Observer based actuator fault tolerant control for nonlinear Takagi-Sugeno systems : an LMI approach

Dalil Ichalal, Benoît Marx, José Ragot and Didier Maquin

Centre de Recherche en Automatique de Nancy (CRAN)
Nancy-Université, CNRS

Mediterranean Conference on Control and Automation, June 23-25, 2010,
Congress Palace, Marrakech, Morocco

Objective of diagnosis and fault tolerant control

- To detect and isolate the (actuator) fault and estimate its magnitude (diagnosis)
- To modify the control law to accomodate the fault

Difficulties

- Taking into account the system complexity in a large operating range
- Nonlinear behavior of the system
- Presence of disturbances

Proposed strategy

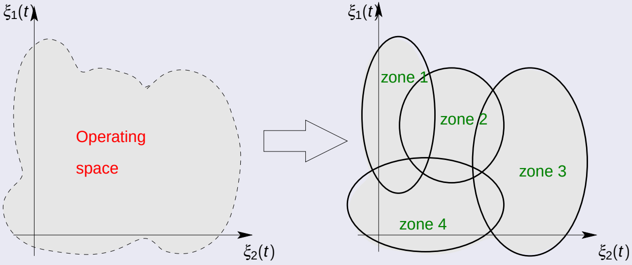
- Takagi-Sugeno representation of nonlinear systems
- Observer-based fault tolerant control design
- Extension of the existing results on linear systems

Plan

1. Takagi-Sugeno approach for modeling
2. Fault tolerant control design
3. Numerical example
4. Conclusions

Takagi-Sugeno approach for modeling

- Operating range decomposition in several local zones.
- A local model represents the behavior of the system in a specific zone.
- The overall behavior of the system is obtained by the aggregation of the sub-models with adequate weighting functions.



Nonlinear system   Multiple Model representation

The main idea of Takagi-Sugeno approach

- Define local models $M_i, \quad i = 1..r$
- Define weighting functions $\mu_i(\xi), \quad 0 \leq \mu_i \leq 1$
- Define an agregation procedure : $M = \sum \mu_i(\xi) M_i$

Interests of Takagi-Sugeno approach

- Simple structure for modeling complex nonlinear systems.
- The specific study of the nonlinearities is not required in some cases.
- Possible extension of the theoretical LTI tools for nonlinear systems.

The difficulties

- How many local models ?
- How to define the domain of influence of each local model ?
- On what variables may depend the weighting functions μ_i ?

Obtaining a Takagi-Sugeno model

- Linearisation of an existing nonlinear model around operating points

  R. Murray-Smith, T. A. Johansen, Multiple model approaches to modelling and control. Taylor & Francis, 1997.

- Direct identification of the model parameters

  K. Gasso, Identification des système dynamiques non linéaires : Approche multimodèle, Ph.D., Institut National Polytechnique de Lorraine, France, 2000.

- Nonlinear transformations of an existing nonlinear model

  A.M. Nagy, G. Mourot, B. Marx, G. Schutz, J. Ragot, Model structure simplification of a biological reactor, 15th IFAC Symp. on System Identification, SYSID'09, 2009

Basic model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) C_i x(t) \end{cases}$$

- Interpolation mechanism $\sum_{i=1}^{r} \mu_i(\xi(t)) = 1$ and $0 \leq \mu_i(\xi(t)) \leq 1, \forall t, \forall i \in \{1, ..., r\}$
- The premise variable $\xi(t)$ are measurable (like $u(t)$, $y(t)$).

A faulty system

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{r} \mu_i(\xi_f(t))(A_i x_f(t) + B_i(u(t) + f(t))) \\ y_f(t) = \sum_{i=1}^{r} \mu_i(\xi_f(t)) C_i x_f(t) \end{cases}$$

- $f(t)$ represents the fault vector (to be detected and accomodated).

Fault tolerant control design

The objective is, in the one hand, to estimate an actuator fault $f(t)$ and the state of the system $x(t)$ with a proportional-integral observer (diagnosis) and, in the other hand, to reconfigure the control law allowing the convergence of $x_f(t)$ to $x(t)$ (FTC).

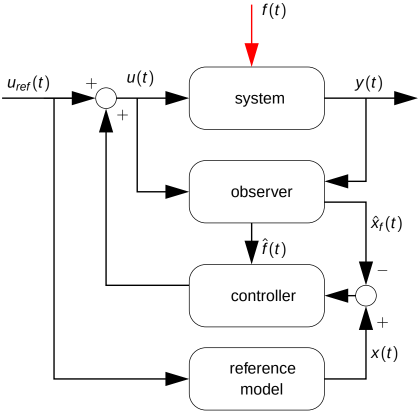


FIGURE: Fault tolerant control scheme

System

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{r} \mu_i(\xi_f(t))(A_i x_f(t) + B_i(u(t) + f(t))) \\ y_f(t) = \sum_{i=1}^{r} \mu_i(\xi_f(t)) C_i x_f(t) \end{cases}$$

Reference model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u_{ref}(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) C_i x(t) \end{cases}$$

Fault tolerant control law

$$u(t) = -\hat{f}(t) + K(x(t) - \hat{x}_f(t)) + u_{ref}(t)$$

Fault tolerant control law

$$u(t) = -\hat{f}(t) + K(x(t) - \hat{x}_f(t)) + u_{ref}(t)$$

System

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{r} \mu_i(\xi_f(t))(A_i x_f(t) + B_i(u(t) + f(t))) \\ y_f(t) = \sum_{i=1}^{r} \mu_i(\xi_f(t)) C_i x_f(t) \end{cases}$$

State and fault estimation

$$\begin{aligned} \dot{\hat{x}}_f(t) &= \sum_{i=1}^{r} \mu_i(\xi_f(t))(A_i \hat{x}_f(t) + B_i(u(t) + \hat{f}(t)) + H_{1i}(y_f(t) - \hat{y}_f(t))) \\ \dot{\hat{f}}(t) &= \sum_{i=1}^{r} \mu_i(\xi_f(t))(H_{2i}(y_f(t) - \hat{y}_f(t))) \\ \hat{y}_f(t) &= \sum_{i=1}^{r} \mu_i(\xi_f(t)) C_i \hat{x}_f(t) \end{aligned}$$

State and fault estimation error + state tracking trajectory error

$$\dot{\tilde{e}}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi_f(t))\mu_j(\xi_f(t))\tilde{A}_{ij}\tilde{e}(t)+\tilde{\Gamma}\delta(t)$$

where

$$\tilde{e}(t)=\begin{pmatrix} x(t)-x_f(t) \\ x_a(t)-\hat{x}_a(t)\end{pmatrix},\ \tilde{\Gamma}=\begin{pmatrix} I_n \\ 0\end{pmatrix},\ \tilde{A}_{ij}=\begin{pmatrix} A_i-B_iK & -\tilde{L}_i \\ 0 & \tilde{A}_i-H_i\tilde{C}_j\end{pmatrix}$$

and

$$x_a(t)=\begin{pmatrix} x_f(t)-\hat{x}_f(t) \\ f(t)-\hat{f}(t)\end{pmatrix},\ \tilde{A}_i=\begin{pmatrix} A_i & B_i \\ 0 & 0\end{pmatrix},\ \tilde{B}_i=\begin{pmatrix} B_i \\ 0\end{pmatrix},\ \tilde{L}_i=\begin{pmatrix} B_iK & B_i\end{pmatrix}$$

$$\delta(t)=\sum_{i=1}^{r}(\mu_i(\xi(t))-\mu_i(\xi_f(t)))(A_ix(t)+B_iu_{ref}(t))$$

- The stability is studied by Lyapunov theory with a quadratic function.
- The $\mathscr{L}_2$ technique is used in order to minimize the effect of $\delta(t)$ on the state and fault estimation errors and the tracking error.

Specifications

$$\dot{\tilde{e}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\xi_f(t))\mu_j(\xi_f(t))\tilde{A}_{ij}\tilde{e}(t) + \tilde{\Gamma}\delta(t)$$

$$\delta(t) = \sum_{i=1}^{r}(\mu_i(\xi(t)) - \mu_i(\xi_f(t)))(A_i x(t) + B_i u_{ref}(t)) \qquad \text{"perturbation"}$$

$$e(t) = x(t) - x_f(t) \qquad \text{tracking error}$$

Objective

- minimize the $\mathscr{L}_2$ gain of the transfer from $\delta(t)$ to $e(t)$

$$\frac{\|e(t)\|_2}{\|\delta(t)\|_2} < \gamma, \quad \|\delta(t)\|_2 \neq 0$$

- ensure asymptotic convergence towards zero of $\delta(t) = 0$

$$\dot{V}(\tilde{e}(t)) + e^T(t)e(t) - \gamma^2\delta^T(t)\delta(t) < 0$$

The system states converge to the reference states and the $\mathscr{L}_2-$gain of the transfer from $\delta(t)$ to $e(t)$ is bounded if there exists symmetric and positive definite matrices X_1, X_2, P_2 and P_3, matrices $\bar{H}_i$ and $\bar{K}$ and positive scalars $\bar{\gamma}$ solution to the following optimization problem

$$\min_{X_1, X_2, P_2, \bar{K}, \bar{H}_i,} \quad \bar{\gamma} \text{ s.t.}$$

$$\begin{aligned} Y_{ii} &< 0,\ i = 1, \dots, r \\ \frac{2}{r-1} Y_{ii} + Y_{ij} + Y_{ji} &< 0,\ i,j = 1, \dots, r,\ i \neq j \end{aligned}$$

where

$$Y_{ij} = \begin{pmatrix} \Psi_i & -B_i M & 0 & I_n & X_1 \\ * & -2\lambda X & \lambda I & 0 & 0 \\ * & * & \Delta_{ij} & 0 & 0 \\ * & * & * & -\bar{\gamma} I_n & 0 \\ * & * & * & * & -I_n \end{pmatrix} < 0$$

- The controller gains and those of the observer are computed from $H_i = \begin{pmatrix} H_{1i} \\ H_{2i} \end{pmatrix} = P_2^{-1} \bar{H}_i$, $K = \bar{K} X_1^{-1}$
- and the attenuation level of the transfer from $\delta(t)$ to $e(t)$ is obtained by $\gamma = \sqrt{\bar{\gamma}}$

- The proposed algorithm of FTC is illustrated by an academic example. Let consider the nonlinear system defined by

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 0 & -3 & 0 \\ 5 & 2 & -4 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad C_1 = C_2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

- The weighting functions μ_i are defined as follows

$$\begin{cases} \mu_1(\xi(t)) = \frac{1-\tanh((\xi(t)))}{2} \\ \mu_2(\xi(t)) = 1 - \mu_1(\xi(t)) \end{cases}$$

- The fault $f(t)$ is time varying and defined by

$$f(t) = \begin{cases} -u(t) & t \geq 10 \\ 0 & t < 10 \end{cases}$$

The first simulation is performed with $\xi(t) = u(t)$.

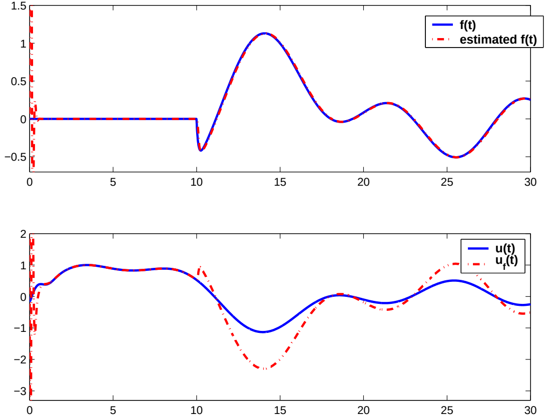


FIGURE: Fault and its estimates (top) Nominal control and FTC (bottom)

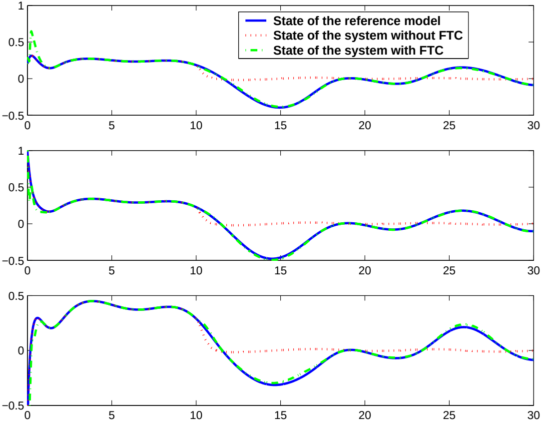


FIGURE: Comparison between states of the system without fault, states with fault and nominal control and states with fault and FTC

An input-to-state stability is obtained because $\mu_i(\xi(t)) - \mu_i(\xi_f(t)) \nrightarrow 0$ $(u(t) \neq u_{ref}(t))$

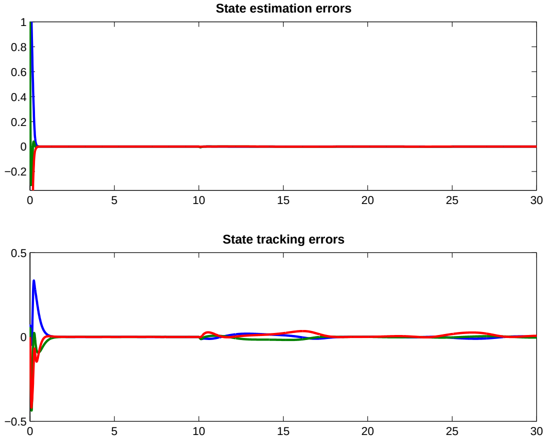


FIGURE: State estimation errors (top) State tracking errors (bottom)

In the second simulation, the premise variable is $\xi(t) = y(t)$. With the FTC law $y_f(t) \to y(t)$ then $\mu_i(\xi(t)) - \mu_i(\xi_f(t)) \to 0$

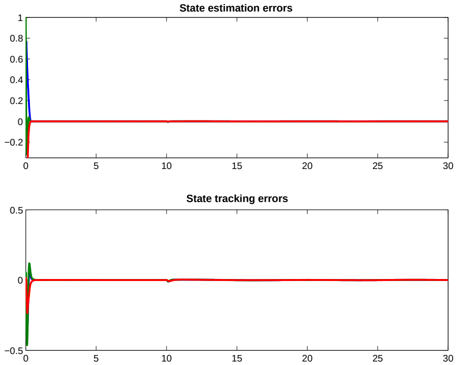


FIGURE: State estimation errors (top) State tracking errors (bottom)

Conclusions

- Active fault tolerant control law for nonlinear systems represented by a Takagi-Sugeno structure.
- State and fault estimation
- Fault tolerant control with reference trajectory tracking
- The problem of FTC design is expressed via an optimization problem subject to LMI (Linear Matrix Inequality) constraints.

Perspectives

- Study of the unmeasurable premise variable case ($\xi(t) = x(t)$).
- Conservatism reduction with Polyquadratic Lyapunov functions and Polya's theorem
- Extension to robust fault tolerant control (disturbances and modeling uncertainties).

Get in touch

Didier Maquin

Professor in Automatic Control
National Polytechnic Institute of Nancy

High School of Electrical and Mechanical Engineering

Research Center for Automatic Control

didier.maquin@ensem.inpl-nancy.fr

More information ?

Personal : `http://www.ensem.inpl-nancy.fr/Didier.Maquin/en/`

Research Lab : `http://www.cran.uhp-nancy.fr/anglais/`