_{2au} & \alpha_{2au} \\ \alpha_{2wu} & -\alpha_3 z^i - \alpha_{2wu} \end{bmatrix} \quad \bar{A} = \begin{bmatrix} -\alpha_1 & 0 \\ 0 & 0 \end{bmatrix} \\ \breve{B}_i &= \begin{bmatrix} -\alpha_{2au} & \alpha_{2au} \\ \alpha_{2wu} & \alpha_3 z^i - \alpha_{2wu} \end{bmatrix} \quad \bar{B} = \begin{bmatrix} \alpha_1 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned} \tag{13}$$

with $i = 1, 2$.

3. NONLINEAR PARAMETER ESTIMATION FOR T-S MODELS

This section summarizes the results in Bezzaoucha et al. (2013) briefly, and then discusses the customization of the results for the application scenarios. Detailed derivation of the results should be referred to in the original paper. For a system defined by (8)-(9), the authors in the original reference propose an observer with a first order structure for the parameter estimation, given by,

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^{2^p} \sum_{j=1}^{2^{n_\theta}} \mu_i^z(z) \mu_j^\theta(\hat{\theta}) [A_{ij}\hat{x} + B_{ij}u + L_{ij}(y(t) - \hat{y}(t))] \\ \dot{\hat{\theta}}(t) &= \sum_{i=1}^{2^p} \sum_{j=1}^{2^{n_\theta}} \mu_i^z(z) \mu_j^\theta(\hat{\theta}) [K_{ij}(y(t) - \hat{y}(t)) - \eta_{ij}\hat{\theta}(t)] \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned} \tag{14}$$

where L_{ij}, K_{ij} and η_{ij} have to be adjusted in order to obtain a good estimate. The premise variables are assumed to be known in this work, which is not the case in the original reference. This however, does not change the steps to derive the results. Given the presence of $\mu_j^\theta(\hat{\theta})$ in the observer model, the comparison of the system (8) and the observer (14) is difficult. The authors in the original work avoid this by representing the original system in an uncertain-like form as,

$$\begin{aligned} \dot{x}(t) = \sum_{i=1}^{2^p} \sum_{j=1}^{2^{n_\theta}} \mu_i^z(z) \mu_j^\theta(\hat{\theta}) [&(A_{ij} + \Delta A(t))x(t) \\ &+ (B_{ij} + \Delta B(t))u(t)] \\ y(t) = Cx(t) + H\nu(t) \quad& \end{aligned} \tag{15}$$

where $\Delta A(t)$ and $\Delta B(t)$ are bounded time varying factors and a function of the differences between the weighting functions $(\mu_j^\theta(\theta) - \mu_j^\theta(\hat{\theta}))$. This representation helps to obtain the observer error dynamics of the form:

$$\dot{e}_a(t) = \sum_{i=1}^{2^p} \sum_{j=1}^{2^{n_\theta}} \mu_i^z(z) \mu_j^\theta(\hat{\theta}) [\Phi_{ij} e_a(t) + \Psi_{ij}(t)\tilde{u}(t)] \tag{16}$$

where $e_a(t) \triangleq [e_x(t)\ e_\theta(t)]^T$ combines the error dynamics of both the states and the parameter, with $\tilde{u} \triangleq \left[x(t)\ \theta(t)\ \dot{\theta}(t)\ u(t)\ \nu(t)\right]^T$. The problem solved here is to determine the observer parameters such that the error is minimized with guaranteed bounds. The time varying terms of $\Psi_{ij}(t)$ are bounded by well known matrix inequality results and enables applying the Bounded Real Lemma on the effective error dynamics. The results are summarized in Theorem 1 and with the key LMI condition of (30) in Bezzaoucha et al. (2013).

Customizations To apply the parameter estimation results to the AHU application problems, a number of customizations were applied. Their motivations and details are summarized as follows:

- L_2 gain: The authors in the reference used the Lyapunov function $V(e_a(t)) = e_a(t)^T P e_a(t)$ for the stability analysis and the employed following inequality to obtain a bounded response from $\tilde{u}(t)$ to $e_a(t)$:
$$\dot{V}(t) + e_a^T(t) P e_a(t) - \tilde{u}^T(t)\Gamma_2 \tilde{u}(t) < 0 \quad (17)$$
Here, Γ_2 is a block diagonal matrix, each block defining the sensitivity of the error dynamics to the particular component of $\tilde{u}$. This leads to an optimization problem to compute β which bounds all the blocks in Γ_2 such that $\Gamma_2^{(k)} < \beta I$ (here (k) corresponds to the block in Γ_2 corresponding to kth entry in $\tilde{u}$). If all the components of $\tilde{u}$ are not in the same amplitude scale, this step may not achieve its purpose. A weighted minimization considering $\Gamma_2^{(k)} < \gamma_k \beta I$ with γ_k adjusting for the amplitude scale, is one option. This could also be replaced with simply choosing some or all the L_2 gains.
- Parameter estimation observer gains: The parameter estimation observer structure in (14) could be viewed simplistically as:
$$\dot{\hat{\theta}}(t) = -\eta\theta(t) + K(y(t) - \hat{y}(t)) \quad (18)$$
It is clear that the ratio of the gains, K/η, needs to be large enough for the estimate not to vanish to zero due to the influence of η. Hence an extra condition is added such that $K/\eta > \rho$. The choice of ρ would depend on the amplitude range of the parameter to be estimated and that of the measurement.
- LMI unknowns: The number of unknown variables in the LMIs is significant. The following parameters are fixed to scalar values (or constant known matrices) to reduce the number of variables:
  - The integral gain (η_{ij}) are chosen and fixed to a scalar value η_0 (or $\eta_0 I_{n_\theta}$ for the general case).
  - The L_2-gain values are also chosen manually based on the input component.
- Measurement noise: The authors in the original reference do not consider measurement noise for the derivation. In this work, the results are extended to include measurement noise and the derivation steps follows that in the original work.

Taking into account the above aspects, the Theorem 1 in Bezzaoucha et al. (2013) could be rewritten as:

Theorem 1. There exists a robust state and parameter observer (14) for the Takagi-Sugeno time varying parameter system (8)-(9) with a bounded gain of $\gamma = [\gamma_x \ \gamma_\theta \ \gamma_{\dot{\theta}} \ \gamma_u \ \gamma_\nu]^T$ from $\tilde{u}(t)$ to $e_a(t)$, if there exists $P_0 = P_0^T > 0$, $P_1 = P_1^T > 0$, $\lambda_1, \lambda_2 > 0$, F_{ij}, R_{ij} such that (for $i = 1.., r$ and $j = 1.., n_\theta$), the LMI (19) is satisfied

$$\begin{bmatrix} T_{11} & T_{12} & 0 & 0 & 0 & 0 & -R_{ij}I_\nu & P_0\mathcal{A} & P_0\mathcal{B} \\ * & T_{22} & 0 & \eta_0 P_1 & P_1 & 0 & -F_{ij}I_\nu & 0 & 0 \\ * & * & T_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma_\theta I_{n_\theta} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma_{\dot{\theta}} I_{n_\theta} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & T_{55} & 0 & 0 & 0 \\ * & * & * & * & * & * & -\gamma_\nu I_\nu & 0 & 0 \\ * & * & * & * & * & * & * & -\lambda_1 I & 0 \\ * & * & * & * & * & * & * & * & -\lambda_2 I \end{bmatrix} < 0$$

$$F_{ij} > P_1 \rho \eta_0 I_{n_\theta} \quad (19)$$

where, $T_{11} = P_0 A_{ij} + A_{ij}^T P_0 - R_{ij}C - C^T R_{ij}^T + I_{n_x}$, $T_{12} = -C^T F_{ij}^T$, $T_{22} = -2\eta_0 P_1 + 1$, $T_{33} = -\gamma_x I_{n_x} + \lambda_1 E_A^T E_A$ and $T_{55} = -\gamma_u I_{n_u} + \lambda_2 E_B^T E_B$. The entry '$*$' refers to the transpose of the corresponding element due to the symmetric nature. The matrices $\mathcal{A}$ and $\mathcal{B}$ are given by,

$$\mathcal{A} \triangleq [A_{11} \ \ldots \ A_{2^p 2^{n_\theta}}]$$
$$\mathcal{B} \triangleq [B_{11} \ \ldots \ B_{2^p 2^{n_\theta}}]$$

The observer gains are given by:

$$\eta_{ij} = \eta_0, \quad L_{ij} = P_0^{-1} R_{ij} \text{ and } K_{ij} = P_1^{-1} F_{ij} \quad (20)$$

for $i = 1, 2, 3, 4$ and $j = 1, 2$.

The proof for this theorem follows that in the original reference.

4. SIMULATION RESULTS AND DISCUSSION

In this section, the discussion on the simulation approach, the results obtained and the inferences are given. A typical set of inputs and premise variables are shown in Fig. 1.

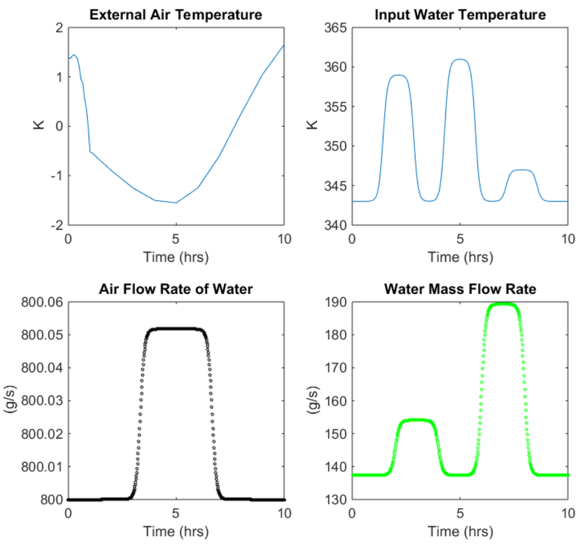

Fig. 1. Typical inputs and premise variables

All the simulations were run over a 10 hour period with a sampling interval of 1 minute. It is to be noted that over a 10 hour period, there would be no noticeable change in the heat transfer coefficient $U_A(t)$. However, for illustration purposes it has been shown to vary over this short time period. The results are expected to hold with appropriate time scaling. All Simulations were carried out in MATLAB with the Yalmip toolbox in conjunction with the LMILab solver from Mathworks.

As discussed in the previous section, the parameters whose values were fixed are given in table 1. For the simulation, the measurement noise added was a zero mean Gaussian noise with a standard deviation of $1K$. The sector bounds considered for the premise variables and the time varying parameters for the simulation are given by: $U_A(t) \in [27, \ 324] \ J/K$, $q_w(t) \in [80, \ 1200] \ g/s$ and $q_a(t) \in [200, \ 1500] \ g/s$.

Table 1. Simulation parameters

Parameters	$\hat{\theta} = \hat{U}_A$	$\hat{\theta} = \hat{q}_w$	$\hat{\theta} = \hat{q}_a$
η_0	10^{-4}	10^{-7}	5×10^{-5}
ρ	10^5	10^7	10^6
γ_θ	0.01	100	100
$\gamma_{\dot{\theta}}$	0.01	10	10
γ_x, γ_u	0.01	1	1
γ_ν	0.01	1	1

4.1 Results

Heat transfer coefficient estimation The state and the parameter estimation results for the heat transfer coefficient estimate problem is given in Fig. 2. The error characteristics are summarized in table 2. The mean and standard deviation values given are for the absolute error over the entire simulation period. The fouling was simulated through a part of a cosine wave as suggested in Guomundsson (2008) and follows closely to the physical characteristics, except the time duration over which it occurs. This simulation however is expected to follow when the appropriate corrections are made to account for slow varying process.

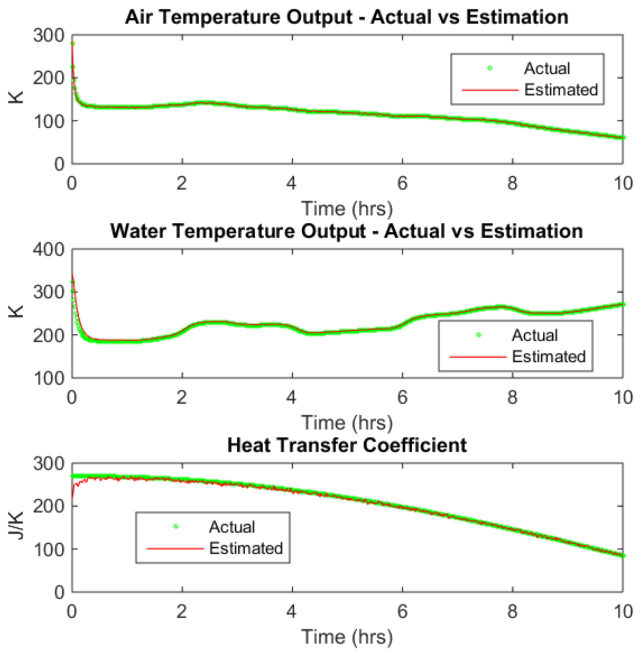

Fig. 2. State and parameter estimates: U_A estimation

Table 2. Simulation Results: $U_A(t)$ Estimation

Error	Mean (%)	Standard Deviation (%)
$\lvert e_{x_1} \rvert$	2.4	2
$\lvert e_{x_2} \rvert$	1.4	0.4
$\lvert e_\theta \rvert$	1.1	1.2

Air mass flow rate estimation The air flow rate estimation results along with the corresponding state estimation are given in Fig 3. The estimation errors' mean and standard deviation are given in table 3. As with the previous case these figures are for a simulation over the 10 hour period.

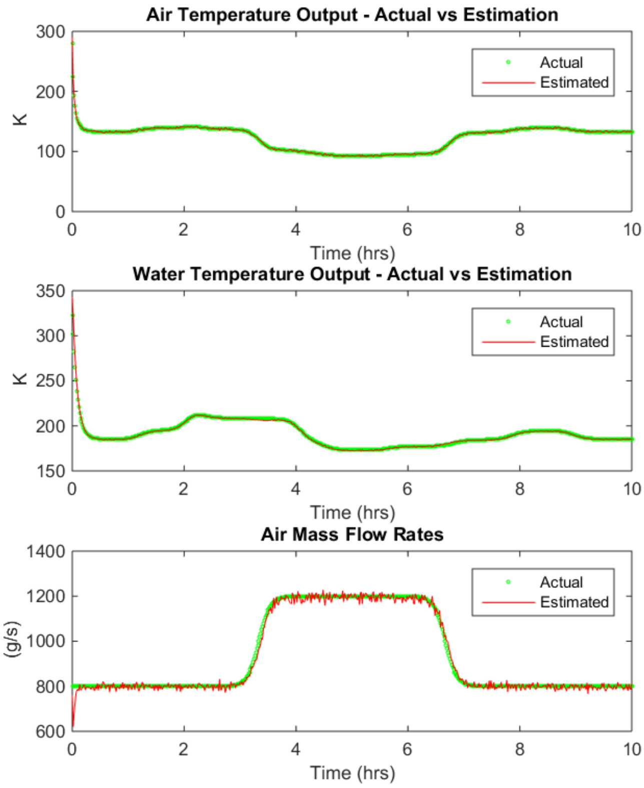

Fig. 3. State and parameter estimates: q_a estimation

Table 3. Simulation Results: $q_a(t)$ Estimation

Error	Mean (%)	Standard Deviation (%)
$\lvert e_{x_1} \rvert$	0.7	0.32
$\lvert e_{x_2} \rvert$	0.57	0.52
$\lvert e_\theta \rvert$	1.3	1.5

Water mass flow rate estimation The estimated water mass flow rate along with the estimated states are given in Fig 4. The mean and standard deviation of the estimation errors for the entire simulation duration is given in table 4. It can be seen that there is considerable error in the parameter estimation in this case. Part of the reason is due to the initial period of oscillations, but also due to model characteristics, further details of which follow in the discussions.

Table 4. Simulation Results: $q_w(t)$ Estimation

Error	Mean (%)	Standard Deviation (%)
$\lvert e_{x_1} \rvert$	0.22	0.64
$\lvert e_{x_2} \rvert$	0.32	1.04
$\lvert e_\theta \rvert$	17	25

4.2 Discussion

The results presented above illustrate the feasibility of parameter estimation on a T-S model to multiple practical scenarios. The simulation lent some observations, which are discussed below.

The simulation results illustrated that the condition $K_{ij}/\eta_{ij} > \rho$ is not sufficient. During the initial simulation period or during sudden changes in the parameter to be estimated, the error $y(t) - \hat{y}(t)$ is significantly high and hence

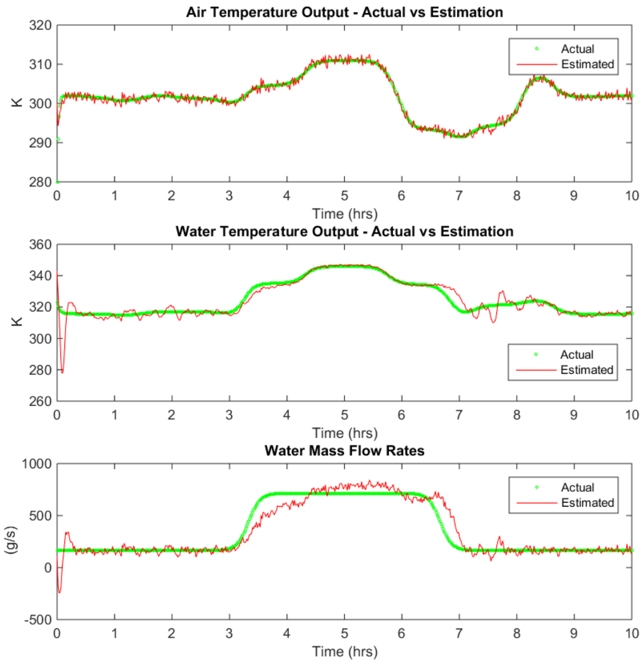

Fig. 4. State and parameter estimates: q_w estimation

a high value of K_{ij} leads to significant oscillations. This oscillations could be damped too slowly for a meaningful response if the ratio (and hence K_{ij}) is too high. This phenomenon could be seen partly in the simulation results in Fig. 3 and Fig. 4. Given the time varying nature of the residue, an appropriate observer will have a K_{ij} value that would be adapted with time and residue amplitude.

The simulations were performed with the unit of *gram* for mass since kg/s introduced stiffness in the mass flow rate estimation models. Such unit changes or using a stiff system compatible *odesolver* may help in many cases. A discrete-time observer model can help alleviate this problem. However, the results in the reference were derived for the continuous-time models and do not directly extend to discrete time.

It is apparent from the results that the estimate of $q_w(t)$ is the least convincing of the three. This could be understood by a simple local algebraic observability analysis (see for example, Sedoglavic (2001)). From (3)-(4), the $U_A(t)$ and $q_a(t)$ estimates can be written as (dropping (t) for simplicity and considering $y = x_1$),

$$U_A = \frac{\dot{y} - \alpha_1 q_a(d-y)}{\alpha_{2a}(u_1 + x_2 - d - y)}$$

$$q_a = \frac{\dot{y} - \alpha_{2a} U_A(u_1 + x_2 - d - y)}{\alpha_1(d-y)}$$

They depend only on $\dot{y}$, and the other unknown x_2 can be obtained from (4) without the need for $\ddot{y}$. However, q_w is given by,

$$q_w = \frac{\dot{x}_2 + \alpha_{2w} U_A(u_1 + x_2 - d - y)}{\alpha_3(u_1 - x_2)} \tag{21}$$

The dependence on $\dot{x}_2$ directly indicates the need for $\ddot{y}$ to compute q_w and hence explains the observation of noise and delay in the estimate of q_w. One possible approach to avoid this could be to design the parameter estimation observer with a filtering mechanism inbuilt to minimize the effect of the measurement noise.

T-S model based observers have been used for residue generation for fault detection, isolation and estimation purposes (see for instance, Ichalal et al. (2014)). The results in this paper suggests that parameter estimation observers could be used in fault detection in heat exchangers.

5. CONCLUDING REMARKS

In this paper, the nonlinear time varying parameter estimation approach in the literature has been adapted for multiple applications in a heat exchanger model. Detailed discussions on the extension and implementation of the existing result in a simulation were given. The simulation results show the promise of this approach. The main future works would be in the direction of those discussed in Sec. 4.2, namely, extending to discrete-time model, adaptive observer gains and enabling filtered measurement to compute residues. From the application perspective, the system model would be expanded to include rooms and other AHU component models.

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