State estimation and fault detection of uncertain systems based on an interval approach

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Overview of the presentation

1.1 Introduction

Context and motivations

- The process should be described in a large operating range
  $\rightarrow$ Nonlinear models
- The knowledge on the process is imperfect
  $\rightarrow$ Uncertain models
- Process evolves in a disturbed environment
  $\rightarrow$ Actuator and measurement noises
- Process can be faulty
  $\rightarrow$ Different operating modes (healthy or not)

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- Process can be faulty
  $\rightarrow$ Different operating modes (healthy or not)

Proposed approach

- Uncertainties and noises are handled by the interval approach
- Fault diagnosis for uncertain nonlinear systems is proposed, based on
  - interval state estimation
  - active mode detection
  - residual generation

Studied systems

The aim is to perform state estimation and fault diagnosis of uncertain nonlinear systems defined by

$$x(k+1) = f(x(k), u(k), \theta(k), v(k))$$
$$y(k) = h(x(k), \theta(k), w(k))$$

where f and g are known nonlinear functions and

- $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^r$ and $y(t) \in \mathbb{R}^p$ are the system state, the input and output
- $v(k) \in \mathbb{R}^{n_v}$ and $w(k) \in \mathbb{R}^{n_w}$ are the state and measurement noises
- $\theta(k) \in \mathbb{R}^{n_\theta}$ is the uncertain parameter

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Notations

- a real interval is defined by $[z]=[z^- \ z^+]=\{z \in \mathbb{R} \mid z^- \leq z \leq z^+\}$
- an interval vector $[z] \in \mathbb{R}^{n_z}$ means that each component is an interval
- $\mathbb{IR}^{n_z}$ denotes the set of all boxes of $\mathbb{R}^{n_z}$

2.1 Principle of the interval state estimation

The state estimation is based on the known data:

- the interval initial condition $x(0) \in \mathbb{IR}^n$
- the measured input and output $u(k) \in \mathbb{R}^r$ and $y_m(k) \in \mathbb{R}^m$
- the time varying lower and upper bounds on the interval noises and uncertainties $v(k)$, $w(k)$ and $\theta(k)$

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- simulating the dynamic equation:
$$([\hat{x}(k-1)], u(k-1), [\theta(k)], [v(k)]) \rightarrow [\hat{x}^+(k)]$$
- merging the two information sources:
$$[\hat{x}^y(k)] \cap [\hat{x}^+(k)] \rightarrow [\hat{x}(k)]$$

2.2 Algorithm of state estimation

- **Step 0.** Initialize $\mathcal{D}^+_{x,0}$ with $\mathcal{D}_0$. Let $k = 1$

Initialization

$\mathcal{D}^+_{x,0}$ is the set of possible values of $x(0)$ deduced by prediction at $k = 0$.
The estimation of $x(0)$ is given by the known initial condition: $[x(0)]$.

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- **Step 0.** Initialize $\mathcal{D}^+_{x,0}$ with $\mathcal{D}_0$. Let $k = 1$
- **Step 1.** Collect the data $u(k)$ and $y_m(k)$

Input and output measurements

At each instant, $u(k)$ and $y_m(k)$ are collected.
It is recalled that $u(k)$ and $y_m(k)$ are not interval.

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- **Step 1.** Collect the data $u(k)$ and $y_m(k)$
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$$\mathcal{D}_{y,k} = \{y \ / \ y - y_m(k) \in [w(k)]\}$$

Output estimation

Based on the known lower and upper bounds of $[w(k)]$, the set of possible values of $y(k)$ is deduced.

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Inverting the observation equation

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- **Step 5.** Reduce the domain complexity: $\hat{\mathcal{D}}_{x,k} \supseteq \mathcal{D}_{x,k}$

Complexity reduction

Intersecting sets may lead to complex shape.
An overestimation (e.g. orthotope) may decrease the shape complexity.

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- **Step 6.** Predict the state set:

$$\mathcal{D}^+_{x,k} = \{f(x, u(k), \theta, v) \;/\; x \in \hat{\mathcal{D}}_{x,k},\;\; v \in [v(k)]\,,\;\; \theta \in [\theta(k)]\}$$

Simulating the dynamic equation

Based on the known lower and upper bounds of $[\theta(k)]$ and $[v(k)]$, the set of possible values of $x(k+1)$, consistent with $x(k) \in \hat{\mathcal{D}}_{x,k}$ is deduced.

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- **Step 7.** Increase $k := k + 1$, go to Step 1

Iteration

The prediction will be faced to measurements collected at step 1., and so on . . .

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- State estimation is based on the information consistency
  → predicted state set $\mathcal{D}^{+}_{x,k-1}$ and observed state set $\mathcal{D}^{y}_{x,k}$ are intersected

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- The presented state estimation can be adapted to fault diagnosis
  - In the framework of supervision, faulty models are available
    $\rightarrow$ fault diagnosis is performed by active mode detection
    $\rightarrow$ state estimation is performed with each model, information inconsistency invalids a model

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  - In the framework of supervision, faulty models are available
    $\rightarrow$ fault diagnosis is performed by active mode detection
    $\rightarrow$ state estimation is performed with each model, information inconsistency invalids a model
  - Fault detection can be made by comparing estimates and measurements
    $\rightarrow$ real state values are not available for comparison with estimated ones
    $\rightarrow$ estimated and measured output are used for residual generation

3.2 Interval fault detection

- The predicted output domain $\mathcal{D}^{+}_{y,k}$ is deduced from the predicted state domain $\mathcal{D}^{+}_{x,k}$

$$\mathcal{D}^{+}_{y,k} = \Big\{ y^{+} \ / \ y^{+} = h(x^{+}, \theta^{+}) + w, \ \ x^{+} = f(x, u(k), \theta, v),$$
$$x \in \mathcal{D}^{+}_{x,k}, \theta^{+} \in [\theta(k+1)], \theta \in [\theta(k)], v \in [v(k)], w \in [w(k)] \Big\}$$

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- A fault indicator is computed

$$r_{k+1} = \mathcal{D}^+_{y,k} \cap \mathcal{D}_{y,k+1}$$

(an overestimation of $\mathcal{D}^+_{y_k}$ can be used to limit the computational load)

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- A fault is detected if $r_{k+1} = \varnothing$

3.3 Change detection of operating mode

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- The state estimation algorithm can also be used for residual generation.

3.4 Algorithm of change detection of operating mode

- **Step 0.** Initialize $\mathcal{D}^{+}_{x,0,i}$ with $\mathcal{D}_0$, for $i = 1, \ldots, N$. Let $k = 1$

Initialization

$\mathcal{D}^{+}_{x,0,i}$ are the sets of possible values of $x_i(0)$ at $k = 0$.
All the state estimates are initialized with: $[x(0)]$.

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Output estimation

Based on the known lower and upper bounds of $[w(k)]$, the set of possible values of $y_i(k)$ is deduced, for each model M_i.

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Merging observation and prediction

The state estimation must be consistent with both observation and prediction, thus the two sets are intersected.

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$$\mathcal{D}_{x,k,i} = \mathcal{D}^+_{x,k-1,i} \cap \mathcal{D}^y_{x,k,i}$$
- **Step 5.** Check for active mode, for $i = 1, \ldots, N$:
if $\mathcal{D}_{x,k-1,i_0} = \varnothing$, then i_0 is not an active mode.

Search for active mode

$\mathcal{D}_{x,k-1,i_0} = \{\varnothing\}$ means that the prediction made with M_{i_0} is inconsistent with measurements $\rightarrow$ the i_0^{th} model is invalidated.

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$$\mathcal{D}^y_{x,k,i} = \{x_i \in \mathbb{R}^n / h_i(x_i, \theta) \in \mathcal{D}_{y,k,i}, \ \theta \in [\theta(k)]\}$$
- **Step 4.** Compute the admissible state domain $\mathcal{D}_{x,k,i}$, for $i = 1, \ldots, N$:
$$\mathcal{D}_{x,k,i} = \mathcal{D}^+_{x,k-1,i} \cap \mathcal{D}^y_{x,k,i}$$
- **Step 5.** Check for active mode, for $i = 1, \ldots, N$:

  if $\mathcal{D}_{x,k-1,i_0} = \varnothing$, then i_0 is not an active mode.
- **Step 6.** Reduce the domain complexity, for $i = 1, \ldots, N$: $\hat{\mathcal{D}}_{x,k,i} \supseteq \mathcal{D}_{x,k,i}$

Complexity reduction

Intersecting sets may lead to complex shape.
An overestimation (e.g. orthotope) may decrease the shape complexity.

3.4 Algorithm of change detection of operating mode

- **Step 0.** Initialize $\mathcal{D}^+_{x,0,i}$ with $\mathcal{D}_0$, for $i = 1, \ldots, N$. Let $k = 1$
- **Step 1.** Collect the data $u(k)$ and $y_m(k)$
- **Step 2.** Compute the output domains $\mathcal{D}_{y,k,i}$, for $i = 1, \ldots, N$:

$$\mathcal{D}_{y,k,i} = \{y_i \ / \ y_i - y_m(k) \in [w(k)]\}$$

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- **Step 6.** Reduce the domain complexity, for $i = 1, \ldots, N$: $\hat{\mathcal{D}}_{x,k,i} \supseteq \mathcal{D}_{x,k,i}$
- **Step 7.** Predict the state set, for $i = 1, \ldots, N$:

$$\mathcal{D}^+_{x,k,i} = \{f_i(x_i, u(k), \theta, v) \ / \ x_i \in \hat{\mathcal{D}}_{x,k,i}, \ v \in [v(k)], \ \theta \in [\theta(k)]\}$$

Simulating the dynamic equation

Knowning lower and upper bounds of $[\theta(k)]$ and $[v(k)]$, the sets of possible values of $x_i(k+1)$, consistent with $x_i(k) \in \hat{\mathcal{D}}_{x,k,i}$ are deduced.

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- **Step 8.** Increase $k := k + 1$, go to Step 1

Iteration

The prediction made with each model will be faced to measurements collected at step 1.

3.5 Algorithm of residual generation

- **Step 0.** Define the initial state domains $\mathcal{D}^{+}_{x,0,i}$, and set $k = 1$.
- **Step 1.** Collect the data $u(k)$ and $y_m(k)$.
- **Step 2.** Characterize the admissible state domains using a prediction based on the i^{th} model:

$$\mathcal{D}^{+}_{x,k,i} = \{f_i(x_i, u(k), \theta, v) / x_i \in \hat{\mathcal{D}}_{x,k,i}, \theta \in [\theta(k)], v \in [v(k)]\}$$

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- **Step 3.** Characterize the admissible output domains $\mathcal{D}^{+}_{y,k,i}$
- **Step 4.** Compute the bounds of the output domains
$$y^{-}_{ij}(k) = \inf y/y \in \mathcal{D}^{+}_{y,k,i} \quad \text{and} \quad y^{+}_{ij}(k) = \sup y/y \in \mathcal{D}^{+}_{y,k,i}$$
where the j is the number of the component output.
- **Step 5.** Compute the interval residuals
$$[r_{ij}(k)] = [y^{-}_{ij}(k) - y_{mj}(k),\ y^{+}_{ij}(k) - y_{mj}(k)]$$

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$$[r_{ij}(k)] = [y^-_{ij}(k) - y_{mj}(k),\ y^+_{ij}(k) - y_{mj}(k)]$$
- **Step 6.** Test the residual by checking if: $0 \in [r_{ij}(k)]$
- **Step 7.** Increase $k = k + 1$ and go to Step 1.

Active mode detection

For a given k, the i^{th} mode is said

- not active, if $0 \notin [r_{ij}(k)]$, $\exists j \in \{1, \dots, p\}$
- active, if $0 \in [r_{ij}(k)]$, $\forall j \in \{1, \dots, p\}$

4.1 Example of search for active mode

- Let consider a system, with a normal operation mode ($i = 0$) and two abnormal modes ($i = 1, 2$), defined by

$$M_i \begin{cases} x(k+1) & = \begin{pmatrix} 0.6 & 0 \\ -0.2 & 0.5 \end{pmatrix} x(k) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(k) + \begin{pmatrix} 0.05 & 0 \\ 0 & 0.05 \end{pmatrix} v(k) \\ y_i(k) & = h_i(x(k), \theta(k)) + w(k) \end{cases}$$

with

$$v_i(k) \in [-1 \; 1] \qquad w_i(k) \in [-0.04 \; 0.04]$$

$$\theta_1(k) \in [0.8 \; 1.2] \qquad \theta_2(k) \in [1.3 \; 1.7]$$

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$$h_0^T(x(k), \theta(k)) = \begin{pmatrix} \frac{x_1(k)+\theta_1(k)}{1+\theta_2(k)x_1(k)} & \frac{x_1(k)+x_2(k)+\theta_2(k)}{\theta_1(k)+x_2^2(k)} \end{pmatrix} \quad (normal)$$

$$h_1^T(x(k), \theta(k)) = \begin{pmatrix} \frac{x_1(k)+0.5+\theta_1(k)}{1+(\theta_2(k)-0.5)x_1(k)} & \frac{x_1(k)+x_2(k)-0.5+\theta_2(k)}{0.5+\theta_1(k)+x_2^2(k)} \end{pmatrix} \quad (abnormal)$$

$$h_2^T(x(k), \theta(k)) = \begin{pmatrix} \frac{x_1(k)+1.5\theta_1(k)}{1+(0.5+3\theta_2(k))x_1(k)} & \frac{x_1(k)+x_2(k)+0.5+3\theta_2(k)}{1.5\theta_1(k)+x_2^2(k)} \end{pmatrix} \quad (abnormal)$$

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- Active mode detection is desired, despite the noises and unknown varying parameters.

4.2 Generation of the active mode indicators

- Switching law of the system:
  - at $k = 15$: $M_0 \rightarrow M_1$
  - at $k = 30$: $M_1 \rightarrow M_2$
  - at $k = 45$: $M_2 \rightarrow M_0$

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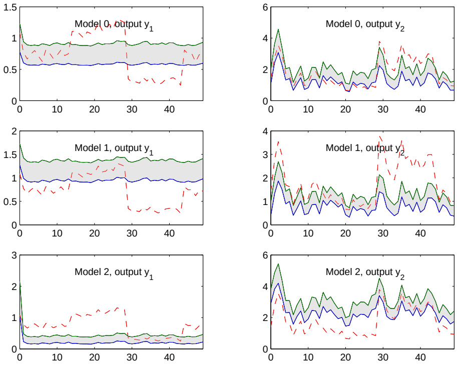
  - at $k = 30$: $M_1 \rightarrow M_2$
  - at $k = 45$: $M_2 \rightarrow M_0$
- At each instant k, the bounds of the N interval outputs are computed :

$$y_{ij}^{-}(k) = \inf_{y_{ij}(k) \in \mathcal{D}_{y,k,i}} y_{ij}(k)$$

$$y_{ij}^{+}(k) = \sup_{y_{ij}(k) \in \mathcal{D}_{y,k,i}} y_{ij}(k)$$

where i is is the number of the model and j the number of the output component.

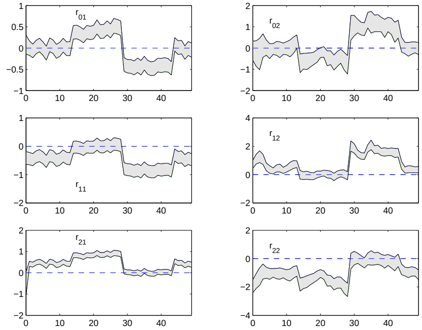


- y_{mj} are drawn in red
- y_{ij}^{-} are drawn in blue
- y_{ij}^{+} are drawn in green

4.3 Generation of the active mode indicators

- Residual generation:

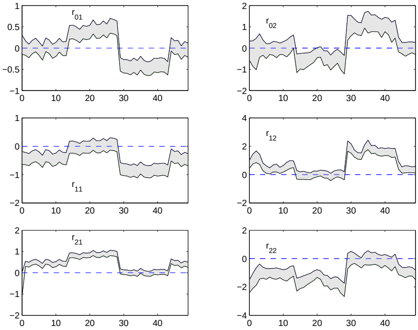
$$r_{ij} = \left[y_{ij}^{-} - y_{mj} \quad y_{ij}^{+} - y_{mj} \right]$$



4.3 Generation of the active mode indicators

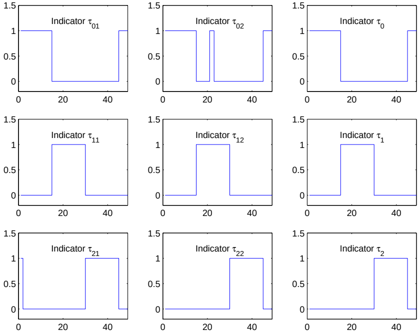
- Residual generation:

$$r_{ij} = \begin{bmatrix} y_{ij}^{-} - y_{mj} & y_{ij}^{+} - y_{mj} \end{bmatrix}$$



- Active mode indicator:

$$\tau_{ij} = \frac{1 - sgn(y_{ij}^{-} - y_j)(y_{ij}^{+} - y_j)}{2}$$



Conclusion and perspectives

- State estimation and fault detection have been proposed for
  - nonlinear systems
  - uncertain and disturbed systems
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  - nonlinear systems
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- The proposed approach is based on interval analysis
- Further works should improve some restrictions
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THANK YOU FOR YOUR ATTENTION

State estimation and fault detection of uncertain systems based on an interval approach

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