Fault diagnosis for nonlinear systems represented by heterogeneous multiple models

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Introduction

Motivations

- Fault detection and isolation (FDI) is increasingly integrated in many real-world applications
- Fault indicators can be obtained with the help of an analytic redundancy
- Analytic redundancy given for example from the state estimation provided by an observer
- FDI: testing the coherency between the measured and the estimated variables

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Main difficulties

- The state estimation quality depends on the model accuracy in a large operating range of the system (global modelling)
- Nonlinear models are often unavoidable for this purpose
- The model complexity makes it difficult the design of appropriate observers
- Consequently: we seek “simple” models with high accuracy degree !

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Proposed strategy

- System modelling using the multiple model approach (complexity/accuracy)
- Extension of the classic linear estimation techniques to multiple models
- Fault indicator generation using observer bank

Plan

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1. Multiple models basis
2. State estimation techniques
   - Proportional gain observer design
   - Proportional-Integral gain Observer design
3. Diagnosis procedures
4. Conclusions and perspectives

Plan

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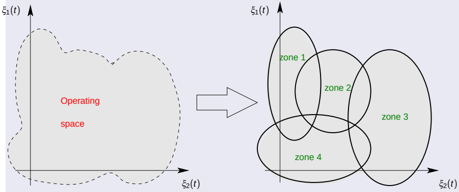
Introduction – philosophy

Multiple models basis: divide and conquer philosophy



Nonlinear system

Multiple model representation

Multiple model = interpolation of a set of linear submodels

- Good trade-off: complexity/accuracy
- Extension of some linear results to nonlinear systems
- The specific analysis of non-linearities can be avoided

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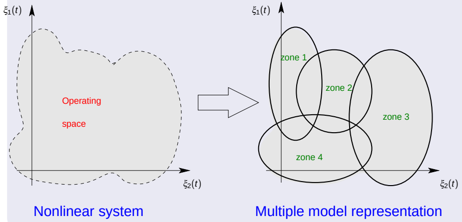

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How the submodels can be interconnected ?

Classic structure
Takagi-Sugeno multiple model

- A state vector is common to all submodels
- Homogeneous submodels: same state dimension

Proposed structure
Heterogeneous multiple model

- A state vector for each submodel
- Heterogeneous submodels: different state dimensions
- Poorly investigated !

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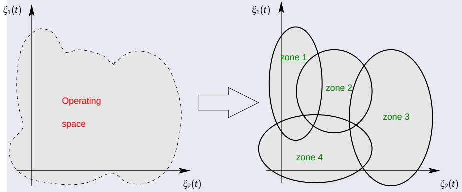

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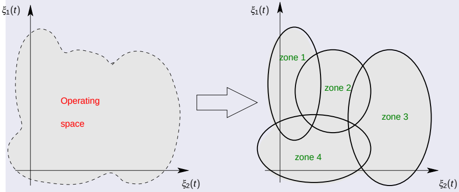

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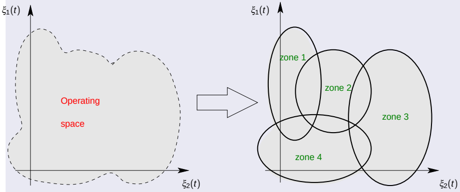

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Heterogeneous multiple model

Used structure

Heterogeneous multiple model :

Collection of submodels

$$\begin{cases} x_i(t+1) & = & A_i x_i(t) + B_i u(t) \\ y_i(t) & = & C_i x_i(t) \end{cases}$$

Characteristics

- The multiple model output is a weighted sum of the submodel outputs
- The state vector dimension of submodels can be different (heterogeneous submodels)

Benefits and motivations

- Modelling of complex variable structure systems (good flexibility and generality)
- Only few studies are devoted to the state estimation and/or FDI strategies

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Interpolation mechanism
$$y(t) = \sum_{i=1}^{L} \mu_i(\xi(t)) y_i(t)$$

$\xi(t)$: measurable decision variable $\mu_i(\xi(t))$: weighting functions

$$\sum_{i=1}^{L} \mu_i(\xi(t)) = 1 \quad \text{and} \quad 0 \leq \mu_i(\xi(t)) \leq 1 \quad \forall i \in 1,\ldots,L \quad \forall t$$

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State estimation: P observer

Structure of the proportional gain observer (PO)

- Multiple model system characterisation

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- Proportional observer: copy of the system with a proportional correction action K_i

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Computation of the gains K_i to ensure the exponential convergence of $x_i(t)$ towards $\hat{x}_i(t)$

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State estimation: P observer design

Rewriting the multiple model equations

1. Let us introduce an augmented state vector

$$x(t) = \left[x_1^T(t) \cdots x_i^T(t) \cdots x_L^T(t)\right]^T \in \mathbb{R}^n, \quad n = \sum_{i=1}^{L} n_i$$

2. The new form of the multiple model is

$$\begin{aligned} x(t+1) &= \tilde{A}x(t) + \tilde{B}u(t) + \tilde{K}(y(t) - \hat{y}(t)) \\ y(t) &= \tilde{C}(t)x(t) \end{aligned}$$

with

$$\begin{cases} \tilde{A} = diag\{A_1, \cdots, A_n\} \in \mathbb{R}^{n \times n} \\ \tilde{B} = [B_1^T, \cdots, B_n^T]^T \in \mathbb{R}^{n \times m} \\ \tilde{C}(t) = [\mu_1(t)C_1, \cdots, \mu_L(t)C_L] \in \mathbb{R}^{p \times n} \\ \tilde{K} = [K_1^T, \cdots, K_L^T]^T \in \mathbb{R}^{n \times p} \end{cases}$$

3. The time-varying matrix $\tilde{C}(t)$ can be rewritten as follows:

$$\begin{aligned} \tilde{C}(t) &= \sum_{i=1}^{L} \mu_i(t)\tilde{C}_i \\ \tilde{C}_i &= \left[0 \quad \cdots \quad C_i \quad \cdots \quad 0\right] \end{aligned}$$

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State estimation: P observer design

Design methodology: Lyapunov theory

(i) Determining the estimation error vector $e(t) = x(t) - \hat{x}(t)$

(ii) A Lyapunov function must be chosen e.g.

$$V(t) = e^T(t)Pe(t) \text{ where } P \text{ is a matrix } P = P^T > 0$$

(iii) The convergence of $e(t)$ is ensured if:

$$\Delta V(t) + 2\alpha V(t) < 0, \quad \Delta V(t) = V(t+1) - V(t)$$

(iv) The design goal is guaranteed if there exists a matrix P (conditions under a LMI form)

LMI conditions to be satisfied for PO design

The state estimation error converges exponentially towards zero if there exists a symmetric, positive definite matrix P and a matrix G solution of the constrained LMI problem:

$$\begin{bmatrix} (1-2\alpha)P & \tilde{A}^T P - \tilde{C}_i^T G^T \\ P\tilde{A} - G\tilde{C}_i & P \end{bmatrix} > 0, \qquad i = 1, \cdots, L \ ,$$

for a given decay rate $0 < \alpha < 0.5$. The gain is given by $\tilde{K} = P^{-1}G$.

- Numerical resolution using convex optimisation algorithms

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State estimation: PI observer design

Proportional-integral gain observer structure (PIO)

- Unknown inputs $\eta(t)$ (e.g. actuator or sensor faults) are now taken into account

$$\begin{cases} x_i(t+1) = A_i x_i(t) + B_i u(t) + D_i \eta(t) \\ y_i(t) = C_i x_i(t) \\ y(t) = \sum_{i=1}^{L} \mu_i(\xi(t)) y_i(t) + V \eta(t) \end{cases}$$

- PI observer: a proportional correction K_i + an integral correction action K_I

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- Simultaneous state and unknown input estimation i.e. $\hat{x}(t) \to x(t)$ and $\hat{\eta}(t) \to \eta(t)$

Assumption

The unknown input $\eta(t)$ is a constant signal

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State estimation: PI observer design

Design goals

Computation of the gains K_i and K_I to ensure the exponential convergence of:

- $\hat{x}(t)$ towards $x(t)$
- $\hat{\eta}(t)$ towards $\eta(t)$

Design methodology

(i) The same PO design procedure can be applied to PIO design

(ii) Introducing an augmented form for both multiple model and PI observer

(iii) Determining the estimation error vectors $e(t) = x(t) - \hat{x}(t)$ and $\varepsilon(t) = \eta(t) - \hat{\eta}(t)$

(iv) Using the Lyapunov method

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$$\begin{bmatrix} (1-2\alpha)P & A_a^T P - \bar{C}_i^T G^T \\ PA_a - G\bar{C}_i & P \end{bmatrix} > 0, \quad i = 1, \cdots, L,$$

where

$$\bar{C}_i = \begin{bmatrix} \tilde{C}_i & V \end{bmatrix}, \; A_a = \begin{bmatrix} \tilde{A} & \tilde{D} \\ 0 & \mathrm{I} \end{bmatrix}, \; K_a = \begin{bmatrix} \tilde{K} \\ K_I \end{bmatrix}$$

for a given decay rate $0 < \alpha < 0.5$. The PI observer gain is given by $K_a = P^{-1}G$.

State estimation: PI observer design

Design goals

Computation of the gains K_i and K_I to ensure the exponential convergence of:

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Application to diagnosis

Diagnosis goals

- Detection and isolation of faults
- Generation of sensor fault indicators (residual signals)
- The residual signal must be null when the system behaviour is according to the expected behaviour

Proposed observer-based FDI strategies

Generation of sensor fault indicators from:

1. an observer bank using PO (classic Dedicated Observer Scheme)
2. simultaneous state and fault estimation using PIO

These FDI strategies are tested using a simplified bioreactor model

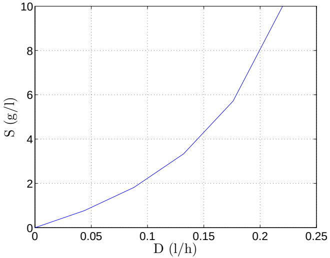
Diagnosis: system presentation

Bioreactor model

The dynamic behaviour of a continuous bioreactor homogeneously mixed and limited by a single substrate can be described by

$$\begin{aligned} \dot{S}(t) &= D(t)(S_{in} - S(t)) - kr(t) \\ \dot{X}(t) &= -D(t)X(t) + r(t) \\ r(t) &= \frac{\mu_{max} S(t)}{K_s + S(t)} X(t) \end{aligned}$$

- $D(t) > 0$ dilution rate (input)
- $S(t)$ carbon substrate and $X(t)$ biomass (outputs)
- $r(t)$ reaction velocity (i.e. the biomass production)
- S_{in}, μ_{max}, K_s and k are some constants



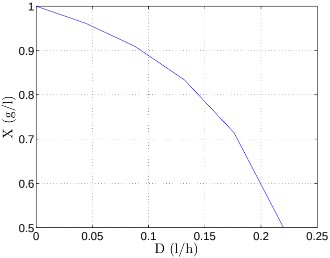


Figure: Static behaviour of the bioreactor

Diagnosis: identification step

Multiple model representation of the bioreactor

- We seek a multiple model representation of the bioreactor system
- The decision variable is the input signal $D(t)$
- Two submodels are identified using a nonlinear identification algorithm

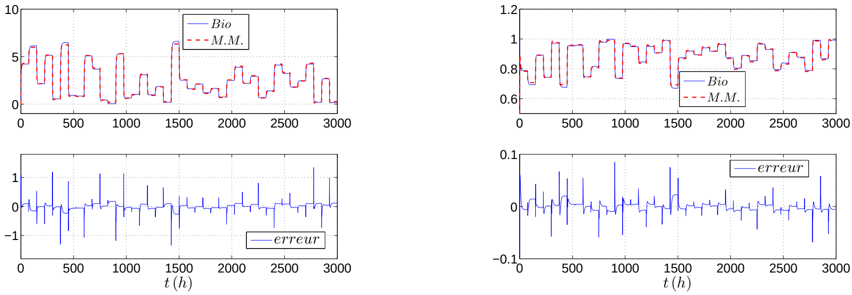
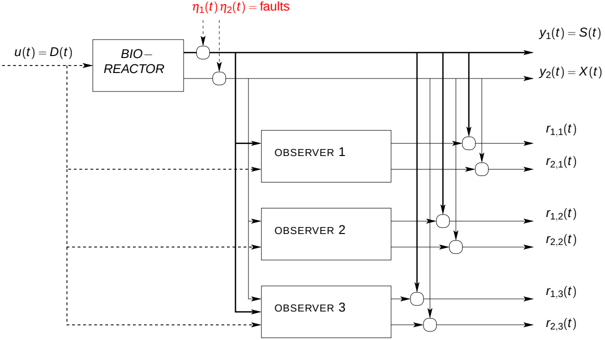


Figure: Carbon substrate $S(t)$ and biomass $X(t)$

Diagnosis: first strategy 1/2

- The residual signals are obtained by comparing the measured and the estimated variables
- Dedicated Observer Scheme is employed to provide structured residual signals
- Decision logic rule is needed via an incidence matrix



	$r_{1,1}$	$r_{2,1}$	$r_{1,2}$	$r_{2,2}$	$r_{1,3}$	$r_{2,3}$
η_1	?	?	1	0	?	?
η_2	0	1	?	?	?	?

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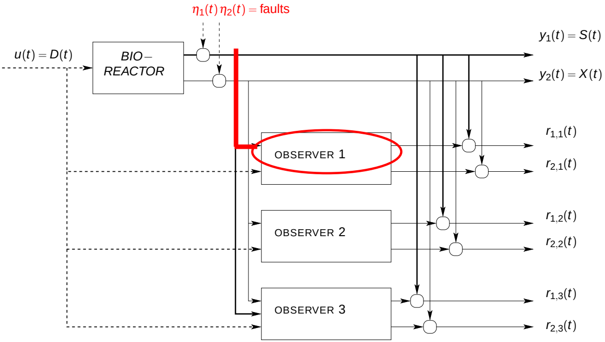

	$r_{1,1}$	$r_{2,1}$	$r_{1,2}$	$r_{2,2}$	$r_{1,3}$	$r_{2,3}$
η_1	?	?	1	0	?	?
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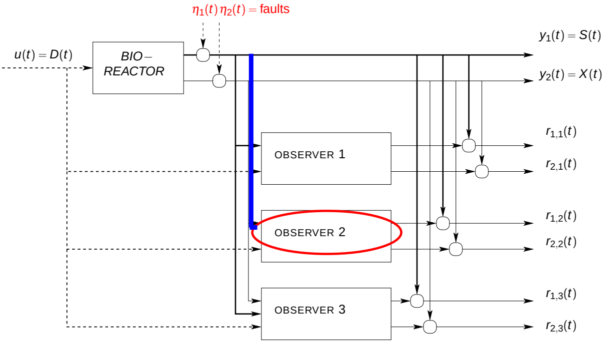

	$r_{1,1}$	$r_{2,1}$	$r_{1,2}$	$r_{2,2}$	$r_{1,3}$	$r_{2,3}$
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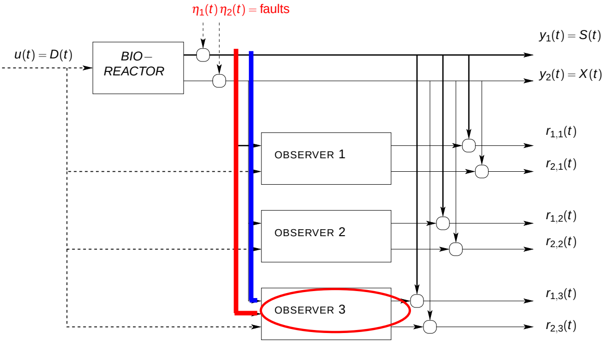

	$r_{1,1}$	$r_{2,1}$	$r_{1,2}$	$r_{2,2}$	$r_{1,3}$	$r_{2,3}$
η_1	?	?	1	0	?	?
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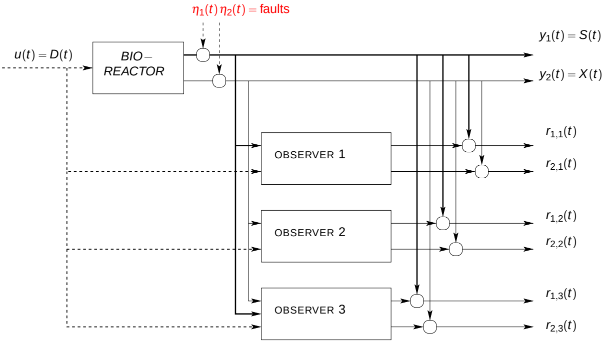

Detection and localisation

	$r_{1,1}$	$r_{2,1}$	$r_{1,2}$	$r_{2,2}$	$r_{1,3}$	$r_{2,3}$
η_1	?	?	1	0	?	?
η_2	0	1	?	?	?	?

Diagnosis: first strategy 1/2

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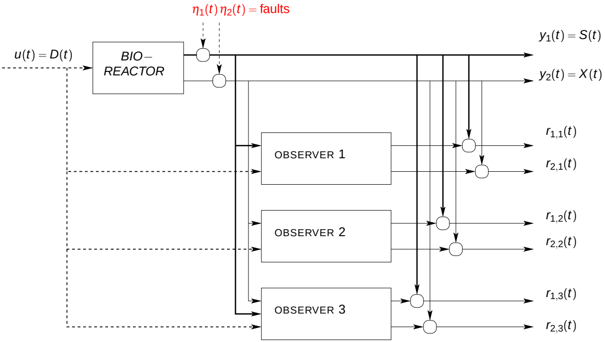

	$r_{1,1}$	$r_{2,1}$	$r_{1,2}$	$r_{2,2}$	$r_{1,3}$	$r_{2,3}$
η_1	?	?	1	0	?	?
η_2	0	1	?	?	?	?

Detection and localisation

Diagnosis: first strategy 2/2

The considered faults acting on the system outputs are step signals of amplitude equal to 10% of the maximal amplitude of each output

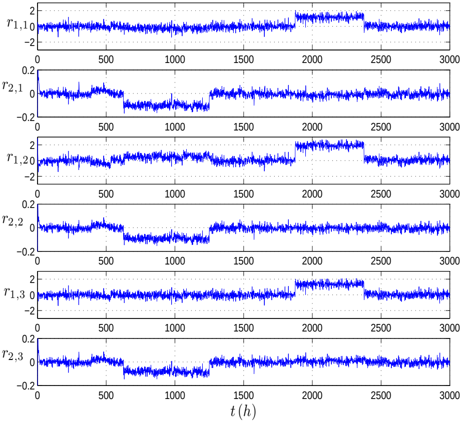

Figure: Residual signals

	$r_{1,1}$	$r_{2,1}$	$r_{1,2}$	$r_{2,2}$	$r_{1,3}$	$r_{2,3}$
η_1	?	?	1	0	?	?
η_2	0	1	?	?	?	?

Table: Incidence matrix

Diagnosis: first strategy 2/2

The considered faults acting on the system outputs are step signals of amplitude equal to 10% of the maximal amplitude of each output

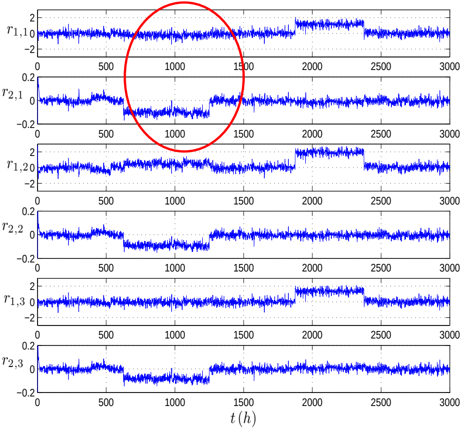

Figure: Residual signals

	$r_{1,1}$	$r_{2,1}$	$r_{1,2}$	$r_{2,2}$	$r_{1,3}$	$r_{2,3}$
η_1	?	?	1	0	?	?
η_2	0	1	?	?	?	?

Table: Incidence matrix

- Configuration $r_{1,1} = 0$ and $r_{2,1} = 1$ consequently a fault on y_2

Diagnosis: first strategy 2/2

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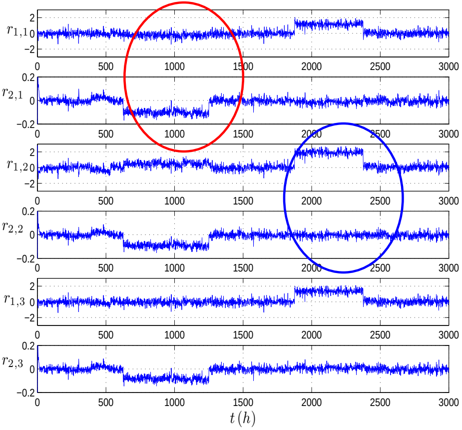

Figure: Residual signals

	$r_{1,1}$	$r_{2,1}$	$r_{1,2}$	$r_{2,2}$	$r_{1,3}$	$r_{2,3}$
η_1	?	?	1	0	?	?
η_2	0	1	?	?	?	?

Table: Incidence matrix

- Configuration $r_{1,1} = 0$ and $r_{2,1} = 1$ consequently a fault on y_2
- Configuration $r_{1,2} = 1$ and $r_{2,2} = 0$ consequently a fault on y_1

Diagnosis: first strategy 2/2

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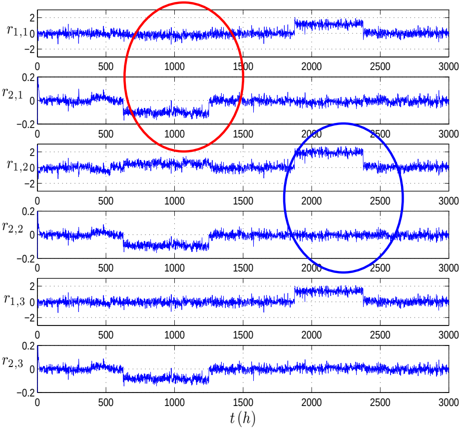

Figure: Residual signals

	$r_{1,1}$	$r_{2,1}$	$r_{1,2}$	$r_{2,2}$	$r_{1,3}$	$r_{2,3}$
η_1	?	?	1	0	?	?
η_2	0	1	?	?	?	?

Table: Incidence matrix

- Configuration $r_{1,1} = 0$ and $r_{2,1} = 1$ consequently a fault on y_2
- Configuration $r_{1,2} = 1$ and $r_{2,2} = 0$ consequently a fault on y_1
- Fault detection and isolation: OK

Diagnosis: first strategy 2/2

The considered faults acting on the system outputs are step signals of amplitude equal to 10% of the maximal amplitude of each output

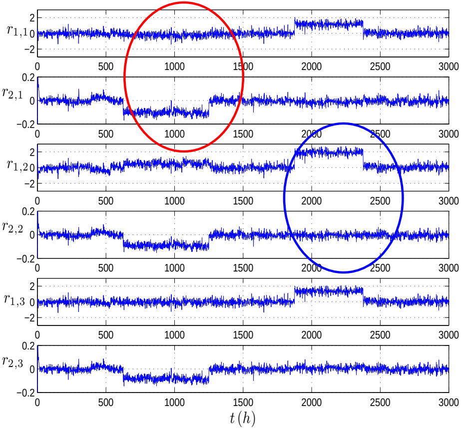

Figure: Residual signals

	$r_{1,1}$	$r_{2,1}$	$r_{1,2}$	$r_{2,2}$	$r_{1,3}$	$r_{2,3}$
η_1	?	?	1	0	?	?
η_2	0	1	?	?	?	?

Table: Incidence matrix

- Configuration $r_{1,1} = 0$ and $r_{2,1} = 1$ consequently a fault on y_2
- Configuration $r_{1,2} = 1$ and $r_{2,2} = 0$ consequently a fault on y_1
- Fault detection and isolation: OK
- Isolation of simultaneous faults: KO
- Fault estimation: KO

Diagnosis: second strategy

- The sensor faults are unknown inputs to be estimated
- Let us consider the proposed PIO
- The fault estimation is directly considered as a residual signal

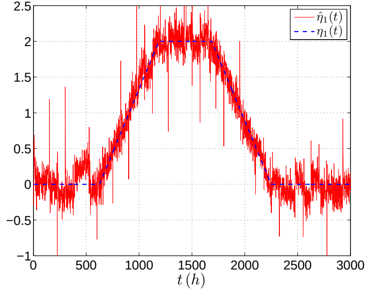

Figure: Fault estimations (residual signals)

- Fault detection, isolation and identification is possible for simultaneous faults
- Good robustness of the PIO w.r.t. time-varying faults

Conclusions and perspectives

Conclusions

- The multiple model approach is a powerful modelling technique (accuracy/complexity)
- Two observers, proportional and proportional-integral, have been proposed for heterogeneous multiple models
- They have been implemented for FDI purposes using two different strategies

Perspectives

- Conservatism reduction of the LMI conditions (use of more complex Lyapunov functions)
- Reduction of the augmented matrix size using other design approaches
- Relaxation of the assumption about the unknown inputs using a multiple integral observer

Fault diagnosis for nonlinear systems represented by heterogeneous multiple models

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