Adaptive observer for fault estimation in nonlinear systems described by a Takagi-Sugeno model

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Outline

1. Multiple model structures
2. Fault estimation
3. Example
4. Conclusion and future works
Multiple model structures
Operating range decomposition into a finite number of operating zones
Each operating zone is modelled using a simple – linear – submodel
The relative contribution of each submodel is quantified by a weighting function
The global nonlinear model is the sum of the submodels weighted by these functions

Advantages of multiple models

Well approximate the behaviour of the complex nonlinear systems
Several results for linear systems can be extended to nonlinear systems through this kind of models
Takagi-Sugeno model

Structure of the model

Multiple model with a unique state

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\
y(t) &= \sum_{i=1}^{M} \mu_i(\xi(t))C_i x(t) = Cx(t)
\end{align*}
\]

\[
\left\{ \begin{array}{l}
\sum_{i=1}^{M} \mu_i(\xi(t)) = 1, \quad \forall t \\
0 \leq \mu_i(\xi(t)) \leq 1, \quad \forall t, \forall i \in \{1, \ldots, M\}
\end{array} \right.
\]

- The local models share the same state vector
- The output equation is frequently assumed linear (all the matrices \(C_i\) are equal)
Fault estimation
Problem formulation

System affected by actuator and sensor faults + output noise

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + E_i f_a(t)) \\
y(t) &= C x(t) + F f_s(t) + D w(t)
\end{align*}
\]

Objective: transform the original sensor faults into actuator faults

\[z(t): \text{filtered version of the output } y(t)\]

\[
\dot{z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(-\bar{A}_i z(t) + \bar{A}_i (C x(t) + F f_s(t) + D w(t)))
\]

\(-\bar{A}_i: \text{stable matrices}\)
Problem formulation

Augmented state and fault vector

\[ X = \begin{bmatrix} x \\ z \end{bmatrix}, \quad f = \begin{bmatrix} f_a \\ f_s \end{bmatrix} \]

Augmented system

\[
\begin{cases}
    \dot{X}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_{ai}X(t) + B_{ai}u(t) + E_{ai}f(t)) + F_{ai}w(t)) \\
    Y(t) = C_aX(t)
\end{cases}
\]

\[
A_{ai} = \begin{bmatrix} A_i & 0 \\ \bar{A}_iC & -\bar{A}_i \end{bmatrix}, \quad B_{ai} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \quad E_{ai} = \begin{bmatrix} E_i & 0 \\ 0 & \bar{A}_iF \end{bmatrix}, \quad F_{ai} = \begin{bmatrix} 0 \\ \bar{A}_iD \end{bmatrix}
\]

\[ C_a = \begin{bmatrix} 0 & 1 \end{bmatrix} \]
Re-written system in an augmented form

\[
\begin{align*}
\dot{X}(t) &= \sum_{i=1}^{M} \mu_i(\xi(t))(A_{ai}X(t) + B_{ai}u(t) + E_{ai}\hat{f}(t) + F_{ai}w(t)) \\
Y(t) &= C_aX(t)
\end{align*}
\]

Proportional integral observer

\[
\begin{align*}
\dot{\hat{X}}(t) &= \sum_{i=1}^{M} \mu_i(\xi(t))(A_{ai}\hat{X}(t) + B_{ai}u(t) + E_{ai}\hat{f}(t) + K_i(Y(t) - \hat{Y}(t))) \\
\dot{\hat{f}}(t) &= \sum_{i=1}^{M} \mu_i(\xi(t))(L_i(Y(t) - \hat{Y}(t))) \\
\hat{Y}(t) &= C_a\hat{X}(t)
\end{align*}
\]
Fault estimation

Estimation errors

\[ \tilde{x}(t) = X(t) - \hat{X}(t) \quad \text{and} \quad \tilde{f}(t) = f(t) - \hat{f}(t) \]

Dynamics of the errors

\[ \dot{\tilde{x}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))((A_{ai} - K_i C_a)\tilde{x}(t) + E_{ai}\tilde{f}(t) + F_{ai}w(t)) \]
\[ \dot{\tilde{f}}(t) = \dot{f}(t) - \sum_{i=1}^{M} \mu_i(\xi(t))L_i C_a \tilde{x}(t) \]

Augmented system of estimation errors

The following vectors are introduced:

\[ \varphi(t) = \begin{bmatrix} \tilde{x}(t) \\ \tilde{f}(t) \end{bmatrix} \quad \text{and} \quad \epsilon(t) = \begin{bmatrix} w(t) \\ \dot{f}(t) \end{bmatrix} \]
Fault estimation

Augmented system of estimation errors

\[ \dot{\varphi}(t) = A_m \varphi(t) + B_m \epsilon(t) \]

with:

\[ A_m = \sum_{i=1}^{M} \mu_i(\xi(t)) \tilde{A}_{0i} \quad \text{and} \quad B_m = \sum_{i=1}^{M} \mu_i(\xi(t)) \tilde{B}_{0i}, \]

\[ \tilde{A}_{0i} = \begin{bmatrix} A_{ai} - K_i C_a E_{ai} & E_{ai} \\ -L_i C_a & 0 \end{bmatrix}, \quad \tilde{B}_{0i} = \begin{bmatrix} F_{ai} & 0 \\ 0 & I \end{bmatrix} \]

Convergence of the generalized estimation error

Let us consider the following quadratic Lyapunov candidate function:

\[ V(t) = \varphi^T(t) P \varphi(t) \]
Robust state and fault estimation

Convergence conditions

The problem of robust state and fault estimation is reduced to find the gains $K_i$ and $L_i$ of the observer to ensure an asymptotic convergence of $\varphi(t)$ towards zero if $\varepsilon(t) = 0$ and to ensure a bounded error in the case where $\varepsilon(t) \neq 0$, i.e.:

$$\lim_{t \to \infty} \varphi(t) = 0 \quad \text{for } \varepsilon(t) = 0$$

$$\| \varphi(t) \|_{Q_\varphi} \leq \lambda \| \varepsilon(t) \|_{Q_\varepsilon} \quad \text{for } \varepsilon(t) \neq 0$$

where $\lambda > 0$ is the attenuation level.

To satisfy these constraints, it is sufficient to find a Lyapunov function $V(t)$ such that

$$\dot{V}(t) + \varphi^T(t) Q_\varphi \varphi(t) - \lambda^2 \varepsilon^T(t) Q_\varepsilon \varepsilon(t) < 0$$
Augmented system

\[
\dot{\phi}(t) = A_m \phi(t) + B_m \epsilon(t)
\]

\[
\dot{V}(t) + \phi^T(t) Q_\phi \phi(t) - \lambda^2 \epsilon^T(t) Q_\epsilon \epsilon(t) < 0
\]

Omitting to denote \( t \), the last inequality can also be written as:

\[
\psi^T \Omega_m \psi < 0
\]

with:

\[
\psi = \begin{bmatrix} \phi \\ \epsilon \end{bmatrix} \quad \text{and} \quad \Omega_m = \begin{bmatrix} A_m^T P + P A_m + Q_\phi & P B_m \\ B_m^T P & -\lambda^2 Q_\epsilon \end{bmatrix}
\]

This inequality holds if \( \Omega_m < 0 \).
Robust state and fault estimation

LMI formulation

\[ A_m = \sum_{i=1}^{M} \mu_i(\xi(t))\tilde{A}_{0i}, \quad \tilde{A}_{0i} = \tilde{A}_i - \tilde{K}_i \tilde{C} \]

where:

\[ \tilde{A}_i = \begin{bmatrix} A_{ai} & E_{ai} \\ 0 & 0 \end{bmatrix}, \quad \tilde{K}_i = \begin{bmatrix} K_i \\ L_i \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} C_a & 0 \end{bmatrix} \]

With the changes of variables \( G_i = P\tilde{K}_i \) and \( m = \lambda^2 \), the matrix \( \Omega_m \) can be written as:

\[ \Omega_m = \sum_{i=1}^{M} \mu_i(\xi(t))\Omega_i \]

\[ \Omega_i = \begin{bmatrix} P\tilde{A}_i + \tilde{A}_i^T P - G_i\tilde{C} - \tilde{C}^T G_i^T + Q_\varphi & P\tilde{B}_{0i} \\ \tilde{B}_{0i}^T P & -mQ_\varepsilon \end{bmatrix} \]
Theorem

The system: \( \dot{\varphi} = A_m \varphi + B_m \varepsilon \) describing the time evolution of the state estimation error \( \tilde{x} \) and the fault estimation error \( \tilde{f} \) is stable and the \( L_2 \)-gain of the transfer from \( \varepsilon(t) \) to \( \varphi(t) \) is bounded, if there exists a symmetric, positive definite matrix \( P \), gain matrices \( G_i, i \in \{1...M\} \) and a positive scalar \( m \) such that the following LMI are verified:

\[
\begin{bmatrix}
    P \tilde{A}_i + \tilde{A}_i^T P - G_i \tilde{C} - \tilde{C}^T G_i^T + Q_\varphi & P \tilde{B}_{0i} \\
    \tilde{B}_{0i}^T P & -mQ_\varepsilon
\end{bmatrix} < 0
\]

The proportional and integral gains of the observer are computed using \( \tilde{K}_{mi} = P^{-1} G_i \) and the attenuation level is given by \( \lambda = \sqrt{m} \).
Example
Example

System matrices

\[ A_1 = \begin{bmatrix} -0.3 & -3 & -0.5 & 0.1 \\ -0.7 & -5 & 2 & 4 \\ 2 & -0.5 & -5 & -0.9 \\ -0.7 & -2 & 1 & -0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.7 & -7 & -1.5 & -7 \\ -0.2 & -2 & 0.6 & 1.3 \\ 5 & -1.5 & -9 & -3.9 \\ -0.4 & -1 & -0.3 & -1 \end{bmatrix} \]

\[ B_1 = \begin{bmatrix} 1 & 2 \\ 5 & 1 \\ 4 & -3 \\ 1 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 2 \\ -1 & -2 \end{bmatrix}, \quad D = \begin{bmatrix} 0.5 & 0.5 \\ 0.2 & 0.2 \\ 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \quad F = \begin{bmatrix} 3.25 & 5 \\ 0 & 0.5 \\ -3.25 & 1.75 \\ 5.75 & 5 \end{bmatrix} \]

\[ E_1 = B_1, \quad E_2 = B_2, \quad C = I, \quad \xi(t) = u(t) \]

\[ Q_\varphi = Q_\epsilon = I, \quad \bar{A}_1 = 5 \times I, \quad \bar{A}_2 = 10 \times I \]

Input signals

\[ u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \]

\[ u_1(t) : \text{telegraph type signal whose amplitude belongs to } [0, 0.5] \]

\[ u_2(t) : 0.4 + 0.25\sin(\pi t) \]
**FIGURE:** System inputs $u(t)$
Actuator and sensor faults

Actuator fault is defined as follows:

\[ f_a(t) = \begin{bmatrix} f_{a1}(t) \\ f_{a2}(t) \end{bmatrix} \]

with:

\[ f_{a1}(t) = \begin{cases} 0.4 \sin(\pi t), & 15 \text{ s} < t < 75 \text{ s} \\ 0, & \text{otherwise} \end{cases}, \quad f_{a2}(t) = \begin{cases} 0, & t < 20 \text{ s} \\ 0.3, & 20 \text{ s} < t < 80 \text{ s} \\ 0.5, & t > 80 \text{ s} \end{cases} \]

and the sensor fault \( f_s(t) \) is defined as follows:

\[ f_s(t) = \begin{bmatrix} f_{s1}(t) \\ f_{s2}(t) \end{bmatrix} \]

with:

\[ f_{s1}(t) = \begin{cases} 0, & t \leq 35 \text{ s} \\ 0.6, & t > 35 \text{ s} \end{cases}, \quad f_{s2}(t) = \begin{cases} 0, & t \leq 25 \text{ s} \\ \sin(0.6\pi t), & t > 25 \text{ s} \end{cases} \]
**Figure:** State estimation errors
Figure: Actuator faults and their estimation
**Figure:** Sensor faults and their estimation
Conclusion and future works
Conclusion

- Original method for state and fault estimation for systems described by nonlinear Takagi-Sugeno models using a proportional integral observer
- Using an adequate rewriting of the system equations, the sensor fault affecting the initial system is transformed into an actuator fault (i.e. into an unknown input); this transformation eases the simultaneous sensor and actuator fault estimation.

Future works

- Fault estimates can be used to conceive a fault tolerant control strategy able to cancel the fault effects on the system performances and behavior
- The proposed method should be extended to TS models with unmeasurable decision variables
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