Design of observers for Takagi-Sugeno fuzzy models for Fault Detection and Isolation

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Goal

- To develop an observer design method for Takagi-Sugeno models subjected to unknown inputs
- To use the proposed observer in a bank of observers for fault detection and isolation

Motivations

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Context

- To take into consideration the complexity of the system in the whole operating range (nonlinear models are needed)
- Observer design problem for generic nonlinear models is very delicate

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Proposed strategy

- Multiple model representation of the nonlinear system
- Design an unknown input observer on the basis of that model
- Convergence conditions are obtained using the Lyapunov method
- Conditions are given under a LMI form

Outline

1. Multiple model approach for modeling
2. Unknown input observer design
   - Structure of the unknown input observer
   - Unknown input observer existence conditions: main result
3. Model of an automatic steering
4. Fault detection and isolation
5. Conclusions

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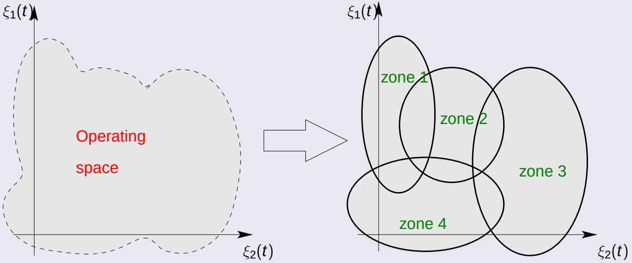
Multiple model approach for modeling

- Operating range decomposition into several local zones.
- A local model represents the behavior of the system in each zone.
- The overall behavior of the system is obtained by the aggregation of the sub-models with adequate weighting functions.



Nonlinear system — Multiple model representation

Why using a multiple model?

- Appropriate tool for modelling complex systems (e.g. black box modelling)
- Tools for linear systems can partially be extended to nonlinear systems
- Specific analysis of the system nonlinearity is avoided

Multiple model approach

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Takagi-Sugeno model

The considered model is described by the following equations

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{N} \mu_i(\xi(t)) \left(A_i x(t) + B_i u(t) + R_i \bar{u}(t) + D_i\right) \\ y(t) = C x(t) \end{cases}$$

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- Interpolation mechanism :

$$\sum_{i=1}^{N} \mu_i(\xi(t)) = 1 \text{ and } 0 \leq \mu_i(\xi(t)) \leq 1, \forall t, \forall i \in \{1, ..., N\}$$

- The premise variable $\xi(t)$ is assumed to be measurable (input $u(t)$ or output $y(t)$).

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- Identification approach
  - Choice of premise variables
  - Choice of the number of modalities of each premise variable
  - Choice of the structure of the local models
  - Parameter identification

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- Identification approach
  - Choice of premise variables
  - Choice of the number of modalities of each premise variable
  - Choice of the structure of the local models
  - Parameter identification
- Transformation of a nonlinear model into a multiple model

$$\left\{ \begin{array}{l} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = Cx(t) \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \dot{x}(t) = \sum_{i=1}^{N} \mu_i(\xi(t))\big(A_i x(t) + B_i u(t) + D_i\big) \\ y(t) = \sum_{i=1}^{N} \mu_i(\xi(t)) C_i x(t) \end{array} \right.$$

  - Exact representation: sector nonlinearity approach
    (the premise variables depend on the state of the system)
  - Linearization of the model around different operating points

Unknown input observer design

Structure of the unknown input observer

Model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{N} \mu_i(\xi) \left(A_i x + B_i u + R_i \bar{u} + D_i \right) \\ y = Cx \end{cases}$$

Structure of the unknown input observer

Model

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Assumption: $\bar{u}(t)$ is bounded

$\|\bar{u}\| < \rho$ where ρ is a positive scalar

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Observer

$$\begin{cases} \dot{\hat{x}} = \sum_{i=1}^{N} \mu_i\left(\xi\right)\left(A_i \hat{x} + B_i u + R_i \hat{\bar{u}}_i + D_i + G_i(y - C\hat{x})\right) \\ \hat{\bar{u}}_i = \gamma W_i\left(y - C\hat{x}\right) \\ \hat{\bar{u}} = \sum_{i=1}^{N} \mu_i\left(\xi\right)\hat{\bar{u}}_i \end{cases}$$

Theorem

The unknown input observer estimates asymptotically with any desired degree of accuracy $\varepsilon > 0$, the state of the T-S model, if there exists symmetric positive definite matrices P and Q and gain matrices G_i and W_i which satisfies the following constraints:

$$\begin{cases} (A_i - G_i C)^T P + P(A_i - G_i C) < -Q \\ W_i C = R_i^T P \end{cases} \quad \forall\, i \in [1, N]$$

The observer is then completely defined by choosing:

$$\gamma \geq \frac{1}{2} \left(\lambda_{\min}(P^{-1} Q)\, \lambda_{\min}(P)\, \varepsilon^2 \right)^{-1} \rho^2$$

and the unknown input estimation is given by

$$\hat{u} = \gamma \sum_{i=1}^{N} \mu_i(\xi)\, W_i \left(y - C\hat{x} \right)$$

Sketch of the proof

(i) Consider the following quadratic Lyapunov function:

$$V = e^T P e$$

(ii) Express the state estimation error $e(t) = x - \hat{x}$ and its derivative w.r.t. time

$$\dot{e} = \sum_{i=1}^{N} \mu_i(\xi) \Big((A_i - G_i C)e + R_i \bar{u} - \gamma R_i W_i C e \Big)$$

(iii) Express the derivative of the Lyapunov function using $\dot{e}$

$$\dot{V} = \sum_{i=1}^{N} \mu_i(\xi) \Big(e^T ((A_i - G_i C)^T P + P(A_i - G_i C))e + 2e^T P R_i \bar{u} - 2\gamma e^T P R_i W_i C e \Big)$$

(iv) Use some upper bound properties and guarantee that $\dot{V} < 0$

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Application to automatic steering of vehicle

Coupling model of longitudinal and lateral motions of a vehicle

$$\dot{u} = vr - fg + \frac{(fk_1 - k_2)}{M}u^2 + c_f\frac{v + ar}{Mu}\delta + \frac{T}{M}$$

$$\dot{v} = -ur - \frac{(c_f + c_r)}{Mu}v + \frac{(bc_r - ac_f)}{Mu}r + \frac{c_f\delta + T\delta}{M}$$

$$\dot{r} = \frac{(bc_r - ac_f)}{I_z u}v - \frac{(b^2c_r + a^2c_f)}{I_z u}r + \frac{aT\delta + ac_f\delta}{I_z}$$

Variables in red

u longitudinal velocity,
v lateral velocity,
r yaw rate,

δ steering angle,
T traction and/or braking force

Representation of the vehicle model by a T-S model

Model parameters

M	Mass of the full vehicle	$1480\ kg$
I_z	Moment of inertia	$2350\ kg.m^2$
g	Acceleration of gravity force	$9.81\ m/s^2$
f	Rotating friction coefficient	0.02
a	Distance from front axle to CG[a]	$1.05\ m$
b	Distance from rear axle to CG	$1.63\ m$
c_f	Cornering stiffness of front tyres	$135000\ N/rad$
c_r	Cornering stiffness of rear tyres	$95000\ N/rad$
k_1	Lift parameter from aerodynamics	$0.005\ Ns^2/m^2$
k_2	Drag parameter from aerodynamics	$0.41\ Ns^2/m^2$

[a] CG: Center of gravity

Representation of the vehicle model by a T-S model

General form of the nonlinear model

$$\dot{x} = F(x,\ w)$$
$$y = Cx$$

where $x = \begin{pmatrix} u \\ v \\ r \end{pmatrix}$, $w = \begin{pmatrix} \delta \\ T \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$,

Takagi-Sugeno model

$$\dot{x} = \sum_{i=1}^{N} \mu_i\left(y_1\right)\left(A_i x + B_i w + D_i\right)$$

$$A_i = \left.\frac{\partial F}{\partial x}\right|_{\substack{x=x^{(i)} \\ w=w^{(i)}}}, \quad B_i = \left.\frac{\partial F}{\partial w}\right|_{\substack{x=x^{(i)} \\ w=w^{(i)}}}, \quad D_i = F(x^{(i)},\ w^{(i)}) - A_i x^{(i)} - B_i w^{(i)}$$

- Membership functions $\mu_i\left(y_1\right)$ are chosen as triangular functions
- The number N of “local models” is chosen by the user (here $N = 3$)
- The operating points around which the linearization is done are optimized.

Takagi-Sugeno model

$$A_i = \begin{pmatrix} A_{11i} & A_{12i} & A_{13i} \\ A_{21i} & A_{22i} & A_{23i} \\ A_{31i} & A_{32i} & A_{33i} \end{pmatrix}$$

$$A_{11i} = \frac{2\left(fk_1 - k_2\right) u_i}{M} - \frac{c_f\left(v_i + ar_i\right)}{Mu_i^2}\delta_i, \quad A_{12i} = r_i + \frac{c_f \delta_i}{Mu_i}, \quad A_{13i} = v_i + \frac{ac_f \delta_i}{Mu_i}$$

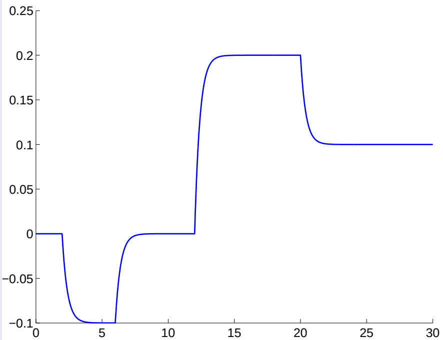
$$A_{21i} = \frac{\left(c_f + c_r\right)}{Mu_i^2} v_i - \frac{\left(bc_r - ac_f\right)}{Mu_i^2} r_i - r_i, \quad A_{22i} = -\frac{\left(c_f + c_r\right)}{Mu_i}, \quad A_{23i} = \frac{\left(bc_r - ac_f\right)}{Mu_i} - u_i$$

$$A_{31i} = \frac{\left(b^2 c_r + a^2 c_f\right)}{I_z u_i^2} r_i - \frac{\left(bc_r - ac_f\right)}{I_z u_i^2} v_i, \quad A_{32i} = \frac{\left(bc_r - ac_f\right)}{I_z u_i}, \quad A_{33i} = -\frac{\left(b^2 c_r + a^2 c_f\right)}{I_z u_i}$$

$$B_i = \begin{pmatrix} c_f \frac{v_i + ar_i}{Mu_i} & \frac{1}{M} \\ \frac{c_f + T_i}{M} & \frac{\delta_i}{M} \\ \frac{aT_i + ac_f}{I_z} & \frac{a\delta_i}{I_z} \end{pmatrix}, \quad D_i = F\left(x_i,\ \delta_i,\ T_i\right) - A_i x_i - B_i \begin{bmatrix} \delta_i \\ T_i \end{bmatrix}$$

Representation of the vehicle model by a T-S model

Takagi-Sugeno model validation



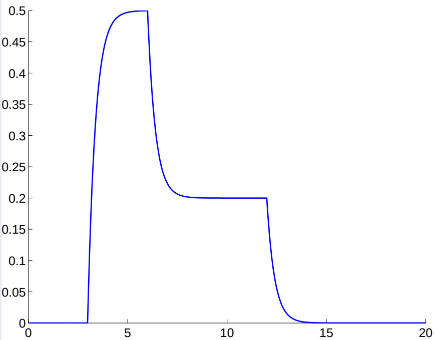


Figure: δ: steering angle T: traction force

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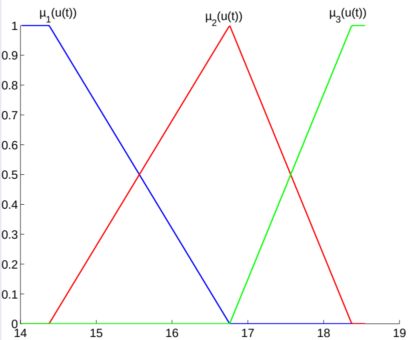

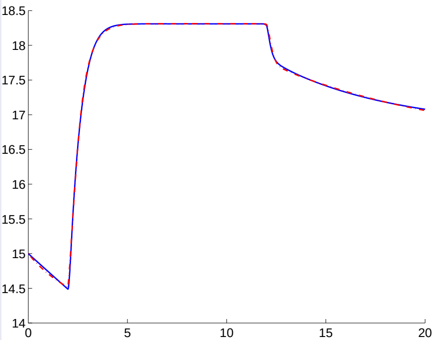

Figure: Membership functions

u issued from nonlinear and TS models

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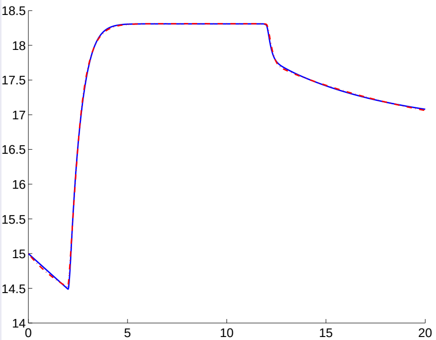


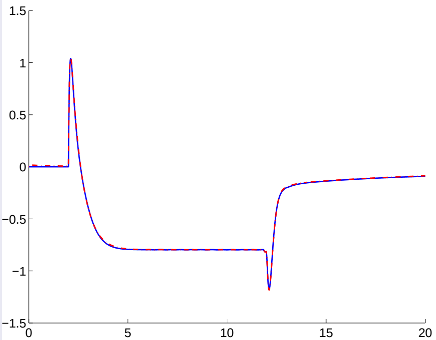


Figure: v issued from nonlinear and TS models r issued from nonlinear and TS models

Fault detection and isolation

System model with additive faults

$$\dot{x} = F(x,\ w) + Rm$$
$$y = Cx + \eta$$

$$\text{with } m = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix},\quad R = \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 0 \end{pmatrix},\quad \eta \sim \mathcal{N}(0, V)$$

m_1 is a piece-wise constant signal with random amplitude.
m_2 is a sinusoïdal signal

Fault detection and isolation

Proposed unknow input observer

$$\begin{cases} \dot{\hat{x}} = \sum_{i=1}^{3} \mu_i(\xi) \left(A_i \hat{x} + B_i u + R\hat{m}_i + D_i + G_i(y - C\hat{x}) \right) \\ \hat{m}_i = \gamma W_i \left(y - C\hat{x} \right) \\ \hat{m} = \sum_{i=1}^{N} \mu_i(\xi) \hat{m}_i \end{cases}$$

Observer parameters

$$G_1 = \begin{pmatrix} 9.22 & -3.88 \\ 0.45 & -1.02 \\ 22.51 & -11.92 \end{pmatrix}, \quad G_2 = \begin{pmatrix} 10.78 & -4.10 \\ 6.64 & 0.55 \\ 27.38 & -16.19 \end{pmatrix}, \quad G_3 = \begin{pmatrix} 8.49 & -4.83 \\ 4.40 & 1.36 \\ 20.27 & -17.07 \end{pmatrix}$$

$$\gamma = 78.12, \qquad W_1 = W_2 = W_3 = \begin{pmatrix} 34.14 & 0 \\ 0 & -10 \end{pmatrix}$$

State estimates

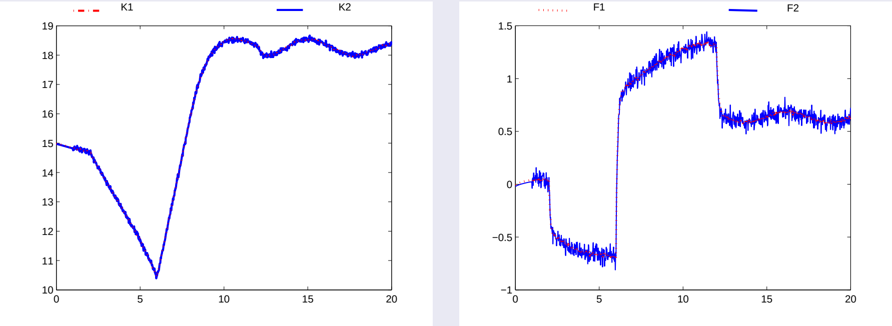
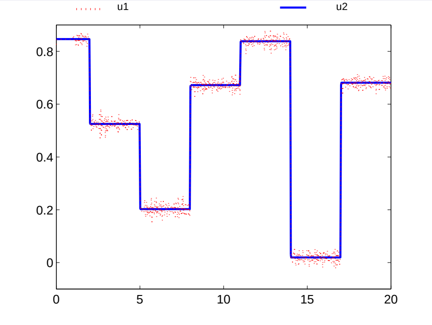


Figure: longitudinal velocity u and lateral velocity v and their estimates

Unknown input estimates



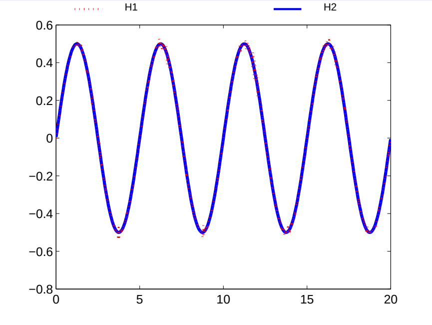


Figure: fault m_1 and its estimate   fault m_2 and its estimate

Conclusions

- A method for the estimation of the state and the unknown input of a nonlinear system has been proposed
- The proposed method uses a Takagi-Sugeno model to represent the behaviour of the nonlinear system
- Feasibility of sensor fault detection on a complex nonlinear model has been shown

Conclusions & Prospects

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- The proposed method uses a Takagi-Sugeno model to represent the behaviour of the nonlinear system
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Futures prospects

- On a theoretical point of view
  - Try to use another Lyapunov function in order to enlarge the stability region
  - More precise analysis of the noise influence (particularly on the unknown input estimation)
- On a practical point of view
  - Apply the proposed method on “real data”
  - Implement the method on a vehicule

Thank you!