Adaptive observer for fault estimation in nonlinear systems described by a Takagi-Sugeno model

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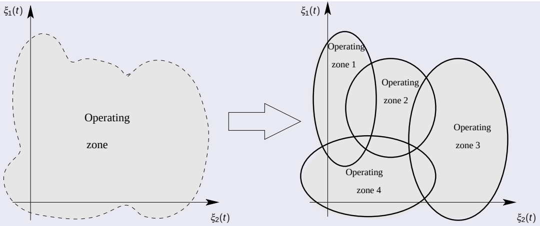
Outline

Multiple model structures

Principle of the multiple models



NON LINEAR SYSTEM
SYSTEME NON LINEAIRE

MULTIPLE MODEL REPRESENTATION
REPRESENTATION MULTIMODELE

- Operating range decomposition into a finite number of operating zones
- Each operating zone is modelled using a simple – linear – submodel
- The relative contribution of each submodel is quantified by a weighting function
- The global nonlinear model is the sum of the submodels weighted by these functions

Advantages of multiple models

- Well approximate the behaviour of the complex nonlinear systems
- Several results for linear systems can be extended to nonlinear systems through this kind of models

Takagi-Sugeno model

Structure of the model

Multiple model with a unique state

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) C_i x(t) = Cx(t) \end{cases}$$

$$\mu_i(\xi(t)) \begin{cases} \sum_{i=1}^{M} \mu_i(\xi(t)) = 1, \quad \forall t \\ 0 \leq \mu_i(\xi(t)) \leq 1, \quad \forall t, \ \forall i \in \{1, ..., M\} \end{cases}$$

$\xi(t)$: *decision variable*

- The local models share the same state vector
- The output equation is frequently assumed linear (all the matrices C_i are equal)

Fault estimation

System affected by actuator and sensor faults + output noise

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + E_i f_a(t)) \\ y(t) = Cx(t) + F f_s(t) + Dw(t) \end{cases}$$

Objective : transform the original sensor faults into actuator faults

$z(t)$: filtered version of the output $y(t)$

$$\dot{z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(-\bar{A}_i z(t) + \bar{A}_i (Cx(t) + F f_s(t) + Dw(t)))$$

$-\bar{A}_i$: stable matrices

Problem formulation

Augmented state and fault vector

$$X = \begin{bmatrix} x \\ z \end{bmatrix}, \quad f = \begin{bmatrix} f_a \\ f_s \end{bmatrix}$$

Augmented system

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_{ai}X(t) + B_{ai}u(t) + E_{ai}f(t)) + F_{ai}w(t)) \\ Y(t) = C_a X(t) \end{cases}$$

$$A_{ai} = \begin{bmatrix} A_i & 0 \\ \bar{A}_i C & -\bar{A}_i \end{bmatrix}, \; B_{ai} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \; E_{ai} = \begin{bmatrix} E_i & 0 \\ 0 & \bar{A}_i F \end{bmatrix}, \; F_{ai} = \begin{bmatrix} 0 \\ \bar{A}_i D \end{bmatrix}$$

$$C_a = \begin{bmatrix} 0 & I \end{bmatrix}$$

Fault estimation

Re-written system in an augmented form

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_{ai}X(t) + B_{ai}u(t) + E_{ai}f(t) + F_{ai}w(t)) \\ Y(t) = C_a X(t) \end{cases}$$

Proportional integral observer

$$\begin{cases} \dot{\hat{X}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_{ai}\hat{X}(t) + B_{ai}u(t) + E_{ai}\hat{f}(t) + K_i(Y(t) - \hat{Y}(t))) \\ \dot{\hat{f}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(L_i(Y(t) - \hat{Y}(t))) \\ \hat{Y}(t) = C_a \hat{X}(t) \end{cases}$$

Fault estimation

Estimation errors

$$\tilde{x}(t) = X(t) - \hat{X}(t) \quad \text{and} \quad \tilde{f}(t) = f(t) - \hat{f}(t)$$

Dynamics of the errors

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))((A_{ai} - K_i C_a)\tilde{x}(t) + E_{ai}\tilde{f}(t) + F_{ai}w(t))$$

$$\dot{\tilde{f}}(t) = \dot{f}(t) - \sum_{i=1}^{M} \mu_i(\xi(t)) L_i C_a \tilde{x}(t)$$

Augmented system of estimation errors

The following vectors are introduced :

$$\varphi(t) = \begin{bmatrix} \tilde{x}(t) \\ \tilde{f}(t) \end{bmatrix} \text{ and } \varepsilon(t) = \begin{bmatrix} w(t) \\ \dot{f}(t) \end{bmatrix}$$

Fault estimation

Augmented system of estimation errors

$$\dot{\varphi}(t) = A_m \varphi(t) + B_m \varepsilon(t)$$

with : $A_m = \sum_{i=1}^{M} \mu_i(\xi(t)) \tilde{A}_{0i}$ and $B_m = \sum_{i=1}^{M} \mu_i(\xi(t)) \tilde{B}_{0i}$,

$$\tilde{A}_{0i} = \begin{bmatrix} A_{ai} - K_i C_a & E_{ai} \\ -L_i C_a & 0 \end{bmatrix}, \; \tilde{B}_{0i} = \begin{bmatrix} F_{ai} & 0 \\ 0 & I \end{bmatrix}$$

Convergence of the generalized estimation error

Let us consider the following quadratic Lyapunov candidate function :

$$V(t) = \varphi^T(t) P \varphi(t)$$

Robust state and fault estimation

Convergence conditions

The problem of robust state and fault estimation is reduced to find the gains K_i and L_i of the observer to ensure an asymptotic convergence of $\varphi(t)$ towards zero if $\varepsilon(t) = 0$ and to ensure a bounded error in the case where $\varepsilon(t) \neq 0$, i.e. :

$$\begin{aligned} &\lim_{t\to\infty} \varphi(t) = 0 && \text{for } \varepsilon(t) = 0 \\ &\|\varphi(t)\|_{Q_\varphi} \leq \lambda \|\varepsilon(t)\|_{Q_\varepsilon} && \text{for } \varepsilon(t) \neq 0 \end{aligned}$$

where $\lambda > 0$ is the attenuation level.

To satisfy these constraints, it is sufficient to find a Lyapunov function $V(t)$ such that

$$\dot{V}(t) + \varphi^T(t) Q_\varphi \varphi(t) - \lambda^2 \varepsilon^T(t) Q_\varepsilon \varepsilon(t) < 0$$

Robust state and fault estimation

Augmented system

$$\dot{\varphi}(t) = A_m \varphi(t) + B_m \varepsilon(t)$$

$$\dot{V}(t) + \varphi^T(t) Q_\varphi \varphi(t) - \lambda^2 \varepsilon^T(t) Q_\varepsilon \varepsilon(t) < 0$$

Omitting to denote t, the last inequality can also be written as :

$$\psi^T \Omega_m \psi < 0$$

with :

$$\psi = \begin{bmatrix} \varphi \\ \varepsilon \end{bmatrix} \text{ and } \Omega_m = \begin{bmatrix} A_m^T P + P A_m + Q_\varphi & P B_m \\ B_m^T P & -\lambda^2 Q_\varepsilon \end{bmatrix}$$

This inequality holds if $\Omega_m < 0$.

Robust state and fault estimation

LMI formulation

$$A_m = \sum_{i=1}^{M} \mu_i(\xi(t))\tilde{A}_{0i}, \qquad \tilde{A}_{0i} = \tilde{A}_i - \tilde{K}_i\tilde{C}$$

where :

$$\tilde{A}_i = \begin{bmatrix} A_{ai} & E_{ai} \\ 0 & 0 \end{bmatrix}, \; \tilde{K}_i = \begin{bmatrix} K_i \\ L_i \end{bmatrix}, \; \tilde{C} = \begin{bmatrix} C_a & 0 \end{bmatrix}$$

With the changes of variables $G_i = P\tilde{K}_i$ and $m = \lambda^2$, the matrix Ω_m can be written as :

$$\Omega_m = \sum_{i=1}^{M} \mu_i(\xi(t))\Omega_i$$

$$\Omega_i = \begin{bmatrix} P\tilde{A}_i + \tilde{A}_i^T P - G_i\tilde{C} - \tilde{C}^T G_i^T + Q_\varphi & P\tilde{B}_{0i} \\ \tilde{B}_{0i}^T P & -mQ_\varepsilon \end{bmatrix}$$

Main result

Theorem

The system : $\dot{\varphi} = A_m \varphi + B_m \varepsilon$ describing the time evolution of the state estimation error $\tilde{x}$ and the fault estimation error $\tilde{f}$ is stable and the $\mathscr{L}_2$–gain of the transfer from $\varepsilon(t)$ to $\varphi(t)$ is bounded, if there exists a symmetric, positive definite matrix P, gain matrices G_i, $i \in \{1...M\}$ and a positive scalar m such that the following LMI are verified :

$$\begin{bmatrix} P\tilde{A}_i + \tilde{A}_i^T P - G_i \tilde{C} - \tilde{C}^T G_i^T + Q_\varphi & P\tilde{B}_{0i} \\ \tilde{B}_{0i}^T P & -mQ_\varepsilon \end{bmatrix} < 0$$

The proportional and integral gains of the observer are computed using $\tilde{K}_{mi} = P^{-1} G_i$ and the attenuation level is given by $\lambda = \sqrt{m}$.

Example

Example

System matrices

$$A_1 = \begin{bmatrix} -0.3 & -3 & -0.5 & 0.1 \\ -0.7 & -5 & 2 & 4 \\ 2 & -0.5 & -5 & -0.9 \\ -0.7 & -2 & 1 & -0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.7 & -7 & -1.5 & -7 \\ -0.2 & -2 & 0.6 & 1.3 \\ 5 & -1.5 & -9 & -3.9 \\ -0.4 & -1 & -0.3 & -1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 & 2 \\ 5 & 1 \\ 4 & -3 \\ 1 & 0 \end{bmatrix}, \; B_2 = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 2 \\ -1 & -2 \end{bmatrix}, \; D = \begin{bmatrix} 0.5 & 0.5 \\ 0.2 & 0.2 \\ 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \; F = \begin{bmatrix} 3.25 & 5 \\ 0 & 0.5 \\ -3.25 & 1.75 \\ 5.75 & 5 \end{bmatrix}$$

$$E_1 = B_1, \quad E_2 = B_2, \quad C = I, \quad \xi(t) = u(t)$$

$$Q_\varphi = Q_\varepsilon = I, \quad \bar{A}_1 = 5 \times I, \quad \bar{A}_2 = 10 \times I$$

Input signals

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \begin{cases} u_1(t) : \text{telegraph type signal whose amplitude belongs to } [0, 0.5] \\ u_2(t) : 0.4 + 0.25\sin(\pi t) \end{cases}$$

Example

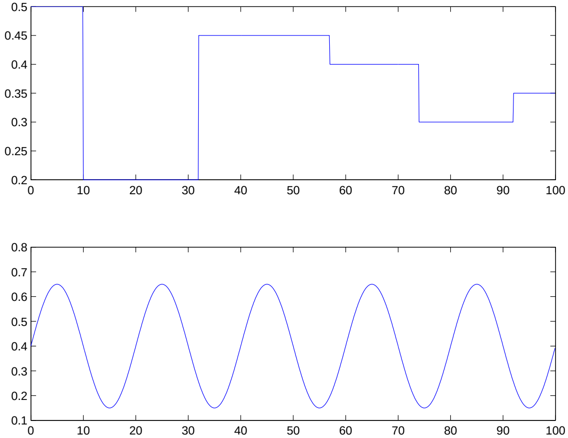


FIGURE: System inputs $u(t)$

Example

Actuator and sensor faults

Actuator fault is defined as follows :

$$f_a(t) = \begin{bmatrix} f_{a1}(t) \\ f_{a2}(t) \end{bmatrix}$$

with :

$$f_{a1}(t) = \begin{cases} 0.4\sin(\pi t), & 15\ s < t < 75\ s \\ 0, & \text{otherwise} \end{cases}, \quad f_{a2}(t) = \begin{cases} 0, & t < 20\ s \\ 0.3, & 20\ s < t < 80\ s \\ 0.5, & t > 80\ s \end{cases}$$

and the sensor fault $f_s(t)$ is defined as follows :

$$f_s(t) = \begin{bmatrix} f_{s1}(t) \\ f_{s2}(t) \end{bmatrix}$$

with :

$$f_{s1}(t) = \begin{cases} 0, & t \leq 35\ s \\ 0.6, & t > 35\ s \end{cases}, \quad f_{s2}(t) = \begin{cases} 0, & t \leq 25\ s \\ \sin(0.6\pi t), & t > 25\ s \end{cases}$$

Example

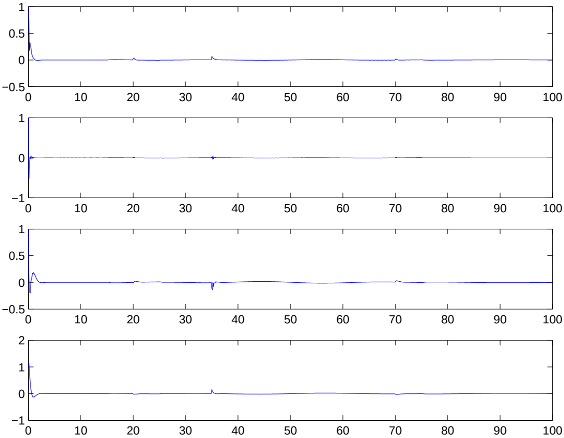


FIGURE: State estimation errors

Example

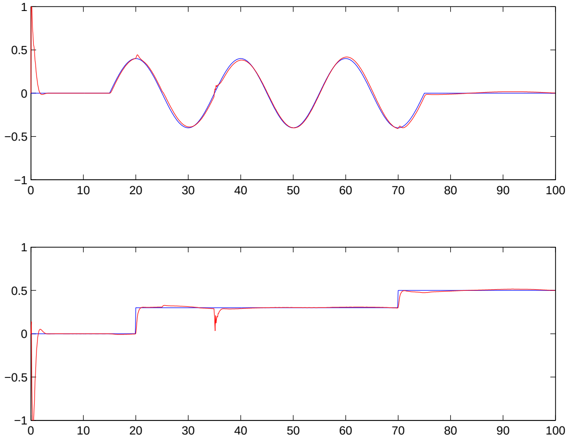


FIGURE: Actuator faults and their estimation

Example

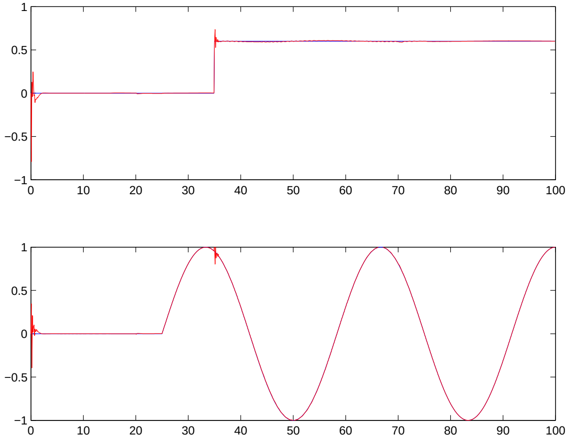


FIGURE: Sensor faults and their estimation

Conclusion and future works

Conclusion and future works

Conclusion

- Original method for state and fault estimation for systems described by nonlinear Takagi-Sugeno models using a proportional integral observer
- Using an adequate rewritting of the system equations, the sensor fault affecting the initial system is transformed into an actuator fault (i.e. into an unknown input) ; this transformation eases the simultaneous sensor and actuator fault estimation.

Future works

- Fault estimates can be used to conceive a fault tolerant control strategy able to cancel the fault effects on the system performances and behavior
- The proposed method should be extended to TS models with unmeasurable decision variables

Get in touch

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