Actuator Fault diagnosis: H_∞ framework with relative degree notion

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Introduction

Consider the linear system

$$\begin{aligned} \dot{x} &= Ax + Ef \\ y &= Cx \end{aligned}$$

where $x \in \mathbb{R}^n$ is the state vector, $f \in \mathbb{R}$ is the fault signal and $y \in \mathbb{R}$ is the output signal.

Objective

The objective is to detect or estimate the fault f from the measurement y.

Problem statement

When the system is affected by a perturbation d, the so-called H_-/H_∞ which consists in computing a residual generator

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t) \\ r(t) = M(y(t) - \hat{y}(t)) \end{cases}$$

with gains L and M in order to satisfy

- Stability of $(A - LC)$
- Minimization of the effect of d on r
- Maximization of the sensitivity of f on r

This is a min/max problem

Problem statement

In order to transform the min/max problem on a simple min problem, a virtual residual generator is defined as follows

$$r_e = r - f = MCe - f$$

where $e = x - \hat{x}$. Then the system generating the state estimation error is given by

$$\begin{cases} \dot{e}(t) = (A - LC)\,e(t) + Ef(t) \\ r_e(t) = MCe(t) - f(t) \end{cases}$$

In standard H_∞ framework, the matrices L and M should be determined in such a way to satisfy the following constraints

$$\begin{cases} \lim\limits_{t \to +\infty} r_e(t) = 0 & \text{if } f(t) = 0 \\ \|r_e(t)\|_2 < \gamma \, \|f(t)\|_2 & \text{if } f(t) \neq 0 \end{cases}$$

Problem statement

A solution can be found by solving an optimization problem under LMI constraints (SISO case)

$$\min_{P,K,M} \quad \gamma$$

s.t.

$$\begin{pmatrix} A^T P + PA - C^T K^T - KC & PE & C^T M^T \\ E^T P & -\gamma & -1 \\ MC & -1 & -\gamma \end{pmatrix} < 0 \qquad (1)$$

where $P = P^T > 0$. After solving the optimization problem, the matrices of the residual generator are obtained by $L = P^{-1}K$ and M is obtained directly. The attenuation level is given by γ.

Problem statement

If the previous optimization problem is solved, then one has:

$$\begin{pmatrix} -\gamma & -1 \\ -1 & -\gamma \end{pmatrix} < 0 \qquad (2)$$

which leads to $\gamma > 1$.
Then the best value for γ is $1 + \epsilon$ where ϵ is a positive small number

H_∞ Residual Generator with relative degree consideration

Let us consider the system

$$\begin{aligned} \dot{x} &= Ax + Ef \\ y &= Cx \end{aligned}$$

where the relative degree is r. This means that

$$y^{(r)}(t) = CA^r x(t) + CA^{r-1}Ef(t)$$

Now, let us consider the new output $\tilde{y}(t)$ defined by

$$\tilde{y}(t) = \begin{pmatrix} y(t) \\ \dot{y}(t) \\ \vdots \\ y^{(r)}(t) \end{pmatrix} = \underbrace{\begin{pmatrix} C \\ CA \\ \vdots \\ CA^r \end{pmatrix}}_{\tilde{C}} x(t) + \underbrace{\begin{pmatrix} 0 \\ 0 \\ \vdots \\ CA^{r-1}E \end{pmatrix}}_{R} f(t)$$

H_∞ Residual Generator with relative degree consideration

The system with the new generated output becomes

$$\begin{cases} \dot{x}(t) = Ax(t) + Ef(t) \\ \tilde{y}(t) = \tilde{C}x(t) + Rf(t) \end{cases}$$

The proposed residual generator is

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + L(\tilde{y}(t) - \hat{\tilde{y}}(t)) \\ \hat{\tilde{y}}(t) = \tilde{C}\hat{x}(t) \\ r(t) = M(\tilde{y}(t) - \hat{\tilde{y}}(t)) \end{cases}$$

H_∞ Residual Generator with relative degree consideration

If there exist a symmetric and positive definite matrix P, gain matrices K and M and a positive scalar γ solution to the following optimization problem

$$\min_{P,K,M} \quad \gamma$$

s.t.

$$\begin{pmatrix} A^T P + PA - \tilde{C}^T K^T - K\tilde{C} & PE - K\tilde{C} & \tilde{C}^T M^T \\ E^T P - \tilde{C}^T K^T & -\gamma & R^T M^T - 1 \\ M\tilde{C} & MR - 1 & -\gamma \end{pmatrix} < 0$$

The gain L of the residual generator is obtained from the equation $L = P^{-1}K$. The attenuation level γ describes the sensitivity of $r(t)$ with respect to $f(t)$. The smallest is γ the greatest is the sensitivity.

H_∞ Residual Generator with relative degree consideration

The negativity of (10) implies that

$$\begin{pmatrix} -\gamma & R^T M^T - 1 \\ MR - 1 & -\gamma \end{pmatrix} < 0$$

which is equivalent to

$$\gamma^2 > \left(R^T M^T - 1\right)(MR - 1)$$

Since this paper considers only systems with single fault and single output, the term $(MR - 1)$ is just a scalar, it follows

$$\gamma > MR - 1 \qquad (3)$$

Since R has full column rank due to the relative degree, it is then possible to chose M such that $(MR - 1) \rightarrow 0$. Thus, the parameter $\gamma > 0$ may takes values small than 1 which enhance the residual sensitivity with respect to the fault compared to the classical approach where $\gamma > 1$.

H_∞ Residual Generator with relative degree consideration (MIMO case)

Under the observability of the pair (C, A) and the relative degree vector $\{r_1, ..., r_{n_y}\}$, the residual generator exists if there exist a symmetric and positive definite matrix P, a gain matrix K and a positive scalar γ solution to the following optimization problem

$$\min_{P,K,M} \quad \gamma$$

s.t.

$$\begin{pmatrix} A^T P + PA - \tilde{C}^T \tilde{K}^T - \tilde{K}\tilde{C} & PE - \tilde{K}\tilde{C} & \tilde{C}^T \tilde{M}^T \\ E^T P - \tilde{C}^T \tilde{K}^T & -\gamma I_{n_f} & R^T \tilde{M}^T - I_{n_f} \\ \tilde{M}\tilde{C} & \tilde{M}R - I_{n_f} & -\gamma I_{n_f} \end{pmatrix} < 0$$

The gain L of the residual generator is obtained from the equation $L = P^{-1}K$. The attenuation level γ describes the sensitivity of $r(t)$ with respect to $f(t)$. The smallest is γ the greatest is the sensitivity.

H_∞ Residual Generator with relative degree consideration (MIMO case)

Notice that the theorem considers the worst case. However, with a simple analysis on the system error dynamics, it can be concluded that: If the condition

$$rank\left(\begin{bmatrix} \tilde{C} & \tilde{R} \\ 0 & I_{n_f} \end{bmatrix}\right) = rank\left(\begin{bmatrix} \tilde{C} & \tilde{R} \end{bmatrix}\right)$$

Then there exists a matrix M such that

$$\begin{cases} M\tilde{C} = 0 \\ M\tilde{R} = I_{n_f} \end{cases}$$

Consequently, the error dynamics becomes

$$\begin{cases} \dot{e}(t) = \left(A - \tilde{L}\tilde{C}\right)e(t) + (E - L\tilde{R})f(t) \\ r_e(t) = 0 \end{cases}$$

Then $r = f$.

H_∞ Residual Generator with relative degree consideration (MIMO case)

If the rank condition above is not satisfied but if

$$rank\left(\begin{bmatrix} \tilde{R} \\ I_{n_f} \end{bmatrix}\right) = rank\left(\tilde{R}\right), rank\left(\begin{bmatrix} E \\ \tilde{R} \end{bmatrix}\right) = rank\left(E\right)$$

Then there exist matrices M and L such that

$$\begin{cases} L\tilde{R} = E \\ M\tilde{R} = I_{n_f} \end{cases}$$

and in addition, the matrix L stabilizes the matrix $A - LC$, the error dynamics becomes

$$\begin{cases} \dot{e}(t) = \left(A - \tilde{L}\tilde{C}\right) e(t) \\ r_e(t) = M\tilde{C}e(t) \end{cases}$$

Then r converges asymptotically to f.

Simulation example

Consider the system with the matrices

$$A = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}, E = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

The system is observable and the output $y(t)$ have a relative degree 2 with respect to the fault $f(t)$.
For the classical approach, solving the optimization problem under the LMI constraint (1) leads to the following solution

$$P = 10^4 \times \begin{pmatrix} 2.082 & -0.0009 \\ -0.0009 & 0.0000 \end{pmatrix},$$

$$L = 10^6 \times \begin{pmatrix} 0.0009 \\ 1.9286 \end{pmatrix}, \ M = -9.2522, \gamma = 1.001$$

Simulation example

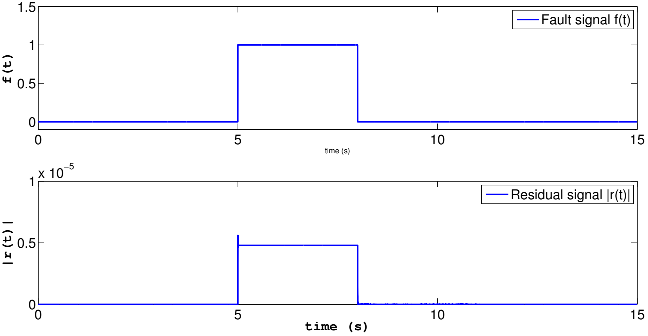

Figure: Fault and residual signal (classical approach)

Simulation example

With the proposed approach, one has

$$P = \begin{pmatrix} 1.1417 & 0 \\ 0 & 1.1417 \end{pmatrix},$$

$$L = 10^3 \times \begin{pmatrix} 1.0005 & 0 & 0 \\ 0.0010 & 1.0005 & 0.0010 \end{pmatrix},\ M = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix}$$

and γ is around 10^{-11} (the rank conditions are satisfied.

Simulation example

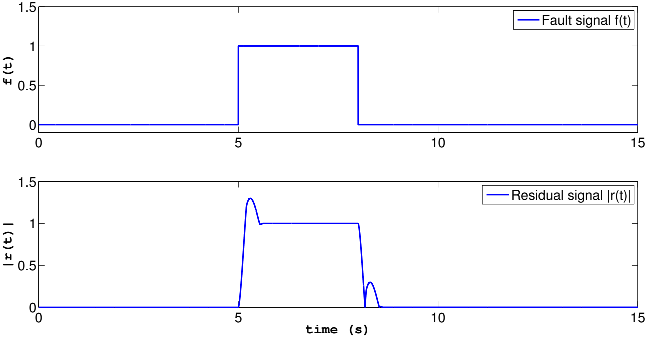

Figure: Fault and residual signal (proposed approach)

Simulation example

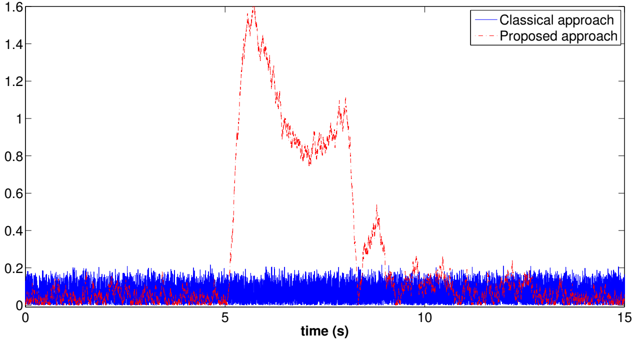

Figure: Residual signals (Comparison)

Conclusion and perspectives

Conclusions

- H_∞ Residual Generator with relative degree consideration
- Rank conditions for exact, asymptotic and bounded fault estimation error convergence

Perspectives

- Including the perturbation affecting the system (Use of Sobolev space and norms)
- Extension to LPV systems

Thank you for your attention