Fault tolerant tracking control for continuous Takagi-Sugeno systems with time varying faults

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Nancy, France

Mediterranean Conference on Control and Automation, June 20-23, 2011
Aquis Corfu Holiday Palace, Corfu, Greece

Objective of diagnosis and fault tolerant control

- To detect and isolate the (actuator) fault and estimate its magnitude (diagnosis)
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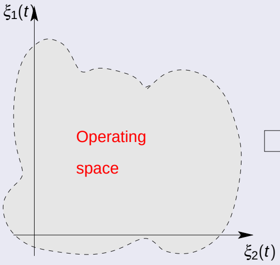
Proposed strategy

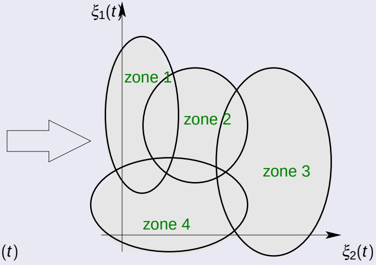
- Takagi-Sugeno representation of nonlinear systems
- Observer-based fault tolerant control design
- Extension of the existing results on linear systems
- Consideration of an *a priori* model of the fault

Outline

- Operating range decomposition in several local zones.
- A local model represents the behavior of the system in a specific zone.
- The overall behavior of the system is obtained by the aggregation of the sub-models with adequate weighting functions.



Nonlinear system



Multiple Model representation

The main idea of Takagi-Sugeno approach

- Define local models $M_i, \quad i = 1..r$
- Define weighting functions $\mu_i(\xi), \quad 0 \leq \mu_i \leq 1$
- Define an agregation procedure : $M = \sum \mu_i(\xi) M_i$

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- Simple structure for modeling complex nonlinear systems.
- The specific study of the nonlinearities is not required.
- Possible extension of the theoretical LTI tools for nonlinear systems.

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The difficulties

- How many local models ?
- How to define the domain of influence of each local model ?
- On what variables may depend the weighting functions μ_i ?

Obtaining a Takagi-Sugeno model

- Identification approach
  - Choice of premise variables
  - Choice of the number of modalities of each premise variables
  - Choice of the structure of the local models
  - Parameter identification
- Transformation of an *a priori* known nonlinear model
  - Linearization around some "well-chosen" points

    Identification of the weighting function parameters to minimize the output error
  - Nonlinear sector approach

    Rewriting of the model in a compact subspace of the state space

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = h(x(t), u(t)) \end{cases} \Rightarrow \begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\big(A_i x(t) + B_i u(t)\big) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\big(C_i x(t) + D_i u(t)\big) \end{cases}$$

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Reference model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(C_i x(t) + D_i u(t)) \end{cases}$$

- Interpolation mechanism $\sum_{i=1}^{r} \mu_i(\xi(t)) = 1$ and $0 \leq \mu_i(\xi(t)) \leq 1, \forall t, \forall i \in \{1, ..., r\}$
- The premise variable $\xi(t)$ are measurable (like $u(t)$, $y(t)$).

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The faulty system

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x_f(t) + B_i u_f(t) + G_i f(t)) \\ y_f(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(C_i x_f(t) + D_i u_f(t) + W_i f(t)) \end{cases}$$

- $f(t)$ represents the fault vector (to be detected and accomodated).

Objective

The objective is, in the one hand, to estimate the actuator fault $f(t)$ and the state of the system $x(t)$ (diagnosis) and, in the other hand, to reconfigure the control law allowing the convergence of $x_f(t)$ to $x(t)$ (FTC).

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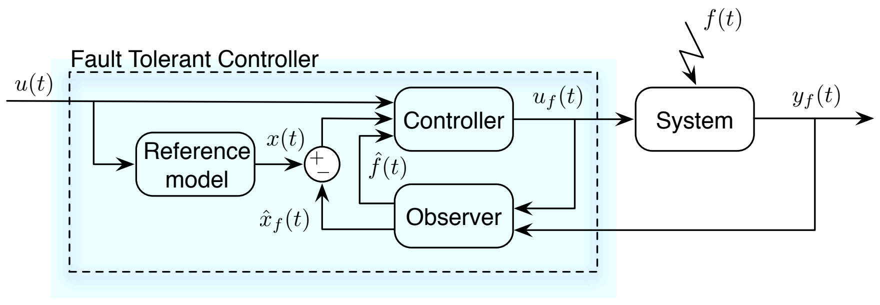
Fault tolerant control scheme



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PI Observer

$$\begin{cases} \dot{\hat{x}}_f(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i \hat{x}_f(t) + B_i u_f(t) + G_i \hat{f}(t) + H_i^1 (y_f(t) - \hat{y}_f(t)) \right) \\ \hat{y}_f(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(C_i \hat{x}_f(t) + D_i u_f(t) + W_i \hat{f}(t) \right) \\ \dot{\hat{f}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(H_i^2 (y_f(t) - \hat{y}_f(t)) - H_i^3 \hat{f}(t) \right) \end{cases}$$

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FTC law

$$u_f(t) = u(t) + \sum_{i=1}^{r} \mu_i(\xi(t)) \left(K_i (x(t) - \hat{x}_f(t)) - K_i^f \hat{f}(t) \right)$$

Exponential faults

$$f_i(t) = e^{\alpha_i t + \beta_i}, \text{ with } \alpha_i, \beta_i \in \mathbb{R}, i = 1, \dots, q$$

$\alpha_i = \alpha_{0,i} + \Delta\alpha_i$
$\alpha_{0,i}$ and $\Delta\alpha_i$ representing respectively the nominal and the uncertain parts of α_i

Let us define :

$$\alpha = \mathrm{diag}(\alpha_1, \dots, \alpha_q)$$
$$\alpha_0 = \mathrm{diag}(\alpha_{0,1}, \dots, \alpha_{0,q})$$
$$\Delta\alpha = \mathrm{diag}(\Delta\alpha_1, \dots, \Delta\alpha_q)$$

The uncertain part can be bounded as :

$$(\Delta\alpha)^T \Delta\alpha \leq v$$

where $v \in \mathbb{R}^{q \times q}$ is a known diagonal positive definite matrix.

Estimation errors

$$
\begin{cases}
e_p(t) = x(t) - x_f(t) & \text{: state tracking error} \\
e_s(t) = x_f(t) - \hat{x}_f(t) & \text{: state estimation error} \\
e_d(t) = f(t) - \hat{f}(t) & \text{: fault estimation error} \\
e_y(t) = y_f(t) - \hat{y}_f(t) & \text{: output error} \\
e_u(t) = u(t) - u_f(t) & \text{: error between nominal and FTC law}
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Notation and hypothesis

$$X_\mu = \sum_{i=1}^{r} \mu(\xi(t)) X_i \qquad \dot{f}(t) = \alpha f(t)$$

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Dynamics of the estimation errors

$$\begin{cases} \dot{e}_p(t) = A_\mu e_p(t) + B_\mu e_u(t) - G_\mu f(t) \\ \dot{e}_s(t) = A_\mu e_s(t) + G_\mu e_d(t) - H_\mu^1 e_y(t) \\ \dot{e}_d(t) = -H_\mu^2 e_y(t) - H_\mu^3 e_d(t) + \left(\alpha + H_\mu^3\right) f(t) \end{cases}$$

Proposed approach

Rather than replacing the expressions of $e_y(t)$ and $e_u(t)$ in the estimation errors, it is preferable to introduce them as static constraints (redundancy approach).

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Introduction of "virtual dynamics"

$$\begin{cases} 0\dot{e}_y(t) = -e_y(t) + C_\mu e_s(t) + W_\mu e_d(t) & 0 \in \mathbb{R}^{p\times p} \\ 0\dot{e}_u(t) = -K_\mu^f f(t) + K_\mu^f e_d(t) + K_\mu e_p(t) + K_\mu e_s(t) + e_u(t) & 0 \in \mathbb{R}^{m\times m} \end{cases}$$

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Advantages

It avoids the introduction of products between unknown matrices leading to BMI.
The estimation error dynamics is described by a single sum rather than a double sum.
It allows to consider nonlinear output equation of TS model without increasing the difficulties.

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The estimation error dynamics is described by a single sum rather than a double sum.
It allows to consider nonlinear output equation of TS model without increasing the difficulties.

Disdvantage

The dynamics of the estimation errors is described by a descriptor system.
However, this singular system is impulse-free.

Estimation error system

$$\tilde{e}^T(t) = \begin{pmatrix} e_p^T(t) & e_s^T(t) & e_d^T(t) & e_y^T(t) & e_u^T(t) \end{pmatrix}$$

$$E\dot{\tilde{e}}(t) = \tilde{A}_\mu \tilde{e}(t) + \tilde{B}_\mu f(t)$$

where $E = \mathrm{diag}\begin{pmatrix} I_n & I_n & I_q & 0_p & 0_m \end{pmatrix}$,

$$\tilde{A}_\mu = \begin{pmatrix} A_\mu & 0 & 0 & 0 & B_\mu \\ 0 & A_\mu & G_\mu & -H_\mu^1 & 0 \\ 0 & 0 & -H_\mu^3 & -H_\mu^2 & 0 \\ 0 & C_\mu & W_\mu & -I & 0 \\ -K_\mu & -K_\mu & -K_\mu^f & 0 & -I \end{pmatrix} \qquad \tilde{B}_\mu = \begin{pmatrix} -G_\mu \\ 0 \\ \alpha + H_\mu^3 \\ 0 \\ K_\mu^f \end{pmatrix}$$

$$E\dot{\tilde{e}}(t) = \tilde{A}_\mu \tilde{e}(t) + \tilde{B}_\mu f(t)$$

Stability guarantee

Quadratic Lyapunov function (tracking, state and fault estimation convergence)

$$V(e_p(t), e_s(t), e_d(t)) = \tilde{e}^T(t) E P \tilde{e}(t)$$

with

$$EP = P^T E \geqslant 0$$

Attenuation of the fault effect

$\mathscr{L}_2$ constraint

$$\int_0^t \tilde{e}^T(\tau) E \tilde{e}(\tau) d\tau \leqslant \gamma^2 \int_0^t f^T(\tau) f(\tau) d\tau$$

where γ represents the attenuation level.

Summary

The tracking error $e_p(t)$, state $e_s(t)$ and fault $e_d(t)$ estimation errors must therefore satisfy the following inequality :

$$\dot{\tilde{e}}^T(t)EP\tilde{e}(t)+\tilde{e}^T(t)EP\dot{\tilde{e}}(t)+\tilde{e}^T(t)E\tilde{e}(t)-\gamma^2 f^T(t)f(t)<0$$

This inequality is fulfilled if :

$$\begin{pmatrix} \tilde{A}_\mu^T P + P^T \tilde{A}_\mu + E & * \\ \tilde{B}_\mu^T P & -\gamma^2 \end{pmatrix} < 0$$

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Structure of the Lyapunov matrix

$$P=\begin{pmatrix} P_1 & 0 & 0 & 0 & 0 \\ 0 & P_7 & 0 & 0 & 0 \\ 0 & 0 & P_{13} & 0 & 0 \\ P_{16} & P_{17} & P_{18} & P_{19} & 0 \\ 0 & 0 & 0 & 0 & P_{25} \end{pmatrix}$$

$P_1 = P_1^T \geqslant 0$, $P_7 = P_7^T \geqslant 0$, $P_{13} = P_{13}^T \geqslant 0$,
P_{16}, P_{17}, P_{18}, P_{19} P_{25} are free slack matrices with appropriate dimensions

Theorem 1

The system that describes the different estimation errors is stable and the $\mathscr{L}_2$-gain from the faults to the state tracking error, the state and fault estimation errors is bounded by $\sqrt{\bar{\gamma}}$, if there exists matrices $P_1 = P_1^T \geqslant 0$, $P_7 = P_7^T \geqslant 0$, $P_{13} = P_{13}^T \geqslant 0$, P_{16}, P_{17}, P_{18}, P_{19}, P_{25}, F_i, R_i, S_i, Q_i and M_i and positive scalars $\bar{\gamma}$ and τ such that the following LMI are verified, for $i = 1, \ldots, r$:

$$\begin{pmatrix} \Upsilon_i^{1,1} & * & * & * & * & * & 0 \\ C_i^T P_{16} & \Upsilon_i^{2,2} & * & * & * & 0 & 0 \\ W_i^T P_{16} & \Upsilon_i^{3,2} & \Upsilon_i^{3,3} & * & * & * & * \\ -P_{16} & \Upsilon_i^{4,2} & \Upsilon_i^{4,3} & \Upsilon^{4,4} & 0 & 0 & 0 \\ \Upsilon_i^{5,1} & -F_i & -Q_i & 0 & \Upsilon^{5,5} & * & 0 \\ -G_i^T P_1 & 0 & \Upsilon_i^{6,3} & 0 & Q_i^T & (\tau - \bar{\gamma})I & 0 \\ 0 & 0 & P_{13} & 0 & 0 & 0 & -\tau \nu I \end{pmatrix} < 0$$

■

Polynomial faults

$$f_i(t) = \lambda_i t + \delta_i, \text{ with } \lambda_i, \delta_i \in \mathbb{R}, i = 1, \ldots, q$$

As well as for exponential function, defining different diagonal matrices, $\lambda = \lambda_0 + \Delta\lambda$, with $\Delta\lambda$ verifying :

$$(\Delta\lambda)^T \Delta\lambda \leq v$$

where $v \in \mathbb{R}^{q \times q}$ is a known diagonal positive definite matrix.

Fault estimation error

$$\dot{e}_d(t) = -H_\mu^2 e_y(t) - H_\mu^3 e_d(t) + H_\mu^3 f(t) + \lambda$$

Estimation error system

$$\tilde{e}^T(t) = \begin{pmatrix} e_p^T(t) & e_s^T(t) & e_d^T(t) & e_y^T(t) & e_u^T(t) \end{pmatrix}$$

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where $E = \operatorname{diag}\begin{pmatrix} I_n & I_n & I_q & 0_p & 0_m \end{pmatrix}$,

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Takagi-Sugeno model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i(\xi(u(t)))(A_i x_f(t) + B_i u_f(t) + G_i f(t)) \\ y(t) = \sum_{i=1}^{2} \mu_i(\xi(u(t)))(C_i x_f(t) + D_i u_f(t) + W_i f(t)) \end{cases}$$

with $A_1 = \begin{pmatrix} -5 & 1 & -3 \\ 1 & -3 & 2 \\ 1 & 1 & -4 \end{pmatrix}$, $A_2 = \begin{pmatrix} -3 & 1 & 1 \\ 0.5 & -3 & 2 \\ 0.5 & 1 & -5 \end{pmatrix}$, $B_1 = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$,

$B_2 = \begin{pmatrix} 5 \\ 4 \\ 5 \end{pmatrix}$, $G_1 = \begin{pmatrix} 5 \\ 5 \\ 0 \end{pmatrix}$, $G_2 = \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}$, $W_1 = \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}$, $W_2 = \begin{pmatrix} -1 \\ 0.5 \end{pmatrix}$,

$C_1 = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0 \end{pmatrix}$, $C_2 = \begin{pmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0 \end{pmatrix}$, $D_1 = \begin{pmatrix} -0.8 \\ 0 \end{pmatrix}$, $D_2 = \begin{pmatrix} -0.8 \\ -0.5 \end{pmatrix}$,

$\mu_1(u(t)) = \frac{1 - \tanh(0.5 - u(t))}{2}$

$\mu_2(u(t)) = 1 - \mu_1(u(t))$.

The nominal input signal is $u(t) = \sin(\cos(0.5t)t)$.

Exponential fault

Actual fault

$$f(t) = e^{1.15t-14} \qquad 8s \leq t \leq 11s$$

The controller and the observer are synthesized for $\alpha_0 = 1$ and $\Delta\alpha = 0.2$.

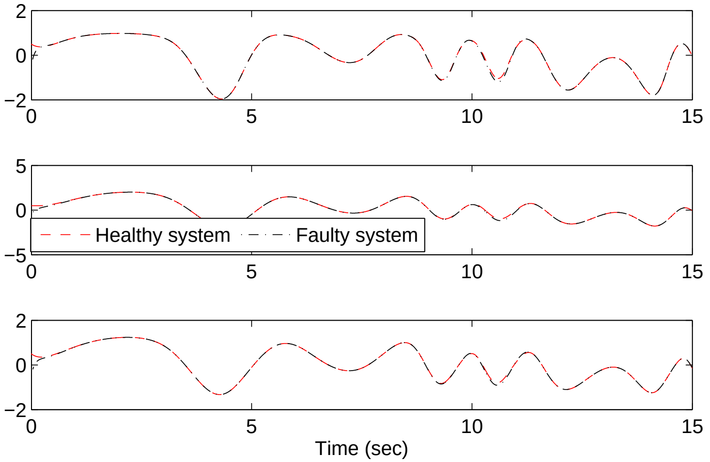


FIGURE: Reference model states vs. faulty system ones with FTC

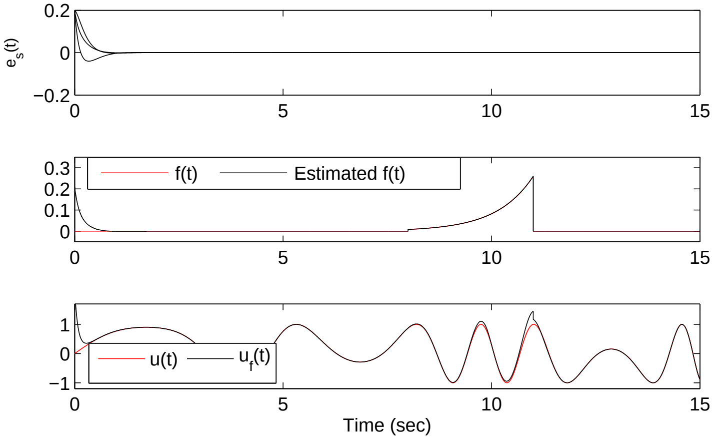


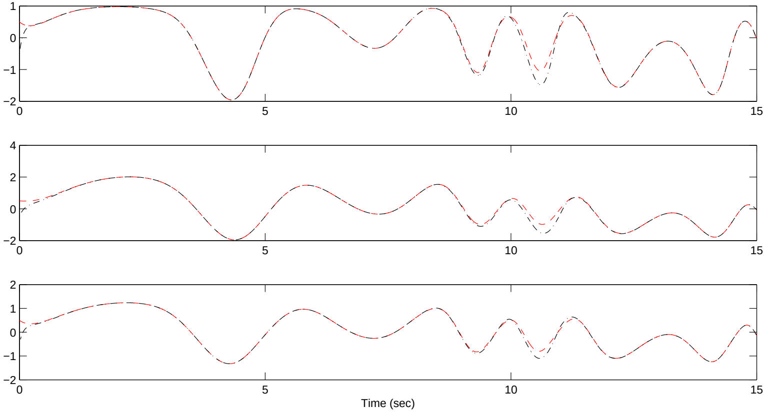
FIGURE: State estimation errors, fault and its estimation, nominal and FTC control inputs

Exponential fault

Actual fault

$$f(t) = e^{1.25t-14} \qquad 8s \leq t \leq 11s$$

The controller and the observer are synthesized for $\alpha_0 = 1$ and $\Delta\alpha = 0.3$.



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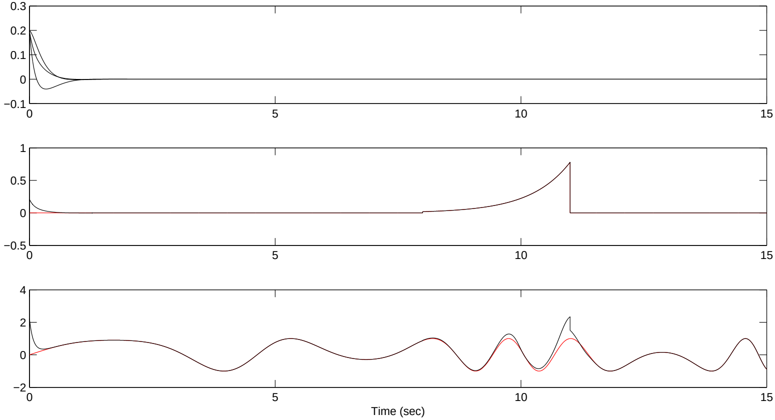

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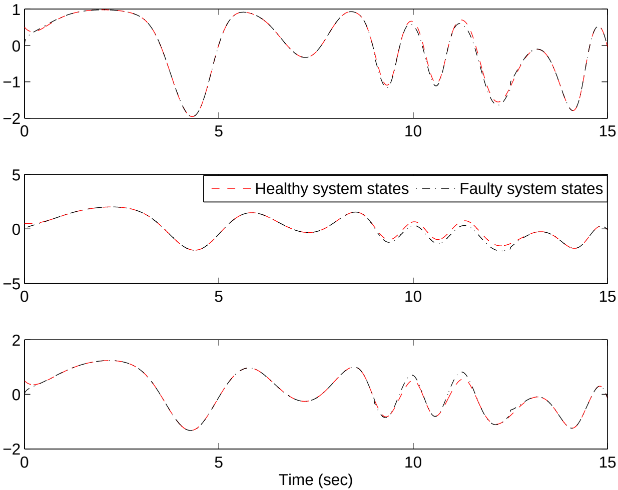
Polynomial fault

Actual fault

$$f(t) = 0.2t - 1 \qquad 9s \leq t \leq 12.5s$$

The controller and the observer are synthesized for $\lambda_0 = 0.11$ and $\Delta\lambda = 0.25$.



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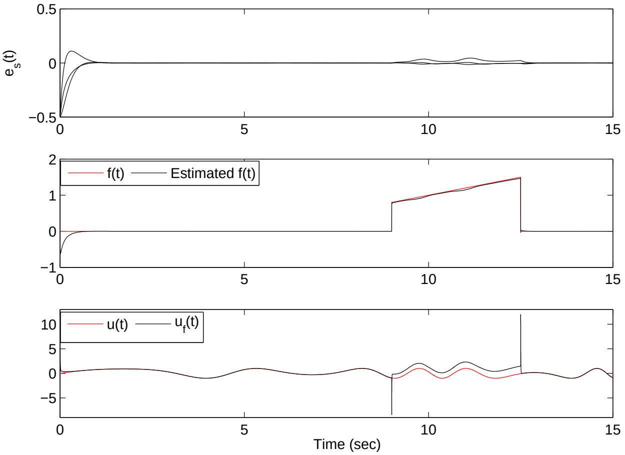

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$$f(t) \equiv 1 \qquad 9s \leq t \leq 12.5s$$

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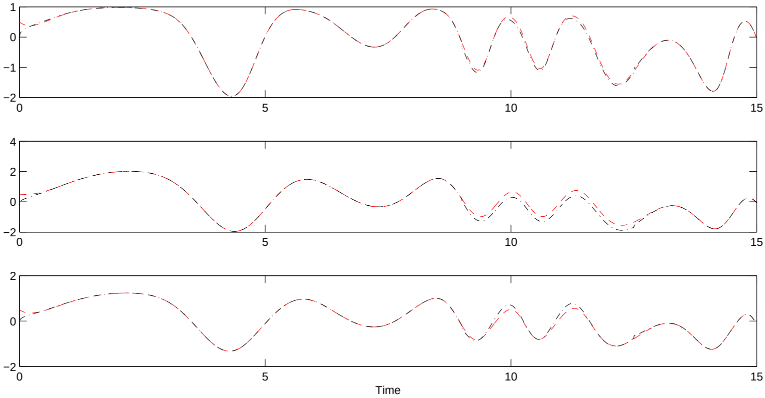

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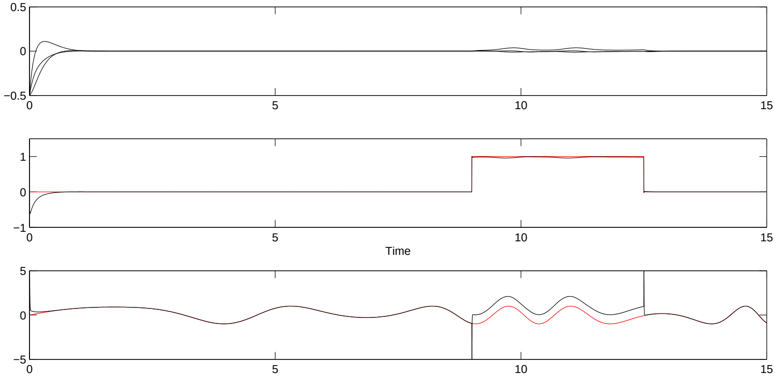

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Polynomial fault

Actual fault

$$f(t) = \sin(4t) \qquad 9s \leq t \leq 12.5s$$

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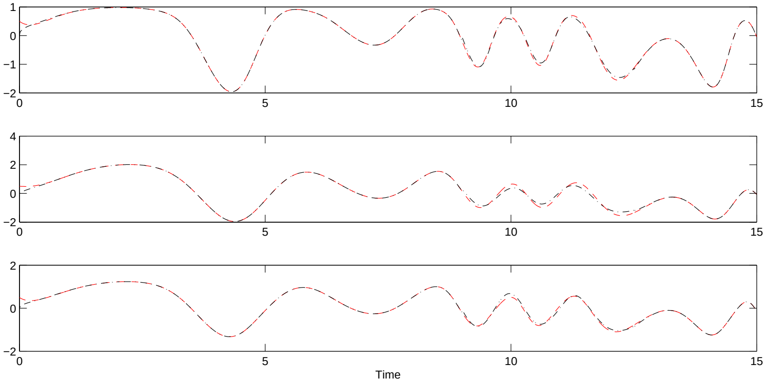

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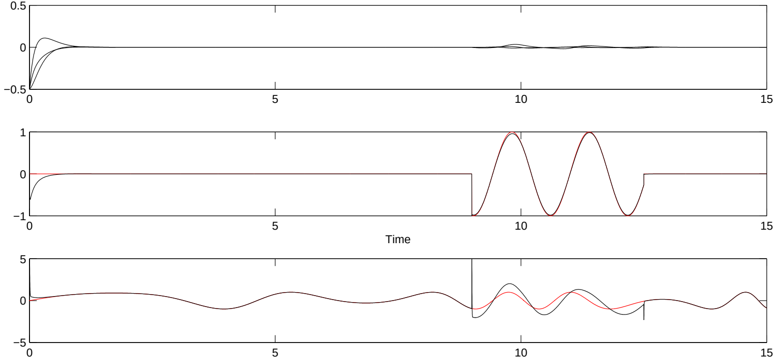

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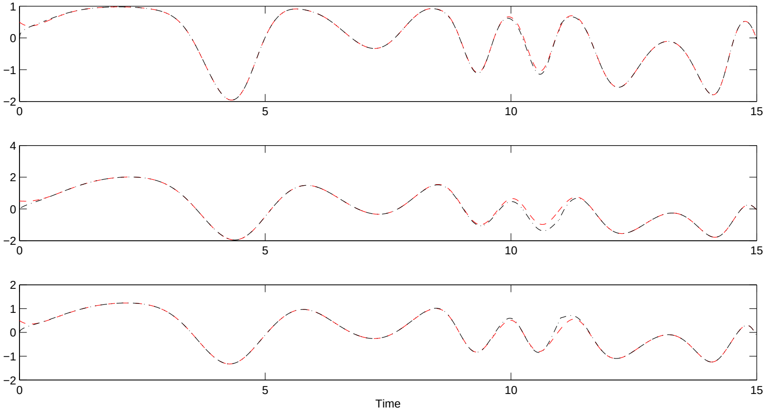

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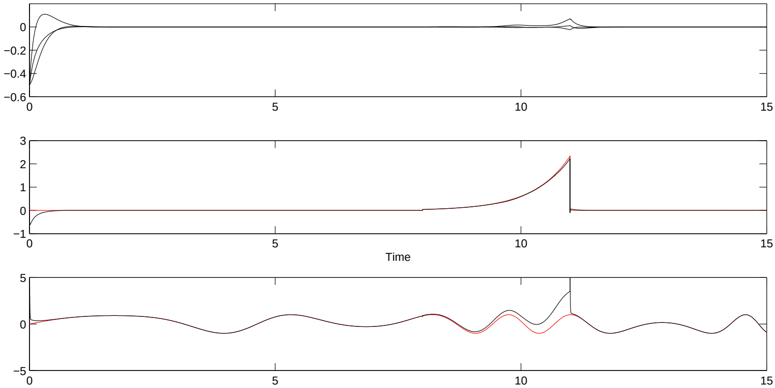

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Conclusions

- Active fault tolerant control law for nonlinear systems represented by a Takagi-Sugeno structure.

Perspectives

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Perspectives

- Study of the unmeasurable premise variable case ($\xi(t) = x(t)$).
- Comparison with multiple integral observer approach
- Implementation of a bank of different controller each of them dedicated to a particular kind of fault and design of a switching control law depending on the measured performances.

Get in touch

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