Observer design for nonlinear systems described by multiple models

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Motivations

Context

- State estimation can be employed as a source of redundancy for fault diagnosis
- Observer design problem for generic nonlinear models is very delicate
- To take into consideration the complexity of the system in the whole operating range (nonlinear models are needed)

Goal

- State estimation of a nonlinear system represented by a multiple model
- Extension of our previous work to improve the dynamic performances of the observer

Proposed strategy

- Multiple model representation of the nonlinear system
- Convergence conditions are obtained using the Lyapunov method
- Conditions are given under a LMI form

Outline

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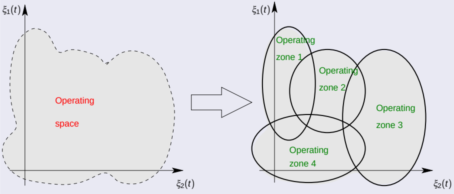
Introduction – philosophy

Basis of multiple model approach: divide and conquer



Nonlinear system   Multiple model representation

Multiple model =interpolation of a set of linear submodels

- Appropriate tool for modelling complex systems
- Specific analysis of the system nonlinearity is avoided
- Tools for linear systems can partially be extended to nonlinear systems

Introduction – philosophy

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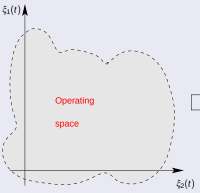
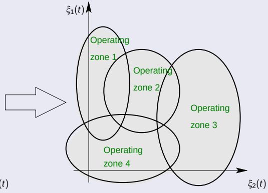


Nonlinear system



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How the submodels can be interconnected?

Classic structure	Proposed structure
Takagi-Sugeno multiple model	Decoupled multiple model

Classic structure — Takagi-Sugeno multiple model

- Common state vector for all submodels
- Dimension of the submodels must be identical (homogeneous)

Proposed structure — Decoupled multiple model

- A different state vector for each submodel
- Dimension of the submodels may be different (heterogeneous)

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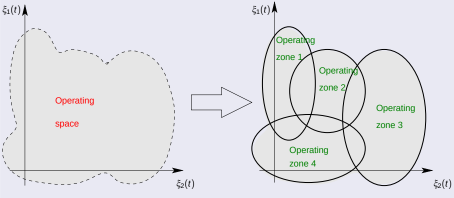

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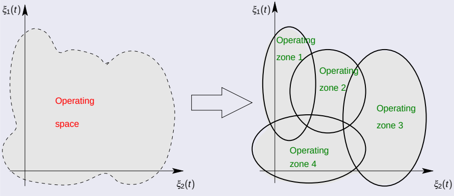

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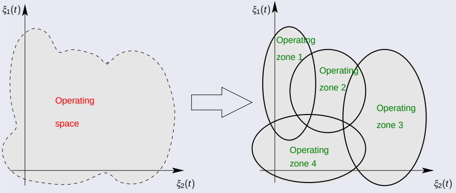

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Decoupled multiple model structure

Employed structure

Decoupled multiple model: multiple model with local state vectors

Collection of submodels

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u(t) \\ y_i(t) = C_i x_i(t) \end{cases}$$

Comments

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Interpolation mechanism

$$y(t) = \sum_{i=1}^{L} \mu_i(\xi(t)) y_i(t)$$

$$\sum_{i=1}^{L} \mu_i(\xi(t)) = 1 \quad \text{and} \quad 0 \leq \mu_i(\xi(t)) \leq 1 \quad \forall i \in 1, \ldots, L \quad \forall t$$

$\xi(t)$: decision variable $\qquad$ $\mu_i(\xi(t))$: weighting functions

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- The multiple model output is given by a weighted sum of the submodel outputs

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Comments

- The multiple model output is given by a weighted sum of the submodel outputs
- Dimension of the submodels can be different !
- This multiple model offers a good flexibility and generality in the modelling stage

Preliminaries and notations

Augmented form of the multiple model

$$\dot{x}_i(t) = A_i x_i(t) + B_i u(t) \quad x_i \in \mathbb{R}^{n_i}$$

$$y_i(t) = C_i x_i(t) \qquad \Leftrightarrow$$

$$y(t) = \sum_{i=1}^{L} \mu_i(\xi(t)) y_i(t)$$

Notations

$$x(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_i(t) \\ \vdots \\ x_L(t) \end{bmatrix} \in \mathbb{R}^n \quad \tilde{A} = \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & A_i & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & A_L \end{bmatrix} \quad \tilde{B} = \begin{bmatrix} B_1 \\ \vdots \\ B_i \\ \vdots \\ B_L \end{bmatrix}$$

$$\tilde{C}(t) = \begin{bmatrix} \mu_1(t) C_1 & \ldots & \mu_i(t) C_i & \ldots & \mu_L(t) C_L \end{bmatrix} \quad \mu(t) = \mu(\xi(t))$$

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$$\dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t) \quad x \in \mathbb{R}^n,\ n = \sum_{i=1}^{L} n_i$$

$$y(t) = \tilde{C}(t)x(t)$$

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State estimation

Observer structure

- Multiple model representation of a nonlinear system

$$\begin{aligned} \dot{x}(t) &= \tilde{A}x(t) + \tilde{B}u(t) \\ y(t) &= \tilde{C}(t)x(t) \end{aligned}$$

- Extension of some LTI results to decoupled multiple models
- Proportional gain observer: $\tilde{K}$ is the observer gain

$$\begin{aligned} \dot{\hat{x}}(t) &= \tilde{A}\hat{x}(t) + \tilde{B}u(t) - \tilde{K}(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= \tilde{C}(t)\hat{x}(t) \end{aligned}$$

Goal

- Determining the gain matrix $\tilde{K}$ such that the estimation error converges toward zero
- Ensuring the observer stability for any combination between the submodels and for any initial conditions
- Dynamic performances of the estimation error must be ensured (e.g. exponential convergence)

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State estimation

Estimation error analysis

- Estimation error:

$$e(t) = x(t) - \hat{x}(t)$$

- Estimation error dynamics:

$$\dot{e}(t) = (\tilde{A} + \tilde{K}\tilde{C}(t))e(t)$$

- Matrix $\tilde{C}(t) = \begin{bmatrix} \mu_1(t)C_1 & \dots & \mu_i(t)C_i & \dots & \mu_L(t)C_L \end{bmatrix}$ can be rewritten as

$$\tilde{C}(t) = \sum_{i=1}^{L} \mu_i(t)\tilde{C}_i \quad \text{where} \quad \tilde{C}_i = \begin{bmatrix} 0 & \dots & C_i & \dots & 0 \end{bmatrix}$$

- Finally

$$\dot{e}(t) = \sum_{i=1}^{L} \mu_i(t)(\tilde{A} + \tilde{K}\tilde{C}_i)e(t) = A_{obs}(t)e(t)$$

- $A_{obs}(t)$ is a time-varying matrix
- Interaction between submodels must be taken into consideration in the observer design!

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Observer design: first strategy

Theorem

The exponential convergence of the estimation error is guaranteed if there exists a symmetric and positive definite matrix P, a matrix G and a positive scalar α such that:

$$(\tilde{A}+\alpha I)^T P + P(\tilde{A}+\alpha I) + (G\tilde{C}_i)^T + G\tilde{C}_i < 0, i = 1...L$$

where α is the decay rate. The observer gain is given by $\tilde{K} = P^{-1}G$.

Comments

1. The choice of the decay rate α is limited because the observability property of matrices $\tilde{A}$ and $\tilde{C}_i$ is not respected.

$$\tilde{A} = \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & A_i & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & A_L \end{bmatrix} \quad \text{and} \quad \tilde{C}_i = [0 \quad \dots \quad C_i \quad \dots \quad 0]$$

2. Eigenvalues assignment is limited !
3. New observer design conditions must be established

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The exponential convergence of the estimation error is guaranteed if there exists a symmetric and positive definite matrix P, a matrix G and a positive scalar α such that:

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Observer design: second strategy

New strategy

- Introduce a new matrix as follows

$$\tilde{C}_0 = \frac{1}{L}\sum_{i=1}^{L}\tilde{C}_i = \frac{1}{L}\begin{bmatrix} C_1 & C_2 & \cdots & C_L \end{bmatrix}.$$

- Estimation error dynamics

$$\begin{aligned}
\dot{e}(t) &= (\tilde{A}+\tilde{K}\tilde{C}(t))e(t) = \sum_{i=1}^{L}\mu_i(t)(\tilde{A}+\tilde{K}\tilde{C}_i)e(t) \\
&= (\tilde{A}+\tilde{K}\sum_{i=1}^{L}\mu_i(t)(\tilde{C}_i \underbrace{+\tilde{C}_0-\tilde{C}_0}_{=0}))e(t) \\
&= (\tilde{A}+\tilde{K}\tilde{C}_0+\tilde{K}\sum_{i=1}^{L}\mu_i(t)(\tilde{C}_i-\tilde{C}_0))e(t) \\
&= (\tilde{A}+\tilde{K}\tilde{C}_0+\sum_{i=1}^{L}\mu_i(t)\tilde{K}\bar{C}_i)e(t) = A_{obs}(t)e(t) \quad \text{where} \quad \bar{C}_i = \tilde{C}_i-\tilde{C}_0
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- $A_{obs}(t)e(t)$ is a constant matrix with an artificial norm-bounded uncertainties due to $\mu_i(t)$

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\end{aligned}$$

constant term

- $A_{obs}(t)e(t)$ is a constant matrix with an artificial norm-bounded uncertainties due to $\mu_i(t)$

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constant term

time-varying term

- $A_{obs}(t)e(t)$ is a constant matrix with an artificial norm-bounded uncertainties due to $\mu_i(t)$

Observer design: new strategy

New proposed theorem

The exponential convergence towards zero of the estimation error is guaranteed if there exists symmetric and positive definite matrices P and Q, a matrix G and a positive scalar α such that:

$$\begin{bmatrix} P(\tilde{A}+\alpha \mathrm{I})+(\tilde{A}+\alpha \mathrm{I})^T P + G\tilde{C}_0 + (G\tilde{C}_0)^T & \bar{G} & \bar{\bar{C}}^T Q \\ \bar{G}^T & -Q & 0 \\ Q\bar{\bar{C}} & 0 & -Q \end{bmatrix} < 0$$

where $\bar{G} = [G \cdots G \cdots G]$ $\quad \bar{\bar{C}} = [\bar{C}_1^T \cdots \bar{C}_i^T \cdots \bar{C}_L^T]^T$ $\quad \bar{C}_i = \tilde{C}_i - \tilde{C}_0$.

where α is the decay rate and the observer gain is given by $\tilde{K} = P^{-1}G$.

1. Consider $V(t) = e^T(t)Pe(t)$ as Lyapunov function
2. Ensuring the following inequality: $\exists \alpha > 0 : \dot{V}(t) + 2\alpha V(t) < 0$
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4. The observability property of matrices $\tilde{A}$ and $\tilde{C}_0$ is now well respected!!
5. Dynamic performances of the observer can be improved ! (decay rate α is not limited)

Observer design: new strategy

New proposed theorem

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Simulation example

Multiple model parameters

$L = 2$ submodels with different dimensions ($n_1 = 3$ and $n_2 = 2$), given by:

$$A_1 = \begin{bmatrix} -2.0 & 0.5 & 0.6 \\ -0.3 & -0.9 & -0.5 \\ -1.0 & 0.6 & -0.8 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.8 & -0.4 \\ 0.1 & -1.0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1.0 & 0.8 & 0.5 \end{bmatrix}^T, \qquad B_2 = \begin{bmatrix} -0.5 & 0.8 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.9 & -0.8 & -0.5 \\ -0.4 & 0.6 & 0.7 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} -0.8 & 0.6 \\ 0.4 & -0.7 \end{bmatrix}.$$

The weighting functions are

$$\mu_i(\xi(t)) = \eta_i(\xi(t)) / \sum_{j=1}^{L} \eta_j(\xi(t)) \quad \text{where} \quad \eta_i(\xi(t)) = \exp\left(-(\xi(t) - c_i)^2 / \sigma^2\right),$$

with $\sigma = 0.5$ and $c_1 = -0.25$ and $c_2 = 0.75$, $\xi(t)$ is the input signal $u(t) \in [-1, 1]$.

Simulation example

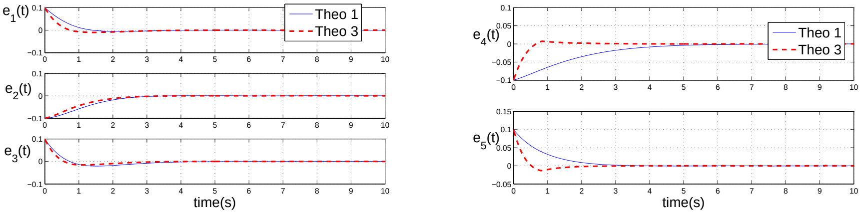


Figure: State estimation errors

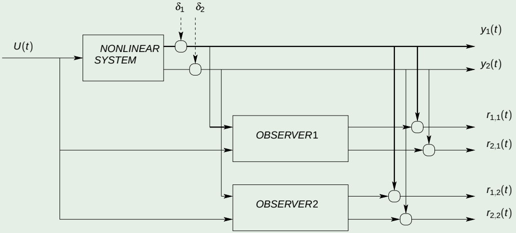
Comments

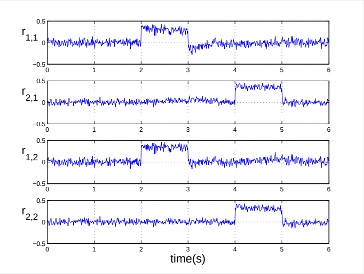
1. Solutions satisfying conditions of the first theorem **are not found** for a decay rate $\alpha > 0.8$
2. Solutions satisfying conditions of the new theorem **are found** for a decay rate $\alpha > 0.8$
3. Good dynamic performances are obtained using new conditions!

Simulation example

Application to sensor fault diagnosis: structuring the residual signals

- Dedicated Observer Scheme is employed for residual signal generation
- An incidence matrix is built
- Configuration of residual signals is used for FDI





	$r_{1,1}$	$r_{2,1}$	$r_{1,2}$	$r_{2,2}$
δ_1	?	?	1	0
δ_2	0	1	?	?

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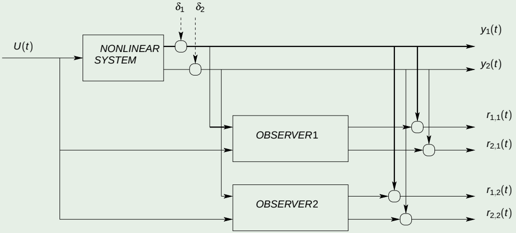

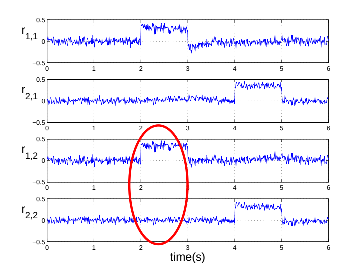

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$\Rightarrow$ Sensor fault on y_1

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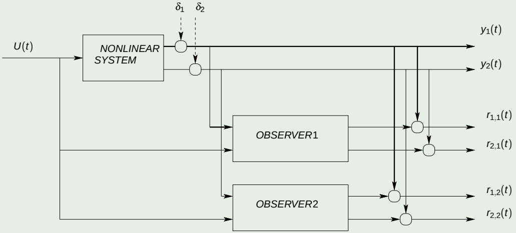

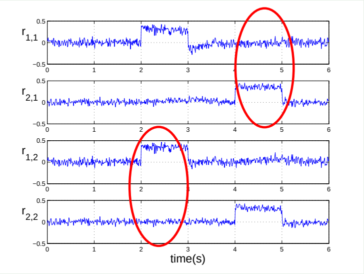

	$r_{1,1}$	$r_{2,1}$	$r_{1,2}$	$r_{2,2}$
δ_1	?	?	1	0
δ_2	0	1	?	?

$\Rightarrow$ Sensor fault on y_1

$\Rightarrow$ Sensor fault on y_2

Conclusions

Conclusions

- State estimation based on a decoupled multiple models is investigated
- Originality: the dimension of each submodel may be different (flexibility in the modelling stage can be provided)
- New convergence conditions for state estimation error are proposed
- Dynamic performances of the observer are improved in this way
- State estimation is employed as a source of redundancy for FDI

Thank you!
comments are welcome!