Fault tolerant control for nonlinear systems subject to different types of sensor faults

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Ideas underlying the contribution

- To compute different state estimates of the nonlinear system (represented by a Takagi-Sugeno model) using a bank of observers fed with different sets of measurements (here a DOS structure for sensor fault)
- To design residual generators able to detect and isolate sensor faults. These residuals help in computing a “confidence level” in the corresponding state estimate
- To design a fault tolerant control law as a weighted sum of state feedback laws ; the weights being indexed on the previous “confidence level” (magnitude of the residual)

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Outline

1. Takagi-Sugeno approach for modeling
2. Observer based state feedback control law
   - Redundant descriptor system approach
   - Relaxed stability conditions : Polya's theorem
3. Residual generation
4. Fault tolerant control design
5. Simulation examples
6. Conclusions

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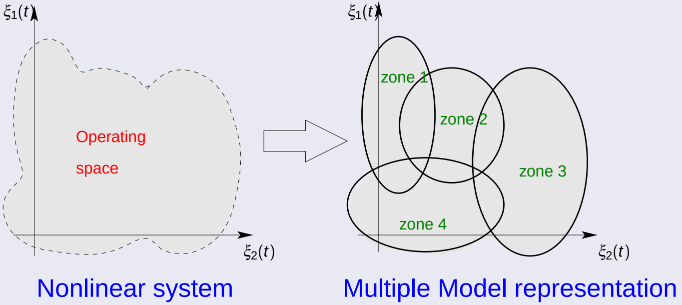
Takagi-Sugeno approach for modeling

Takagi-Sugeno principle

- Operating range decomposition in several local zones.
- A local model represents the behavior of the system in a specific zone.
- The overall behavior of the system is obtained by the aggregation of the sub-models with adequate weighting functions.



The main idea of Takagi-Sugeno approach

- Define local models $M_i, \quad i = 1..r$
- Define weighting functions $\mu_i(\xi), \quad 0 \leq \mu_i \leq 1$
- Define an agregation procedure : $M = \sum_{i=1}^{r} \mu_i(\xi) M_i$

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- Simple structure for modeling complex nonlinear systems.
- The specific study of the nonlinearities is not required.
- Possible extension of the theoretical LTI tools for nonlinear systems.

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The difficulties

- How many local models ?
- How to define the domain of influence of each local model ?
- On what variables may depend the weighting functions μ_i ?

Obtaining a Takagi-Sugeno model

- Identification approach
  - Choice of premise variables
  - Choice of the number of modalities of each premise variables
  - Choice of the structure of the local models
  - Parameter identification
- Transformation of an *a priori* known nonlinear model
  - Linearization around some "well-chosen" points

    Identification of the weighting function parameters to minimize the output error

  - Nonlinear sector approach

    Rewriting of the model in a compact subspace of the state space

$$\begin{cases} \dot{x}(t) = f(x(t),u(t)) \\ y(t) = h(x(t),u(t)) \end{cases} \Rightarrow \begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\big(A_i x(t) + B_i u(t)\big) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\big(C_i x(t) + D_i u(t)\big) \end{cases}$$

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Observer based state feedback control law for Takagi-Sugeno systems

T-S System

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) C_i x(t) \end{cases}$$

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Hypotheses

- The pairs (A_i, B_i) are controllable
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$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))) \\ \hat{y}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) C_i \hat{x}(t) \\ u(t) = -\sum_{i=1}^{r} \mu_i(\xi(t)) K_i \hat{x}(t) \end{cases}$$

PDC control law (Wang et al., 1996)

State estimation error

$$e(t) = x(t) - \hat{x}(t)$$

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Dynamics of the closed-loop system

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\xi(t))\mu_j(\xi(t))\left((A_i - B_iK_j)x(t) + B_iK_je(t)\right) \\ \dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\xi(t))\mu_j(\xi(t))\left(A_i - L_iC_j\right)e(t) \end{cases}$$

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Augmented state

$$x_a(t) = [x^T(t) \;\; e^T(t)]^T$$

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$$\dot{x}_a(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\xi(t))\mu_j(\xi(t)) \begin{pmatrix} A_i - B_i K_j & B_i K_j \\ 0 & A_i - L_i C_j \end{pmatrix} x_a(t)$$

Quadratic Lyapunov function

$$V(x_a(t)) = x_a^T(t) P x_a(t), \quad P = P^T \geq 0, \quad P = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix}$$

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Derivative of the Lyapunov function

$$\dot{V}(x_a(t)) = \dot{x}_a^T(t) P x_a(t) + x_a^T(t) P \dot{x}_a(t)$$

$$\dot{V}(x_a(t)) = x_a^T(t) \left(\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\xi(t)) \mu_j(\xi(t)) \Delta_{ij} \right) x_a(t)$$

$$\Delta_{ij} = \begin{pmatrix} A_i^T P_1 + P_1 A_i - K_j{}^T B_i^T P_1 - P_1 B_i K_j & P_1 B_i K_j \\ K_j{}^T B_i^T P_1 & A_l^T P_2 + P_2 A_l - C_j^T L_i{}^T P_2 - P_2 L_i C_j \end{pmatrix}$$

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Difficulties

$$\Delta_{ij} \leq 0 \Rightarrow \text{Bilinear Matrix Inequalities}$$

Difficult to solve as it corresponds to a non convex optimization problem!

Redundant descriptor system approach

Idea : to introduce a "virtual" dynamics for $u(t)$

$$0 \times \dot{u}(t) = -\sum_{i=1}^{r} \mu_i(\xi(t)) K_i \hat{x}(t) - u(t)$$

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New augmented state

$$\tilde{x}(t) = [x^T(t) \;\; e^T(t) \;\; u^T(t)]^T$$

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Augmented system

$$\begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{pmatrix} \dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\xi(t))\mu_j(\xi(t)) \begin{pmatrix} A_i & 0 & B_i \\ 0 & A_i - L_i C_j & 0 \\ -K_i & K_i & -I \end{pmatrix} \tilde{x}(t)$$

$$E\dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\xi(t))\mu_j(\xi(t)) \tilde{A}_{ij} \tilde{x}(t)$$

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Asymptotic stability

Consider the quadratic Lyapunov function

$$V(\tilde{x}(t)) = \tilde{x}^T(t)E^T P\tilde{x}(t), \quad E^T P = P^T E \geq 0, \quad P = \begin{pmatrix} P_1 & 0 & 0 \\ 0 & P_5 & 0 \\ 0 & 0 & P_9 \end{pmatrix}$$

Derivative of the Lyapunov function

$$\dot{V}(\tilde{x}(t)) = \dot{\tilde{x}}^T(t)E^T P\tilde{x}(t) + \tilde{x}^T(t)PE\dot{\tilde{x}}(t)$$

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Asymptotic stability conditions

The derivative of the Lyapunov function is negative provided $X_{ij} \leq 0$

$$X_{ij} = \begin{pmatrix} P_1 A_i + A_i^T P_1 & 0 & P_1 B_i - {F_i}^T \\ * & P_5 A_i + A_i^T P_5 - M_i C_j - C_j^T {M_i}^T & {F_i}^T \\ * & * & -2P_9 \end{pmatrix}$$

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Solution

The redundant descriptor system approach allows the asymptotic stability conditions to be expressed using LMI that can be easily solved.

Objective

Reduce the conservativeness of the LMI conditions by Polya's theorem

Principle

Let us consider the inequality

$$X_{\xi\xi} = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(t))\mu_j(\xi(t))X_{ij} < 0$$

Knowing that

$$\left(\sum_{i=1}^{r}\mu_i(\xi(t))\right)^p = \sum_{i=1}^{r}\mu_i(\xi(t)) = 1$$

where p is a positive integer, we obtain

$$\left(\sum_{i=1}^{r}\mu_i(\xi(t))\right)^p \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(t))\mu_j(\xi(t))X_{ij} < 0$$

Example

For example, choosing $p = 1$, and $r = 2$, we obtain an equivalent inequality

$$X_{\xi\xi} = \sum_{i_1=1}^{2} \sum_{i_2=1}^{2} \sum_{i_3=1}^{2} \mu_{i_1}\mu_{i_2}\mu_{i_3} X_{i_1 i_2} < 0$$

Consequently, the negativity of $X_{\xi\xi}$ is ensured if

$$\begin{aligned} X_{11} &< 0 \\ X_{22} &< 0 \\ X_{11}+X_{12}+X_{21} &< 0 \\ X_{22}+X_{21}+X_{12} &< 0 \end{aligned}$$

- Remark that the negativity of X_{12} and X_{21} is not required.

- Reduced conditions are obtained by increasing p
- Asymptotic necessary and sufficient conditions can be obtained by choosing $p \to \infty$ (Sala et al. 2007)

Theorem 1

The observer based control law ensures asymptotic stability of the system, if there exists symmetric and positive definite matrices P_1, P_5 and P_9 and gain matrices F_i and M_i such that the following constraints hold

$$\begin{array}{l} X_{ii} < 0, \;\; i = 1, \ldots, r \\ X_{ii} + X_{ji} + X_{ij} < 0, \;\; i, j = 1, \ldots, r, i \neq j \end{array}$$

where

$$X_{ij} = \begin{pmatrix} P_1 A_i + A_i^T P_1 & 0 & P_1 B_i - F_i{}^T \\ * & P_5 A_i + A_i^T P_5 - M_i C_j - C_j^T M_i{}^T & F_i{}^T \\ * & * & -2P_9 \end{pmatrix}$$

The gains of the observer based controller are derived from the following equations

$$K_i = P_9^{-1} F_i, \;\; L_i = P_5^{-1} M_i$$

Residual generation

Considered faulty system

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(C_i x(t) + G_i f(t)) \end{cases}$$

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Residual generator

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\left(A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))\right) \\ \hat{y}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) C_i \hat{x}(t) \\ r(t) = M(y(t) - \hat{y}(t)) \end{cases}$$

Fault tolerant control design

Description

- Use of an observer bank : the k^{th} observer is fed with the input of the system $u(t)$ and the k^{th} output $y_k(t)$ and produces the estimate $\hat{x}^k(t)$;
- The control signal $u(t)$ is a blending of the p observed state feedback controls ;

$$u(t) = -\sum_{j=1}^{r}\sum_{k=1}^{p} h_k(r(t))\mu_j(\xi(t))K_j^k\hat{x}^k(t)$$

- The blending is ensured by the functions $h_k(r(t))$, which are smooth nonlinear ones satisfying the convex sum property ;
- The design of such functions is based on the idea that if the k^{th} sensor is affected by a fault, the residual $r_k(t)$ is non zero then the function $h_k(r(t))$ must be close to zero in order to minimize the influence of $\hat{x}^k(t)$ affected by the k^{th} fault

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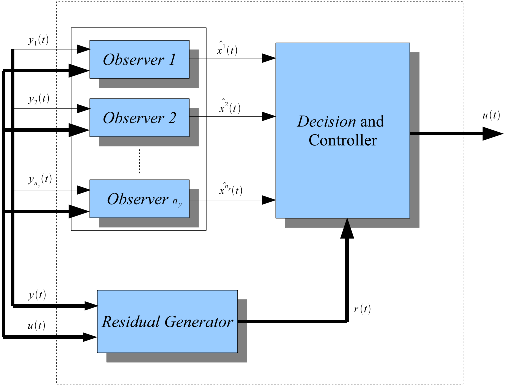
$$u(t) = -\sum_{j=1}^{r}\sum_{k=1}^{p} h_k(r(t))\mu_j(\xi(t))K_j^k\hat{x}^k(t)$$

- The blending is ensured by the functions $h_k(r(t))$, which are smooth nonlinear ones satisfying the convex sum property ;
- The design of such functions is based on the idea that if the k^{th} sensor is affected by a fault, the residual $r_k(t)$ is non zero then the function $h_k(r(t))$ must be close to zero in order to minimize the influence of $\hat{x}^k(t)$ affected by the k^{th} fault



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Closed loop system

$$\dot{x} = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{p} h_k(r)\mu_i(\xi)\mu_j(\xi)\left((A_i - B_i K_j^k)x + B_i K_j^k e^k\right)$$

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where C_j^k is the k^{th} row of the matrix C_j.

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Augmented system

$$(x_a^k)^T = [x^T \;\; (e^k)^T]$$

$$\dot{x}_a^k = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{p} h_k(r)\mu_i(\xi)\mu_j(\xi)\begin{pmatrix} A_i - B_i K_j^k & B_i K_j^k \\ 0 & A_i - L_i^k C_j^k \end{pmatrix} x_a^k$$

- The proposed algorithm of FTC is illustrated by an academic example. Let consider the nonlinear system represented by two submodels defined by

$$A_1 = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 1 & 2 & -4 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 1 \\ 5 \\ 0.5 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

- The weighting functions μ_i are defined as follows

$$\begin{cases} \mu_1(y(t)) = \dfrac{1 - \tanh(y_2(t))}{2} \\ \mu_2(y(t)) = 1 - \mu_1(y(t)) \end{cases}$$

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- Blending functions for the FTC law

$$\omega_k(r_k(t)) = \exp(-r_k^2(t)/\sigma_k)$$

$$h_k(r(t)) = \frac{\omega_k(r_k(t))}{\sum_{\ell=1}^{p} \omega_\ell(r_\ell(t))}$$

- Structure of the control law

$$u(t) = -\sum_{k=1}^{2}\sum_{j=1}^{2} h_k(r(t))\mu_j(\xi(t))K_j^k \hat{x}^k(t) + ref(t)$$

$ref(t)$ is a given reference signal.

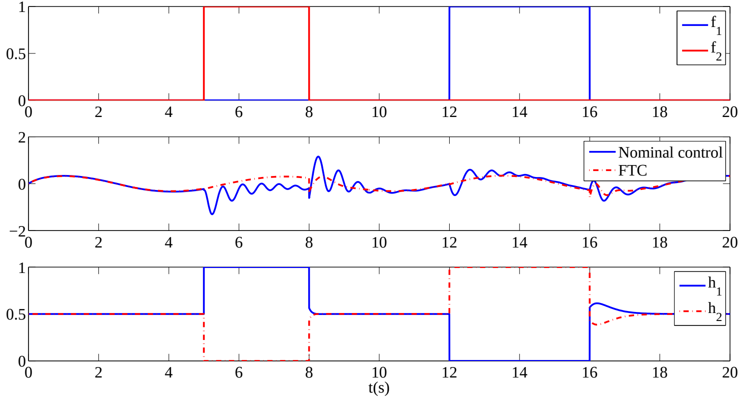


FIGURE: Fault and control signals

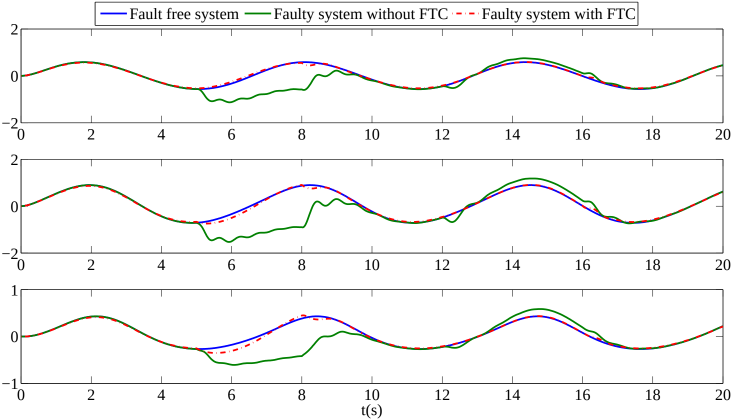
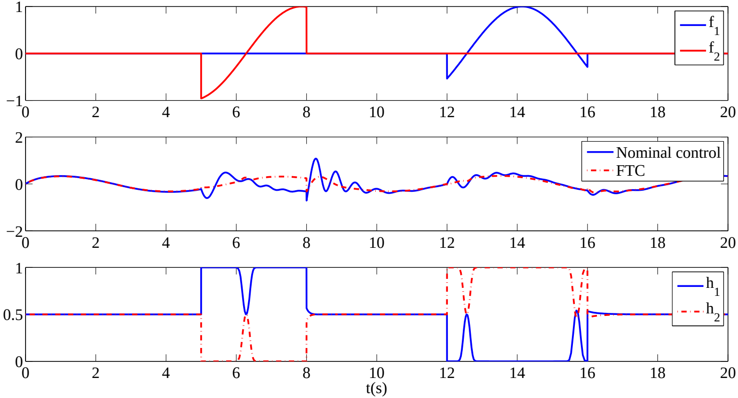


FIGURE: State comparison



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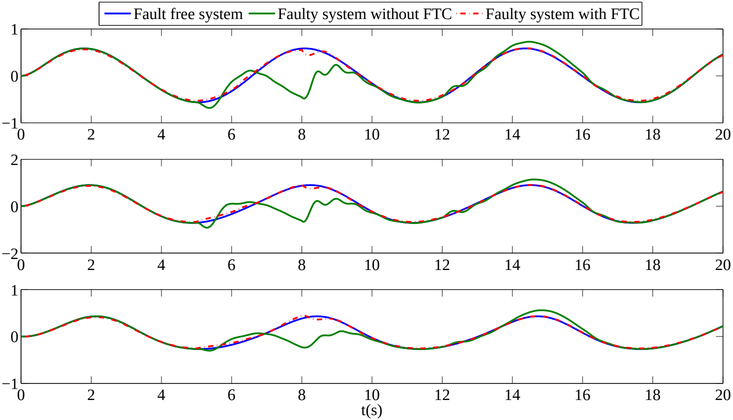

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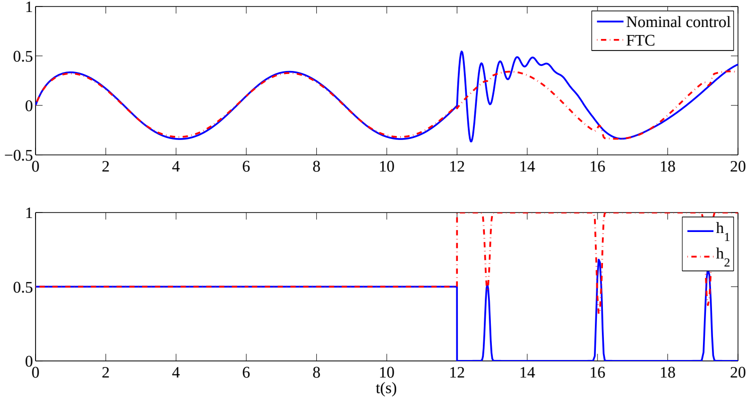

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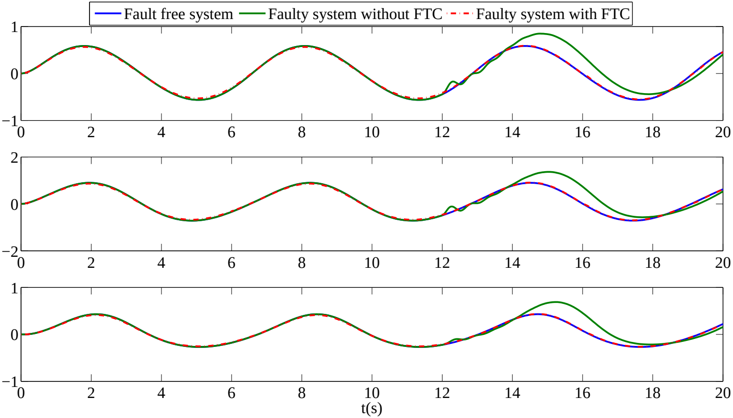

FIGURE: State comparison

Conclusions

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Perspectives

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- Extension to robust fault tolerant control (disturbances and modeling uncertainties).

Thank you for attention !

Get in touch

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