

Linear Feedback Control Input under Actuator Saturation : a Takagi-Sugeno Approach

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Objective

Design a stabilizing static state feedback control
ensuring the stability of the system under actuator saturation

Contribution

Polytopic representation of the nonlinear behaviour (saturation)

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- 1 Objective and main contribution
- 2 Takagi-Sugeno approach for modeling
- 3 Bounded control (actuator saturation)
 - Scalar case
 - Vectorial case
 - Saturated state feedback control input
 - Numerical example
- 4 Conclusion and Perspectives

■ The Takagi-Sugeno structure

$$\begin{cases} \dot{x}(t) &= \sum_{i=1}^n \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) &= \sum_{i=1}^n \mu_i(\xi(t))(C_i x(t) + D_i u(t)) \end{cases}$$

$x(t) \in \mathbb{R}^{n_x}$ is the system state variable, $u(t) \in \mathbb{R}^{n_u}$ is the control input and $y(t) \in \mathbb{R}^m$ is the system output. $\xi(t) \in \mathbb{R}^q$ is the decision variable vector assumed to be measurable (as the system output) or known (as the system input).

- Based on a nonlinear interpolation between certain linear submodels with adequate weighting functions $\mu_i(\xi(t))$ satisfying the convex sum property

$$\begin{cases} \sum_{i=1}^n \mu_i(\xi(t)) = 1 & \text{et} \\ 0 \leq \mu_i(\xi(t)) \leq 1, & i = 1, \dots, n, \quad \forall t \end{cases}$$

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How to systematically obtain a T-S model from a given NL system ?

- **Sector nonlinearity approach** : a systematic procedure which guarantees an exact model construction for nonlinear systems with bounded nonlinearities.
- Starting with a general form of nonlinear systems, a quasi-LPV state representation is realized. The T-S form is obtained by using the convex polytopic transformation. Each vertex defines a linear submodel and nonlinear parts are rejected into the weighting functions.

$$\text{NL} \left\{ \begin{array}{l} \dot{x}(t) = f_x(x(t), u(t)) \\ y(t) = f_y(x(t), u(t)) \end{array} \right. \Rightarrow$$

$$\text{Quasi-LPV} \left\{ \begin{array}{l} \dot{x}(t) = A(x(t), u(t))x(t) + B(x(t), u(t))u(t) \\ y(t) = C(x(t), u(t))x(t) + D(x(t), u(t))u(t) \end{array} \right. \Rightarrow$$

$$\text{T-S model} \left\{ \begin{array}{l} \dot{x}(t) = \sum_{i=1}^n \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^n \mu_i(\xi(t))(C_i x(t) + D_i u(t)) \end{array} \right.$$

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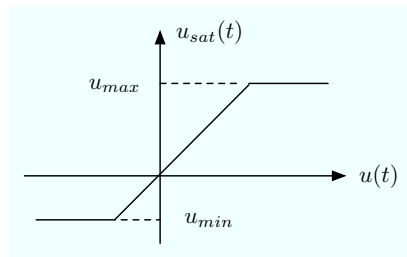
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Takagi-Sugeno saturation control

Objective

Model the nonlinear actuator saturation using the T-S representation

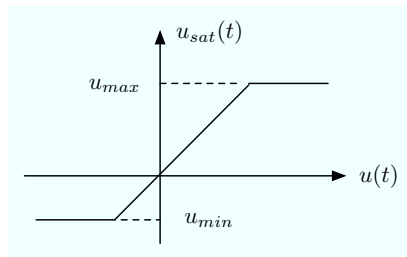
$$u_{\text{sat}} = \begin{cases} u(t) & \text{if } u_{\min} \leq u(t) \leq u_{\max} \\ u_{\max} & \text{if } u(t) \geq u_{\max} \\ u_{\min} & \text{if } u(t) \leq u_{\min} \end{cases}$$



$$\begin{cases} u_{\text{sat}}(t) &= \mu_1(t)u_{\min} + \mu_2(t)u(t) + \mu_3(t)u_{\max} \\ \mu_1(t) &= \frac{1 - \text{sign}(u(t) - u_{\min})}{2} \\ \mu_2(t) &= \frac{\text{sign}(u(t) - u_{\min}) - \text{sign}(u(t) - u_{\max})}{2} \\ \mu_3(t) &= \frac{1 + \text{sign}(u(t) - u_{\max})}{2} \end{cases}$$

$$\Rightarrow \begin{cases} u_{\text{sat}}(t) = \sum_{i=1}^3 \mu_i(t)(\lambda_i u(t) + \gamma_i) \\ \lambda_1 = 0 \quad \lambda_2 = 1 \quad \lambda_3 = 0 \\ \gamma_1 = u_{\min} \quad \gamma_2 = 0 \quad \gamma_3 = u_{\max} \end{cases}$$

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■ Illustrative example : $n_u = 2$:

$$u_{\text{sat}}(t) = \begin{pmatrix} \sum_{i=1}^3 \mu_i^1(u_1(t))(\lambda_i^1 u_1 + \gamma_i^1) \\ \sum_{i=1}^3 \mu_i^2(u_2(t))(\lambda_i^2 u_2 + \gamma_i^2) \end{pmatrix}$$

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$$u_{\text{sat}}(t) = \sum_{i=1}^9 \mu_i(t)(\Lambda_i u(t) + \Gamma_i)$$

For each submodel i ($1 : 9$) corresponds a couple (σ_j^1, σ_j^2) ($j = 1 : 3$) s.t.

$$\Lambda_i = \text{diag}(\lambda_j^1, \lambda_j^2)$$

$$\Gamma_i = [\gamma_j^1 \quad \gamma_j^2]^T$$

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T-S representation for the saturation

- General case : $u(t) \in \mathbb{R}^{n_u}$, $u_j(t)$ (resp. $u_{sat}^j(t)$) the j^{th} component of $u(t)$ (resp. $u_{sat}(t)$)

$$u(t) \rightarrow u_{sat}(t) = \sum_{i=1}^{3^{n_u}} \mu_i^{sat}(t) (\Lambda_i u(t) + \Gamma_i)$$

$$\begin{cases} \mu_i^{sat}(t) &= \prod_{j=1}^{n_u} \mu_{\sigma_i^j}^j(u_j(t)) \\ \Lambda_i &= \text{diag}(\lambda_{\sigma_i^1}^1, \dots, \lambda_{\sigma_i^{n_u}}^{n_u}) \\ \Gamma_i &= \left(\gamma_{\sigma_i^1}^1 \quad \dots \quad \gamma_{\sigma_i^{n_u}}^{n_u} \right)^T \end{cases}$$

where the indexes σ_i^j ($i = 1, \dots, 3^{n_u}$ and $j = 1, \dots, n_u$), equal to 1, 2 or 3, indicate which partition of the j^{th} input (μ_1^j, μ_2^j or μ_3^j) is involved in the i^{th} submodel.

The relations between the i^{th} submodel and the σ_i^j indices are given by the following equation

$$i = 3^{n_u-1} \sigma_i^1 + 3^{n_u-2} \sigma_i^2 + \dots + 3^0 \sigma_i^{n_u} - (3^1 + 3^2 + \dots + 3^{n_u-1})$$

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Saturated state feedback control input

Problem statement

$$\dot{x}(t) = Ax(t) + Bu_{sat}(t)$$

$$u_{sat}(t) = \sum_{i=1}^{3^{n_u}} \mu_i^{sat}(t) (\Lambda_i u(t) + \Gamma_i)$$

$$u(t) = -Kx(t)$$

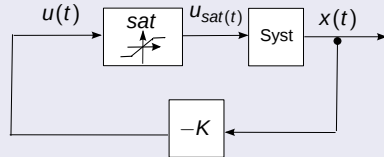


FIGURE: Closed loop system

Objectives

Find a control gain K ensuring the stability of the system
in the presence of control input saturation

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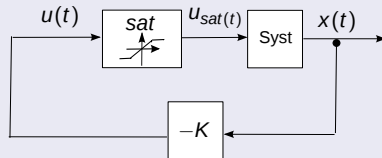


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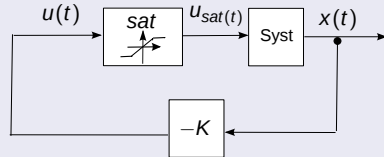


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Proposed solution

- T-S representation :

$$\dot{x}(t) = \sum_{i=1}^{3^{n_u}} \mu_i^{sat}(t) ((A - B\Lambda_i K)x(t) + B\Gamma_i)$$

- Optimisation problem under LMI constraints : $\min_K ||x(t)||$

Lyapunov approach

$$V(x(t)) = x^T(t)Px(t), \quad P = P^T > 0$$

$$\dot{V}(x(t)) \leq \sum_{i=1}^{3^{n_u}} \mu_i(t) (x^T(t) ((A - B\Lambda_i K)^T P + P(A - B\Lambda_i K) + P\Sigma^{-1}P)x(t) + \Gamma_i^T B^T \Sigma B \Gamma_i)$$

Let us define :

$$\mathcal{Q}_i = (A - B\Lambda_i K)^T P + P(A - B\Lambda_i K) + P\Sigma^{-1}P$$

$$\varepsilon = \min_{i=1:3^{n_u}} \lambda_{\min}(-\mathcal{Q}_i)$$

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Condition

$$\dot{V}(t) < -\varepsilon \|x\|^2 + \delta$$

$x(t)$ is uniformly bounded and converges to a small origin-centred ball of radius $\sqrt{\frac{\delta}{\varepsilon}}$ if :

$$\mathcal{Q}_i < 0 \quad \text{and} \quad \|x\|^2 > \frac{\delta}{\varepsilon}$$

LMI to solve

$$\mathcal{Q}_i = (A - B\Lambda_i K)^T P + P(A - B\Lambda_i K) + P\Sigma^{-1}P$$

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Control gain :

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Objective : minimize the ball radius

- $x(t)$ is uniformly bounded and converges to a small origin-centred ball of radius $\sqrt{\frac{\delta}{\varepsilon}}$
- How to minimize the radius ?

$$\begin{cases} \delta < \beta \\ \varepsilon > 1/\beta \end{cases} \Rightarrow \text{radius} < \beta \Rightarrow \min(\text{radius}) \equiv \min \beta$$

 δ and ε optimisation

$$\delta = \max_{i=1:3^{n_u}} \Gamma_i^T B^T \Sigma B \Gamma_i \quad \text{with} \quad \delta < \beta \Rightarrow \Gamma_i B^T \Sigma B \Gamma_i < \beta$$

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Theorem

There exists a static state feedback controller for a saturated input system such that the system state converges toward an origin-centred ball of radius bounded by β if there exists matrices $P_1 = P_1^T > 0, R, \Sigma = \Sigma^T > 0$ solutions of the following optimization problem ($i = 1 : 3^{n_u}$)

$$\min_{P_1, R, \Sigma} \beta$$

$$\Gamma_i B^T \Sigma B \Gamma_i < \beta, \quad \begin{pmatrix} Q_i & I \\ I & -\beta I \end{pmatrix} < 0, \quad Q_i = \begin{pmatrix} P_1 A^T + A P_1 - R^T \Lambda_i^T B^T - B \Lambda_i R & I \\ I & -\Sigma \end{pmatrix}$$

The controller gain is given by

$$K = R P_1^{-1}$$

Numerical example

Linear system under actuator saturation

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu_{sat}(t) \\ u_{sat}(t) = \sum_{i=1}^{3^{n_u}} \mu_i^{sat}(t)(\Lambda_i u(t) + \Gamma_i) \\ u(t) = -Kx(t) \end{cases}$$

$$\dot{x}(t) = \sum_{i=1}^{3^{n_u}} \mu_i(t)((A - B\Lambda_i K)x(t) + B\Gamma_i)$$

$$A = \begin{pmatrix} -2 & 0.1 & 0 \\ 0 & -0.5 & 0 \\ 0.2 & 0.1 & -3 \end{pmatrix}, B = \begin{pmatrix} 0.1 & 1 \\ 0.5 & 1 \\ 0.8 & 0.6 \end{pmatrix}, u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{cases} u_{1max} & = & 1 \\ u_{1min} & = & -1 \\ u_{2max} & = & 1.5 \\ u_{2min} & = & -1.5 \end{cases}$$

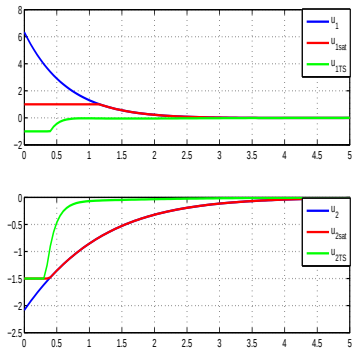
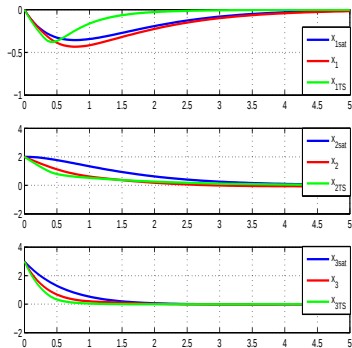


FIGURE: System states (left) - input (right) for nominal, nominal saturated and T-S saturated controllers

Conventional anti-windup controller

The conventional anti-windup controller uses the difference $v - v_{sat}$ as an input for a large gain matrix $\Psi = \alpha I$ where $\alpha \gg 1$ is a scalar

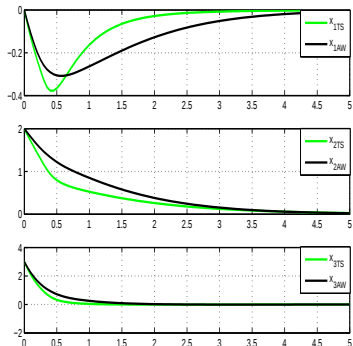
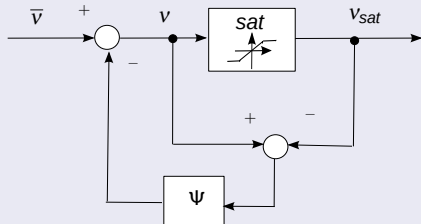


FIGURE: Conventional AW controller (left) - system states for AW and T-S saturated controllers (right)

Conclusions

- New systematic approach for the saturation representation.
- Procedure based on convex polytopic transformation.
- Convergences conditions expressed in an optimisation problem with LMI constraints.

Perspectives

- Extension to nonlinear systems (incertain) with a T-S representation.
- Extension to the output feedback controller .
- Diagnostic application and FTC control.

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Thanks for your attention

Somes references about the T-S representation

- 1 K. Tanaka, H.O. Wang. Fuzzy Control Systems Design and Analysis : A Linear Matrix Inequality Approach. Ed. John Wiley and Sons, Inc, 2001.
- 2 A. Akhenak, M. Chadli, J. Ragot, D. Maquin. Estimation d'état et d'entrées inconnues d'un système non linéaire représenté sous forme multimodèle. CIFA, 2004.
- 3 J. Li, C. Bo, J. Zhang and J. Du. Fault Diagnosis and Accommodation Based on Online Multi-model for Nonlinear Process. Lecture Notes in Computer Science, 2006.
- 4 Z. Petres. Polytopic Decomposition of Linear Parameter-Varying Models by Tensor-Product Model Transformation. Ph.D. Dissertation, 2006.
- 5 R. Orjuela, D. Maquin, J. Ragot. Nonlinear system identification using uncoupled state multiple-model approach. Workshop on Advanced Control and Diagnosis, 2006.
- 6 D. Saïfia, M. Chadli, S. Iabiod. H_∞ Control of Multiple Model Subject to Actuator Saturation : Application to Quarter-Car Suspension System. Journal of Analog Integrated Circuits and Signal Processing, 2011.
- 7 E. Naderi, N. Meskin, K. Khorasani. Nonlinear Fault Diagnosis of Jet Engines by Using a Multiple Model-Based Approach, J. Eng. Gas Turbines Power, 2012.
- 8 A.M. Nagy, G. Mourot, G. Schutz, J. Ragot. Exact activated sludge reactor modeling using a multiple model. Industrial & Engineering Chemistry Research, 2010.
- 9 S. Tarbouriech, G. Garcia, J.M. Gomes da Silva Jr. I. Queinnec. Stability and Stabilization of Linear Systems with Saturating Actuators. Springer-Verlag, 2011.
- 10 A. Benzaouia. Saturated Switching Systems. Springer Verlag, 2012.