State estimation and fault detection for systems described by Takagi-Sugeno nonlinear models

Didier Maquin

Centre de Recherche en Automatique de Nancy – France
UMR 7039, Nancy-Université – CNRS

STA'2009, Hammamet, Tunisia
December 20-22, 2009

Outline of the talk

1. FDI and fault estimation of nonlinear systems
   - Context and problem position
   - Basis of multiple model approach

Outline of the talk

1. FDI and fault estimation of nonlinear systems
   - Context and problem position
   - Basis of multiple model approach
2. Observer design for TS model – an introduction

Outline of the talk

1. FDI and fault estimation of nonlinear systems
   - Context and problem position
   - Basis of multiple model approach

2. Observer design for TS model – an introduction

3. Observer design for TS model with unmeasurable decision variables
   - First approach based on Lipschitz hypotheses
   - Second approach relying on the perturbation attenuation

Outline of the talk

1. FDI and fault estimation of nonlinear systems
   - Context and problem position
   - Basis of multiple model approach
2. Observer design for TS model – an introduction
3. Observer design for TS model with unmeasurable decision variables
   - First approach based on Lipschitz hypotheses
   - Second approach relying on the perturbation attenuation
4. Design of unknown input observers
   - Partial decoupling of fault
   - Design of proportional integral observer

Outline of the talk

1. FDI and fault estimation of nonlinear systems
   - Context and problem position
   - Basis of multiple model approach
2. Observer design for TS model – an introduction
3. Observer design for TS model with unmeasurable decision variables
   - First approach based on Lipschitz hypotheses
   - Second approach relying on the perturbation attenuation
4. Design of unknown input observers
   - Partial decoupling of fault
   - Design of proportional integral observer
5. Fault detection and isolation

Outline of the talk

1. FDI and fault estimation of nonlinear systems
   - Context and problem position
   - Basis of multiple model approach
2. Observer design for TS model – an introduction
3. Observer design for TS model with unmeasurable decision variables
   - First approach based on Lipschitz hypotheses
   - Second approach relying on the perturbation attenuation
4. Design of unknown input observers
   - Partial decoupling of fault
   - Design of proportional integral observer
5. Fault detection and isolation
6. Fault tolerant control

Outline of the talk

1. FDI and fault estimation of nonlinear systems
   - Context and problem position
   - Basis of multiple model approach
2. Observer design for TS model – an introduction
3. Observer design for TS model with unmeasurable decision variables
   - First approach based on Lipschitz hypotheses
   - Second approach relying on the perturbation attenuation
4. Design of unknown input observers
   - Partial decoupling of fault
   - Design of proportional integral observer
5. Fault detection and isolation
6. Fault tolerant control
7. Conclusion and prospects

Outline of the talk

1. FDI and fault estimation of nonlinear systems
   - Context and problem position
   - Basis of multiple model approach
2. Observer design for TS model – an introduction
3. Observer design for TS model with unmeasurable decision variables
   - First approach based on Lipschitz hypotheses
   - Second approach relying on the perturbation attenuation
4. Design of unknown input observers
   - Partial decoupling of fault
   - Design of proportional integral observer
5. Fault detection and isolation
6. Fault tolerant control
7. Conclusion and prospects

Context and problem position

- Despite a lot of works in this area, model-based FDI for nonlinear systems remains a difficult task
- Observer-based approach using bank of observers is among the more used methods
- Neccessity to be able to design nonlinear observers (possibly with unknown inputs)

Context and problem position

- Despite a lot of works in this area, model-based FDI for nonlinear systems remains a difficult task
- Observer-based approach using bank of observers is among the more used methods
- Neccessity to be able to design nonlinear observers (possibly with unknown inputs)

Proposed approach and requirements

- Nonlinear systems are modeled using the so-called “multiple model approach”
- Classical model-based approaches are adapted to that kind of models

Context and problem position

- Despite a lot of works in this area, model-based FDI for nonlinear systems remains a difficult task
- Observer-based approach using bank of observers is among the more used methods
- Neccessity to be able to design nonlinear observers (possibly with unknown inputs)

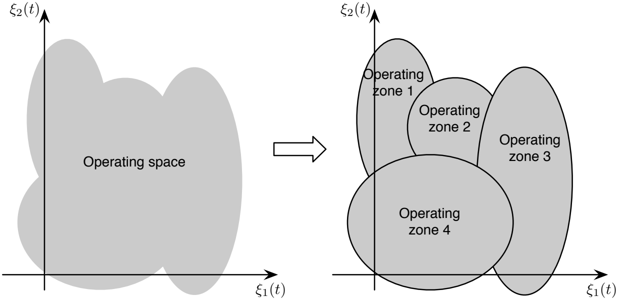
Proposed approach and requirements

- Nonlinear systems are modeled using the so-called "multiple model approach"
- Classical model-based approaches are adapted to that kind of models

Why using a multiple model ?

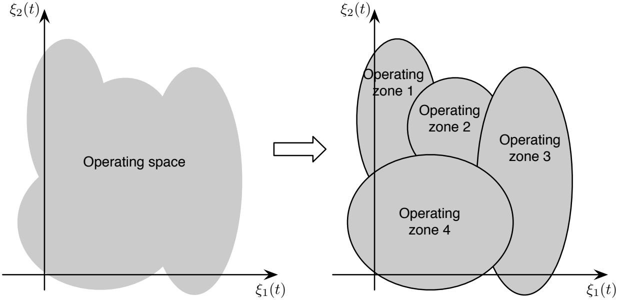
- Appropriate tool for modelling complex systems (black box or "exact" modelling)
- Tools for linear systems can partially be extended to nonlinear systems
- Specific analysis of the system nonlinearity is avoided

- Decomposition of the operating space into different operating zones
- Modelling the behaviour in each zone by a single submodel
- Quantifying, using weighting functions, the contribution of each submodel



Nonlinear systems — Multiple model representation

- Decomposition of the operating space into different operating zones
- Modelling the behaviour in each zone by a single submodel
- Quantifying, using weighting functions, the contribution of each submodel



Nonlinear systems   Multiple model representation

Multiple model : association of a set of submodels
blended by an interpolation mechanism

Connection of submodels – Decoupled multiple model

Model structure

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u(t) \\ y_i(t) = C_i x_i(t) \end{cases} \qquad i = 1, ..., r$$

Model structure

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u(t) \\ y_i(t) = C_i x_i(t) \end{cases} \qquad i = 1, ..., r$$

$$y(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) y_i(t)$$

Connection of submodels – Decoupled multiple model

Model structure

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u(t) \\ y_i(t) = C_i x_i(t) \end{cases} \qquad i = 1, ..., r$$

$$y(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) y_i(t)$$

- The multiple model output is a weighted sum of the submodel outputs

Connection of submodels – Decoupled multiple model

Model structure

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u(t) \\ y_i(t) = C_i x_i(t) \end{cases} \qquad i = 1, \ldots, r$$

$$y(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) y_i(t)$$

- The multiple model output is a weighted sum of the submodel outputs
- Convex sum property of the weighting function $\mu_i(\xi(t))$

$$\sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \;\text{ and }\; 0 \leq \mu_i(\xi(t)) \leq 1, \;\forall t, \;\forall i \in \{1, \ldots, r\}$$

Connection of submodels – Decoupled multiple model

Model structure

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u(t) \\ y_i(t) = C_i x_i(t) \end{cases} \qquad i = 1, ..., r$$

$$y(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) y_i(t)$$

- The multiple model output is a weighted sum of the submodel outputs
- Convex sum property of the weighting function $\mu_i(\xi(t))$

$$\sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \text{ and } 0 \leq \mu_i(\xi(t)) \leq 1, \ \forall t, \ \forall i \in \{1, ..., r\}$$

- Each "local model" has its own state vector $x_i(t)$

Connection of submodels – Decoupled multiple model

Model structure

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u(t) \\ y_i(t) = C_i x_i(t) \end{cases} \qquad i = 1, ..., r$$

$$y(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) y_i(t)$$

- The multiple model output is a weighted sum of the submodel outputs
- Convex sum property of the weighting function $\mu_i(\xi(t))$

$$\sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \;\text{ and }\; 0 \leq \mu_i(\xi(t)) \leq 1, \; \forall t, \; \forall i \in \{1, ..., r\}$$

- Each "local model" has its own state vector $x_i(t)$
- Dimension of the submodels can be different

Connection of submodels – Decoupled multiple model

Model structure

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u(t) \\ y_i(t) = C_i x_i(t) \end{cases} \qquad i = 1, ..., r$$

$$y(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) y_i(t)$$

- The multiple model output is a weighted sum of the submodel outputs
- Convex sum property of the weighting function $\mu_i(\xi(t))$

$$\sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \text{ and } 0 \leq \mu_i(\xi(t)) \leq 1, \; \forall t, \; \forall i \in \{1, ..., r\}$$

- Each "local model" has its own state vector $x_i(t)$
- Dimension of the submodels can be different

- Orjuela, R. (2008). Contribution à l'estimation d'état et au diagnostic des systèmes représentés par des multimodèles. Thèse de doctorat, Institut National Polytechnique de Lorraine, Nancy, France.

Takagi-Sugeno model structure

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\xi(t))\big(A_i x(t) + B_i u(t)\big) \\ y(t) &= \sum_{i=1}^{r} \mu_i(\xi(t)) C_i x(t)\end{aligned}$$

Takagi-Sugeno model structure

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\xi(t))\big(A_i x(t) + B_i u(t)\big) \\ y(t) &= \sum_{i=1}^{r} \mu_i(\xi(t)) C_i x(t) \end{aligned}$$

- Convex sum property of the weighting function $\mu_i(\xi(t))$

$$\sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \ \text{ and } \ 0 \leq \mu_i(\xi(t)) \leq 1, \ \forall t, \ \forall i \in \{1, ..., r\}$$

Takagi-Sugeno model structure

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\big(A_i x(t) + B_i u(t)\big)$$

$$y(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) C_i x(t)$$

- Convex sum property of the weighting function $\mu_i(\xi(t))$

$$\sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \text{ and } 0 \leq \mu_i(\xi(t)) \leq 1, \ \forall t, \ \forall i \in \{1, ..., r\}$$

- System is described by a unique state $x(t)$

Construction of TS models – 3 main approaches

Construction of TS models – 3 main approaches

- Identification approach
  - Choice of premise variables
  - Choice of the number of modalities of each premise variable
  - Choice of the structure of the local models
  - Parameter identification

Construction of TS models – 3 main approaches

- Identification approach
  - Choice of premise variables
  - Choice of the number of modalities of each premise variable
  - Choice of the structure of the local models
  - Parameter identification

- Transformation of a nonlinear model into a multiple model

Construction of TS models – 3 main approaches

- Identification approach
  - Choice of premise variables
  - Choice of the number of modalities of each premise variable
  - Choice of the structure of the local models
  - Parameter identification

- Transformation of a nonlinear model into a multiple model
  - Linearization around some "well-chosen" points

    Identification of the weighting function parameter to minimize the ouput error

Construction of TS models – 3 main approaches

- Identification approach
  - Choice of premise variables
  - Choice of the number of modalities of each premise variable
  - Choice of the structure of the local models
  - Parameter identification

- Transformation of a nonlinear model into a multiple model
  - Linearization around some "well-chosen" points

    Identification of the weighting function parameter to minimize the ouput error

  - Nonlinear sector approach

    Rewriting of the model in a compact subspace of the state space

$$\left\{\begin{array}{l} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = h(x(t), u(t)) \end{array}\right. \Rightarrow \left\{\begin{array}{l} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\big(A_i x(t) + B_i u(t)\big) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\big(C_i x(t) + D_i u(t)\big) \end{array}\right.$$

Nonlinear sector approach

Example

$$\begin{cases} \dot{x}_1(t) = -x_1(t)\cos(x_1(t)) + u(t) \\ \dot{x}_2(t) = x_1^2(t) - x_2(t) \end{cases}$$

Example

$$\begin{cases} \dot{x}_1(t) = -x_1(t)\cos(x_1(t)) + u(t) \\ \dot{x}_2(t) = x_1^2(t) - x_2(t) \end{cases} \Rightarrow \dot{x}(t) = \begin{pmatrix} -\cos(x_1(t)) & 0 \\ x_1(t) & -1 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$$

Example

$$\begin{cases} \dot{x}_1(t) = -x_1(t)\cos(x_1(t)) + u(t) \\ \dot{x}_2(t) = x_1^2(t) - x_2(t) \end{cases} \Rightarrow \dot{x}(t) = \begin{pmatrix} -\cos(x_1(t)) & 0 \\ x_1(t) & -1 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$$

$-1 \leq \cos(x_1) \leq 1$

$x_1 \cos(x_1) = F_1^1(x_1)x_1 - F_2^1(x_1)x_1$, with $F_1^1(x_1) = \frac{\cos(x_1)+1}{2}$ and $F_2^1(x_1) = \frac{1-\cos(x_1)}{2}$

Example

$$\begin{cases} \dot{x}_1(t) = -x_1(t)\cos(x_1(t)) + u(t) \\ \dot{x}_2(t) = x_1^2(t) - x_2(t) \end{cases} \Rightarrow \dot{x}(t) = \begin{pmatrix} -\cos(x_1(t)) & 0 \\ x_1(t) & -1 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$$

$-1 \leq \cos(x_1) \leq 1$
$x_1 \cos(x_1) = F_1^1(x_1)x_1 - F_2^1(x_1)x_1$, with $F_1^1(x_1) = \frac{\cos(x_1)+1}{2}$ and $F_2^1(x_1) = \frac{1-\cos(x_1)}{2}$

$-1 \leq x_1 \leq 1$
$x_1^2 = F_1^2(x_1)x_1 - F_2^2(x_1)x_1$, with $F_2^1(x_1) = \frac{x_1+1}{2}$ and $F_2^2(x_1) = \frac{1-x_1}{2}$

Example

$$\begin{cases} \dot{x}_1(t) = -x_1(t)\cos(x_1(t)) + u(t) \\ \dot{x}_2(t) = x_1^2(t) - x_2(t) \end{cases} \Rightarrow \dot{x}(t) = \begin{pmatrix} -\cos(x_1(t)) & 0 \\ x_1(t) & -1 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$$

$-1 \leq \cos(x_1) \leq 1$

$x_1 \cos(x_1) = F_1^1(x_1)x_1 - F_2^1(x_1)x_1,$ with $F_1^1(x_1) = \frac{\cos(x_1)+1}{2}$ and $F_2^1(x_1) = \frac{1-\cos(x_1)}{2}$

$-1 \leq x_1 \leq 1$

$x_1^2 = F_1^2(x_1)x_1 - F_2^2(x_1)x_1,$ with $F_2^1(x_1) = \frac{x_1+1}{2}$ and $F_2^2(x_1) = \frac{1-x_1}{2}$

$$\mu_1(x_1) = F_1^1(x_1)F_1^2(x_1), \quad \mu_2(x_1) = F_1^1(x_1)F_2^2(x_1)$$
$$\mu_3(x_1) = F_2^1(x_1)F_1^2(x_1), \quad \mu_4(x_1) = F_2^1(x_1)F_2^2(x_1)$$

Nonlinear sector approach

Example

$$\begin{cases} \dot{x}_1(t) = -x_1(t)\cos(x_1(t)) + u(t) \\ \dot{x}_2(t) = x_1^2(t) - x_2(t) \end{cases} \Rightarrow \dot{x}(t) = \begin{pmatrix} -\cos(x_1(t)) & 0 \\ x_1(t) & -1 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$$

$-1 \leq \cos(x_1) \leq 1$
$x_1 \cos(x_1) = F_1^1(x_1)x_1 - F_2^1(x_1)x_1,$ with $F_1^1(x_1) = \frac{\cos(x_1)+1}{2}$ and $F_2^1(x_1) = \frac{1-\cos(x_1)}{2}$

$-1 \leq x_1 \leq 1$
$x_1^2 = F_1^2(x_1)x_1 - F_2^2(x_1)x_1,$ with $F_2^1(x_1) = \frac{x_1+1}{2}$ and $F_2^2(x_1) = \frac{1-x_1}{2}$

$$\mu_1(x_1) = F_1^1(x_1)F_1^2(x_1), \quad \mu_2(x_1) = F_1^1(x_1)F_2^2(x_1)$$
$$\mu_3(x_1) = F_2^1(x_1)F_1^2(x_1), \quad \mu_4(x_1) = F_2^1(x_1)F_2^2(x_1)$$

$$A_1 = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix},\ A_2 = \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix},\ A_3 = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix},\ A_4 = \begin{pmatrix} 1 & 0 \\ -1 & -1 \end{pmatrix},\ B_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Nonlinear sector approach

Example

$$\begin{cases} \dot{x}_1(t) = -x_1(t)\cos(x_1(t)) + u(t) \\ \dot{x}_2(t) = x_1^2(t) - x_2(t) \end{cases} \Rightarrow \dot{x}(t) = \begin{pmatrix} -\cos(x_1(t)) & 0 \\ x_1(t) & -1 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$$

$-1 \leq \cos(x_1) \leq 1$

$x_1\cos(x_1) = F_1^1(x_1)x_1 - F_2^1(x_1)x_1$, with $F_1^1(x_1) = \frac{\cos(x_1)+1}{2}$ and $F_2^1(x_1) = \frac{1-\cos(x_1)}{2}$

$-1 \leq x_1 \leq 1$

$x_1^2 = F_1^2(x_1)x_1 - F_2^2(x_1)x_1$, with $F_2^1(x_1) = \frac{x_1+1}{2}$ and $F_2^2(x_1) = \frac{1-x_1}{2}$

$$\mu_1(x_1) = F_1^1(x_1)F_1^2(x_1), \quad \mu_2(x_1) = F_1^1(x_1)F_2^2(x_1)$$
$$\mu_3(x_1) = F_2^1(x_1)F_1^2(x_1), \quad \mu_4(x_1) = F_2^1(x_1)F_2^2(x_1)$$

$$A_1 = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, \ A_2 = \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}, \ A_3 = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}, \ A_4 = \begin{pmatrix} 1 & 0 \\ -1 & -1 \end{pmatrix}, \ B_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{cases} \dot{x}_1(t) = -x_1(t)\cos(x_1(t)) + u(t) \\ \dot{x}_2(t) = -x_1^2(t) - x_2(t) \end{cases} \Rightarrow \dot{x}(t) = \sum_{i=1}^{4} \mu_i(x_1(t))(A_i x(t) + B_i u(t))$$

Outline of the talk

Observer design for TS model – an introduction

Takagi-Sugeno model

$$\begin{cases} \dot{x}(t) = \displaystyle\sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) = Cx(t) \end{cases}$$

Observer design for TS model – an introduction

Takagi-Sugeno model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i x(t) + B_i u(t) \right) \\ y(t) = Cx(t) \end{cases}$$

Hypotheses

1. The ouput equation is linear with regard to the system state
2. The decision variables are accessible $\Rightarrow \xi(t) = u(t)$ or $\xi(t) = y(t)$

Observer design for TS model – an introduction

Takagi-Sugeno model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i x(t) + B_i u(t) \right) \\ y(t) = C x(t) \end{cases}$$

Hypotheses

1. The ouput equation is linear with regard to the system state
2. The decision variables are accessible $\Rightarrow \xi(t) = u(t)$ or $\xi(t) = y(t)$

Observer

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i \hat{x}(t) + B_i u(t) + L_i (y(t) - \hat{y}(t)) \right) \\ \hat{y}(t) = C \hat{x}(t) \end{cases}$$

Takagi-Sugeno model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) = Cx(t) \end{cases}$$

Hypotheses

1. The ouput equation is linear with regard to the system state
2. The decision variables are accessible $\Rightarrow \xi(t) = u(t)$ or $\xi(t) = y(t)$

Observer

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t)\right) \\ \hat{y}(t) = C\hat{x}(t) \end{cases}$$

The gain L_i can be easily designed.

Observer design for TS model – an introduction

Takagi-Sugeno model

$$\begin{cases} \dot{x}(t) = \displaystyle\sum_{i=1}^{r} \mu_i(\xi(t))\left(A_i x(t) + B_i u(t)\right) \\ y(t) = Cx(t) \end{cases}$$

Observer design for TS model – an introduction

Takagi-Sugeno model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) = Cx(t) \end{cases}$$

Observer

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))\right) \\ \hat{y}(t) = C\hat{x}(t) \end{cases}$$

Observer design for TS model – an introduction

Takagi-Sugeno model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) = Cx(t) \end{cases}$$

Observer

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))\right) \\ \hat{y}(t) = C\hat{x}(t) \end{cases}$$

State estimation error

$$e(t) = x(t) - \hat{x}(t)$$

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i - L_i C\right) e(t)$$

State estimation error

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\,(A_i - L_i C)\,e(t)$$

Observer design for TS model – an introduction

State estimation error

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\,(A_i - L_i C)\,e(t)$$

Convergence analysis – Lyapunov approach

Quadratic Lyapunov function

$$V(e(t)) = e^T(t)Pe(t), \text{ with } P = P^T > 0$$

The state estimation error asymptotically converges towards zero if

$$\dot{V}(e(t)) < 0$$

$$\dot{V}(e(t)) = \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) < 0$$

$$\dot{V}(e(t)) = e^T(t)\left(\sum_{i=1}^{r} \mu_i(\xi(t))\left((A_i - L_i C)^T P + P(A_i - L_i C)\right)\right)e(t) < 0$$

Observer design for TS model – an introduction

State estimation error and derivative of Lyapunov function

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i - L_i C\right) e(t)$$

$$\dot{V}(e(t)) = e^T(t) \left(\sum_{i=1}^{r} \mu_i(\xi(t)) \left((A_i - L_i C)^T P + P(A_i - L_i C) \right) \right) e(t) < 0$$

Observer design for TS model – an introduction

State estimation error and derivative of Lyapunov function

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i - L_i C\right) e(t)$$

$$\dot{V}(e(t)) = e^T(t) \left(\sum_{i=1}^{r} \mu_i(\xi(t)) \left((A_i - L_i C)^T P + P(A_i - L_i C) \right) \right) e(t) < 0$$

Sufficient convergence conditions

Convex sum property of μ_i functions implies that $\dot{V}(e(t)) < 0$ if :

$$(A_i - L_i C)^T P + P(A_i - L_i C) < 0, \quad \forall i$$

Observer design for TS model – an introduction

State estimation error and derivative of Lyapunov function

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \, (A_i - L_i C) \, e(t)$$

$$\dot{V}(e(t)) = e^T(t) \left(\sum_{i=1}^{r} \mu_i(\xi(t)) \left((A_i - L_i C)^T P + P(A_i - L_i C) \right) \right) e(t) < 0$$

Sufficient convergence conditions

Convex sum property of μ_i functions implies that $\dot{V}(e(t)) < 0$ if :

$$(A_i - L_i C)^T P + P(A_i - L_i C) < 0, \quad \forall i$$

These conditions are expressed using **bilinear** matrix inequalities

Observer design for TS model – an introduction

State estimation error and derivative of Lyapunov function

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\,(A_i - L_i C)\,e(t)$$

$$\dot{V}(e(t)) = e^T(t)\left(\sum_{i=1}^{r} \mu_i(\xi(t))\left((A_i - L_i C)^T P + P(A_i - L_i C)\right)\right) e(t) < 0$$

Sufficient convergence conditions

Convex sum property of μ_i functions implies that $\dot{V}(e(t)) < 0$ if :

$$(A_i - L_i C)^T P + P(A_i - L_i C) < 0, \quad \forall i$$

These conditions are expressed using **bilinear** matrix inequalities

Transformation into LMI

With the bijective change of variable $K_i = PL_i$ (i.e. $L_i = P^{-1}K_i$)

$$(A_i^T P + PA_i - C^T K_i^T - K_i C) < 0, \quad \forall i$$

Outline of the talk

Takagi-Sugeno model with unmeasured decision variables $x(t)$

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) = Cx(t) \end{cases}$$

Observer design for TS model with unmeasurable decision variables

Takagi-Sugeno model with unmeasured decision variables $x(t)$

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) \right) \\ y(t) = C x(t) \end{cases}$$

Observer – weighting functions depend now on the estimated state $\hat{x}(t)$

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left(A_i \hat{x}(t) + B_i u(t) + L_i(y - \hat{y}(t)) \right) \\ \hat{y}(t) = C \hat{x}(t) \end{cases}$$

Observer design for TS model with unmeasurable decision variables

Takagi-Sugeno model with unmeasured decision variables $x(t)$

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) = Cx(t) \end{cases}$$

Observer – weighting functions depend now on the estimated state $\hat{x}(t)$

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left(A_i \hat{x}(t) + B_i u(t) + L_i(y - \hat{y}(t)\right) \\ \hat{y}(t) = C\hat{x}(t) \end{cases}$$

The gain L_i are considerably more difficult to design !

Observer design for TS model with unmeasurable decision variables

Takagi-Sugeno model with unmeasured decision variables $x(t)$

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) = Cx(t) \end{cases}$$

Observer – weighting functions depend now on the estimated state $\hat{x}(t)$

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left(A_i \hat{x}(t) + B_i u(t) + L_i(y - \hat{y}(t)\right) \\ \hat{y}(t) = C\hat{x}(t) \end{cases}$$

The gain L_i are considerably more difficult to design !

There exists very few results in the litterature dealing with that problem which is essentially in the FDI framework

Previous studies in this area

1. Thau, F. (1973). Observing the state of non-linear dynamic systems. *International Journal of Control*, 17(3) :471-479.

Previous studies in this area

1. Thau, F. (1973). Observing the state of non-linear dynamic systems. *International Journal of Control*, 17(3) :471-479.

Nonlinear model

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t), u(t)) \\ y(t) = Cx(t) \end{cases}$$

Previous studies in this area

1. Thau, F. (1973). Observing the state of non-linear dynamic systems. *International Journal of Control*, 17(3) :471-479.

Nonlinear model

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t), u(t)) \\ y(t) = Cx(t) \end{cases}$$

Observer

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + f(\hat{x}(t), u(t)) + L(y(t) - \hat{y}(t)) \\ y(t) = C\hat{x}(t) \end{cases}$$

Previous studies in this area

1. Thau, F. (1973). Observing the state of non-linear dynamic systems. *International Journal of Control*, 17(3) :471-479.

Nonlinear model

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t), u(t)) \\ y(t) = Cx(t) \end{cases}$$

Observer

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + f(\hat{x}(t), u(t)) + L(y(t) - \hat{y}(t)) \\ y(t) = C\hat{x}(t) \end{cases}$$

State estimation error $(e(t) = x(t) - \hat{x}(t))$

$$\dot{e}(t) = (A - LC)e(t) + f(x(t), u(t)) - f(\hat{x}(t), u(t))$$

Previous studies in this area

1. Thau, F. (1973). Observing the state of non-linear dynamic systems. *International Journal of Control*, 17(3) :471-479.

Nonlinear model

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t), u(t)) \\ y(t) = Cx(t) \end{cases}$$

Observer

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + f(\hat{x}(t), u(t)) + L(y(t) - \hat{y}(t)) \\ y(t) = C\hat{x}(t) \end{cases}$$

State estimation error $(e(t) = x(t) - \hat{x}(t))$

$$\dot{e}(t) = (A - LC)e(t) + f(x(t), u(t)) - f(\hat{x}(t), u(t))$$

Lipschitz condition on the nonlinear part

$$\|f(x(t), u(t)) - f(\hat{x}(t), u(t))\| < \gamma \|x(t) - \hat{x}(t)\|$$

1. Thau, F. (1973). Observing the state of non-linear dynamic systems. *International Journal of Control*, 17(3) :471-479.

Nonlinear model

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t), u(t)) \\ y(t) = Cx(t) \end{cases}$$

Observer

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + f(\hat{x}(t), u(t)) + L(y(t) - \hat{y}(t)) \\ y(t) = C\hat{x}(t) \end{cases}$$

State estimation error $(e(t) = x(t) - \hat{x}(t))$

$$\dot{e}(t) = (A - LC)e(t) + f(x(t), u(t)) - f(\hat{x}(t), u(t))$$

Lipschitz condition on the nonlinear part

$$\|f(x(t), u(t)) - f(\hat{x}(t), u(t))\| < \gamma \|x(t) - \hat{x}(t)\|$$

Result : the state estimation error converges asymptotically towards 0

if $(A - LC)^T P + P(A - LC) = -Q$ with $\gamma < \dfrac{\lambda_{max}(Q)}{\lambda_{min}(P)}, P = P^T > 0, Q = Q^T > 0$

1. Thau, F. (1973). Observing the state of non-linear dynamic systems. *International Journal of Control*, 17(3) :471-479.

Nonlinear observer design based on Lipchitz assumption

Previous studies in this area

1. Thau, F. (1973). Observing the state of non-linear dynamic systems. *International Journal of Control*, 17(3) :471-479.

   Nonlinear observer design based on Lipchitz assumption

2. Raghavan, S. and Hedrick, J. K. (1994). Observer design for a class of nonlinear systems. *International Journal of Control*, 59(2) :515-528.

   Provide an algorithm to compute the observer gain L

Previous studies in this area

1. Thau, F. (1973). Observing the state of non-linear dynamic systems. *International Journal of Control*, 17(3) :471-479.

Nonlinear observer design based on Lipchitz assumption

2. Raghavan, S. and Hedrick, J. K. (1994). Observer design for a class of nonlinear systems. *International Journal of Control*, 59(2) :515-528.

Provide an algorithm to compute the observer gain L

3. Bergsten, P. and Palm, R. (2000). Thau-Luenberger observers for TS fuzzy systems. In 9th IEEE International Conference on Fuzzy Systems, FUZZ IEEE, San Antonio, TX, USA.
4. Bergsten, P., Palm, R., and Driankov, D. (2001). Fuzzy observers. In IEEE International Fuzzy Systems Conference, Melbourne, Australia.
5. Bergsten, P., Palm, R., and Driankov, D. (2002). Observers for Takagi-Sugeno fuzzy systems. *IEEE Transactions on Systems, Man, and Cybernetics - Part B : Cybernetics*, 32(1) :114-121.

7. Yoneyama, J. (2008). H_∞ output feedback control for fuzzy systems with immeasurable premise variables : discrete-time case. *Applied Soft Computing*, 8(2) :949-958.

8. Yoneyama, J. (2009). H_∞ filtering for fuzzy systems with immeasurable premise variables : an uncertain system approach. *Fuzzy Sets and Systems*, 160(12) :1738-1748.

Previous studies in this area

7. Yoneyama, J. (2008). H_∞ output feedback control for fuzzy systems with immeasurable premise variables : discrete-time case. *Applied Soft Computing*, 8(2) :949-958.
8. Yoneyama, J. (2009). H_∞ filtering for fuzzy systems with immeasurable premise variables : an uncertain system approach. *Fuzzy Sets and Systems*, 160(12) :1738-1748.

---

9. Ichalal D., Marx B., Ragot J., and Maquin D. (2008). Design of observers for Takagi-Sugeno systems with immeasurable premise variables : an L_2 approach. in 17th IFAC World Congress, Seoul, Korea.
10. Ichalal D., Marx B., Ragot J., and Maquin D. (2009). State estimation of nonlinear systems using multiple model approach. In American Control Conference, ACC'2009, St. Louis, Missouri, USA.
11. Ichalal D., Marx B., Ragot J., and Maquin D. (2009). Fault diagnosis in Takagi-Sugeno nonlinear systems. In 7th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes, SafeProcess'2009, Barcelona, Spain.
12. Ichalal D., Marx B., Ragot J., and Maquin D. (2009). An approach for the state estimation of Takagi-Sugeno models and application to sensor fault diagnosis. 48th IEEE Conference on Decision and Control, Shanghai, P.R. China.

Non-perturbed TS model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) \right) \\ y(t) = Cx(t) \end{cases}$$

Non-perturbed TS model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) = Cx(t) \end{cases}$$

Rewritting of the model

$$\begin{cases} \dot{x}(t) = A_0 x(t) + \sum_{i=1}^{r} \mu_i(x(t))(\bar{A}_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases}$$

with

$$A_0 = \frac{1}{r} \sum_{i=1}^{r} A_i \quad \text{or} \quad A_0 = A_j \text{ (dominant local model } j\text{)}$$

$$\bar{A}_i = A_i - A_0$$

Non-perturbed TS model

$$\begin{cases} \dot{x}(t) = A_0 x(t) + \sum_{i=1}^{r} \mu_i(x(t))(\bar{A}_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases}$$

First approach based on Lipschitz hypotheses

Non-perturbed TS model

$$\begin{cases} \dot{x}(t) = A_0 x(t) + \sum_{i=1}^{r} \mu_i(x(t))(\bar{A}_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases}$$

Structure of the proposed observer

$$\begin{cases} \dot{\hat{x}}(t) = A_0 \hat{x}(t) + \sum_{i=1}^{r} \mu_i(\hat{x}(t))(\bar{A}_i \hat{x}(t) + B_i u(t)) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases}$$

First approach based on Lipschitz hypotheses

Non-perturbed TS model

$$\begin{cases} \dot{x}(t) = A_0 x(t) + \sum_{i=1}^{r} \mu_i(x(t))(\bar{A}_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases}$$

Structure of the proposed observer

$$\begin{cases} \dot{\hat{x}}(t) = A_0 \hat{x}(t) + \sum_{i=1}^{r} \mu_i(\hat{x}(t))(\bar{A}_i \hat{x}(t) + B_i u(t)) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases}$$

State estimation error

$$e(t) = x(t) - \hat{x}(t)$$

$$\dot{e}(t) = (A_0 - LC)e(t) + \Delta(x, \hat{x}, u)$$

where :

$$\Delta(x, \hat{x}, u) = \sum_{i=1}^{r} (\bar{A}_i(\mu_i(x(t))x(t) - \mu_i(\hat{x}(t))\hat{x}(t)) + B_i(\mu_i(x(t)) - \mu_i(\hat{x}(t))u(t)))$$

First approach based on Lipschitz hypotheses

State estimation error

$$\dot{e}(t) = (A_0 - LC)e(t) + \Delta(x, \hat{x}, u)$$

where :

$$\Delta(x, \hat{x}, u) = \sum_{i=1}^{r} \left(\bar{A}_i(\mu_i(x(t))x(t) - \mu_i(\hat{x}(t))\hat{x}(t)) + B_i(\mu_i(x(t)) - \mu_i(\hat{x}(t))u(t))\right)$$

State estimation error

$$\dot{e}(t) = (A_0 - LC)e(t) + \Delta(x, \hat{x}, u)$$

where :

$$\Delta(x, \hat{x}, u) = \sum_{i=1}^{r} \left(\bar{A}_i(\mu_i(x(t))x(t) - \mu_i(\hat{x}(t))\hat{x}(t)) + B_i(\mu_i(x(t)) - \mu_i(\hat{x}(t))u(t))\right)$$

Lipschitz hypotheses

- **A1.** $|\mu_i(x(t))x(t) - \mu_i(\hat{x}(t))\hat{x}(t)| < \alpha_i |x(t) - \hat{x}(t)| \quad \alpha_i > 0$
- **A2.** $|B_i(\mu_i(x(t)) - \mu_i(\hat{x}(t)))| < \beta_i |x(t) - \hat{x}(t)| \quad \beta_i > 0$
- **A3.** $|u(t)| < \rho \quad \rho > 0$

First approach based on Lipschitz hypotheses

State estimation error

$$\dot{e}(t) = (A_0 - LC)e(t) + \Delta(x, \hat{x}, u)$$

where :

$$\Delta(x, \hat{x}, u) = \sum_{i=1}^{r} \left(\bar{A}_i(\mu_i(x(t))x(t) - \mu_i(\hat{x}(t))\hat{x}(t)) + B_i(\mu_i(x(t)) - \mu_i(\hat{x}(t))u(t))\right)$$

Lipschitz hypotheses

- **A1.** $|\mu_i(x(t))x(t) - \mu_i(\hat{x}(t))\hat{x}(t)| < \alpha_i \, |x(t) - \hat{x}(t)| \quad \alpha_i > 0$
- **A2.** $|B_i(\mu_i(x(t)) - \mu_i(\hat{x}(t)))| < \beta_i \, |x(t) - \hat{x}(t)| \quad \beta_i > 0$
- **A3.** $|u(t)| < \rho \quad \rho > 0$

Fallout

$$|\Delta(x, \hat{x}, u)| < \gamma \, |x(t) - \hat{x}(t)|$$

where :

$$\gamma = \sum_{i=1}^{r} (\bar{\sigma}(\bar{A}_i)\alpha_i + \beta_i \rho), \qquad \bar{\sigma}(\bar{A}_i) : \text{maximum singular value of } (\bar{A}_i)$$

First approach based on Lipschitz hypotheses

State estimation error

$$\dot{e}(t) = (A_0 - LC)e(t) + \Delta(x, \hat{x}, u)$$

where :

$$|\Delta(x, \hat{x}, u)| < \gamma \, |x(t) - \hat{x}(t)|$$

First approach based on Lipschitz hypotheses

State estimation error

$$\dot{e}(t) = (A_0 - LC)e(t) + \Delta(x, \hat{x}, u)$$

where :
$$|\Delta(x, \hat{x}, u)| < \gamma \, |x(t) - \hat{x}(t)|$$

Convergence analysis – Lyapunov approach

Quadratic Lyapunov function

$$V(e(t)) = e^T(t)Pe(t), \text{ with } P = P^T > 0$$

The state estimation error asymptotically converges towards zero if

$$\dot{V}(t) = e^T(t)\left(\Phi^T P + P\Phi\right)e(t) + 2e^T(t)P\Delta(x, \hat{x}, u) < 0$$

where : $\Phi = A_0 - LC$

First approach based on Lipschitz hypotheses

State estimation error

$$\dot{e}(t) = (A_0 - LC)e(t) + \Delta(x, \hat{x}, u)$$

where : $$|\Delta(x, \hat{x}, u)| < \gamma\, |x(t) - \hat{x}(t)|$$

Convergence analysis – Lyapunov approach

Quadratic Lyapunov function

$$V(e(t)) = e^T(t)Pe(t), \text{ with } P = P^T > 0$$

The state estimation error asymptotically converges towards zero if

$$\dot{V}(t) = e^T(t)\left(\Phi^T P + P\Phi\right)e(t) + 2e^T(t)P\Delta(x, \hat{x}, u) < 0$$

where : $\Phi = A_0 - LC$

Non quadratic terms : $e^T P\Delta$

Lemma 1

For two matrices X and Y with appropriate dimensions, the following property holds :

$$X^T Y + XY^T < X^T \Omega^{-1} X + Y \Omega Y^T, \quad \Omega > 0$$

Some useful tools in LMI framework

Lemma 1

For two matrices X and Y with appropriate dimensions, the following property holds :

$$X^T Y + XY^T < X^T \Omega^{-1} X + Y \Omega Y^T, \quad \Omega > 0$$

Lemma 2 : Schur complement

Let us consider three matrices $Q(x) = Q^T(x)$, $R(x) = R^T(x)$ and $S(x)$ of compatible dimensions depending linearly on the variable x. The following LMIs are equivalent :

1. $\begin{pmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{pmatrix} > 0$
2. $R(x) > 0, Q(x) - S(x)R^{-1}(x)S^T(x) > 0$

Convergence analysis – Lyapunov approach

$$\dot{V}(t) = e^T(t)\left(\Phi^T P + P\Phi\right)e(t) + 2e^T(t)P\Delta(x, \hat{x}, u)$$

$$\text{with } |\Delta(x, \hat{x}, u)| < \gamma\, |e(t)|$$

$$e^T(t)(\Phi^T P + P\Phi + PQ^{-1}P)e(t) + \underbrace{\Delta^T(x, \hat{x}, u)Q\Delta(x, \hat{x}, u)}_{< e^T(t)(\gamma^2 Q)e(t)} < 0$$

Convergence analysis – Lyapunov approach

$$\dot{V}(t) = e^T(t)\left(\Phi^T P + P\Phi\right)e(t) + 2e^T(t)P\Delta(x,\hat{x},u)$$

$$\text{with } |\Delta(x,\hat{x},u)| < \gamma\,|e(t)|$$

$$e^T(t)(\Phi^T P + P\Phi + PQ^{-1}P)e(t) + \underbrace{\Delta^T(x,\hat{x},u)Q\Delta(x,\hat{x},u)}_{<e^T(t)(\gamma^2 Q)e(t)} < 0$$

Finally, the convergence condition can be written as

$$(A_0 - LC)^T P + P(A_0 - LC) + PQ^{-1}P + \gamma^2 Q < 0$$

First approach based on Lipschitz hypotheses

Convergence analysis – Lyapunov approach

$$\dot{V}(t) = e^T(t)\left(\Phi^T P + P\Phi\right)e(t) + 2e^T(t)P\Delta(x,\hat{x},u)$$

$$\text{with } |\Delta(x,\hat{x},u)| < \gamma\,|e(t)|$$

$$e^T(t)(\Phi^T P + P\Phi + PQ^{-1}P)e(t) + \underbrace{\Delta^T(x,\hat{x},u)Q\Delta(x,\hat{x},u)}_{<e^T(t)(\gamma^2 Q)e(t)} < 0$$

Finally, the convergence condition can be written as

$$(A_0 - LC)^T P + P(A_0 - LC) + PQ^{-1}P + \gamma^2 Q < 0$$

With the change of variable $K = PL$ and using the Schur complement

$$\begin{bmatrix} A_0^T P + PA_0 - C^T K^T - KC + \gamma^2 Q & P \\ P & -Q \end{bmatrix} < 0$$

Non-perturbed TS model

$$\begin{cases} \dot{x}(t) = A_0 x(t) + \sum_{i=1}^{r} \mu_i(x(t))(\bar{A}_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

Structure of the proposed observer

$$\begin{cases} \dot{\hat{x}}(t) = A_0 \hat{x}(t) + \sum_{i=1}^{r} \mu_i(\hat{x}(t))(\bar{A}_i \hat{x}(t) + B_i u(t)) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \quad (2)$$

Theorem 1

The state estimation error between the TS model (1) and its observer (2) converges asymptotically toward zero, if there exists a matrix $P = P^T > 0$, a diagonal positive matrix Q and a gain matrix K such that the following condition holds :

$$\begin{bmatrix} A_0^T P + PA_0 - C^T K^T - KC + \gamma^2 Q & P \\ P & -Q \end{bmatrix} < 0 \quad (3)$$

The gain of the observer is computed by $L = P^{-1}K$. ■

First approach based on Lipschitz hypotheses

Perturbed TS model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i \omega(t) \right) \\ y(t) = Cx(t) \end{cases}$$

First approach based on Lipschitz hypotheses

Perturbed TS model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i \omega(t) \right) \\ y(t) = C x(t) \end{cases}$$

Hypothesis & design goals

- $\omega(t)$: exogeneous disturbance such that $\omega(t) \in \mathcal{L}_2$
- determine the observer gain L such that the observer error dynamics is bounded and the $\mathcal{L}_2$ gain of the transfer from $\omega(t)$ to $e(t)$ is below a given threshold

$$\frac{\|e(t)\|_2}{\|\omega(t)\|_2} < \xi, \quad \xi > 0$$

First approach based on Lipschitz hypotheses

Perturbed TS model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i \omega(t)\right) \\ y(t) = Cx(t) \end{cases}$$

Hypothesis & design goals

- $\omega(t)$: exogeneous disturbance such that $\omega(t) \in \mathcal{L}_2$
- determine the observer gain L such that the observer error dynamics is bounded and the $\mathcal{L}_2$ gain of the transfer from $\omega(t)$ to $e(t)$ is below a given threshold

$$\frac{\|e(t)\|_2}{\|\omega(t)\|_2} < \xi, \quad \xi > 0$$

Convergence analysis of the state estimation error

The condition which guarantees the boundedness of the $\mathcal{L}_2$ norm of the transfer from $\omega(t)$ to $e(t)$ is given by :

$$\dot{V}(t) + e^T(t)e(t) - \xi^2 \omega^T(t)\omega(t) < 0$$

$\mathcal{L}_2$ gain

Consider the linear system :

$$\begin{cases} \dot{x}(t) & = & Ax(t) + Bu(t) \\ y(t) & = & Cx(t) + Du(t) \end{cases}$$

If the system is stable and $u(t)$ is bounded then there exists $\xi > 0$ such that :

$$\int_0^{+\infty} y^T(t)y(t)dt \leq \xi^2 \int_0^{+\infty} u^T(t)u(t)dt$$

The constant ξ is called $\mathcal{L}_2$ gain of the system.

Some useful tools in LMI framework

$\mathcal{L}_2$ gain

Consider the linear system :

$$\begin{cases} \dot{x}(t) & = & Ax(t) + Bu(t) \\ y(t) & = & Cx(t) + Du(t) \end{cases}$$

If the system is stable and $u(t)$ is bounded then there exists $\xi > 0$ such that :

$$\int_0^{+\infty} y^T(t)y(t)dt \leq \xi^2 \int_0^{+\infty} u^T(t)u(t)dt$$

The constant ξ is called $\mathcal{L}_2$ gain of the system.

Lemma 3 : Bounded real lemma

The previous constraint holds for any bounded $u(t)$ with $u(t) \neq 0$ if and only if there exists a matrix P such that :

$$\begin{pmatrix} A^T P + PA + C^T C & PB + C^T D \\ B^T P + D^T C & D^T D - \xi^2 I \end{pmatrix} < 0$$

Perturbed TS model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i \omega(t)\right) \\ y(t) = Cx(t) \end{cases} \qquad (4)$$

Theorem 2

The robust observer for the system (4) satisfying the perturbation attenuation constraint is determined by minimizing the real positive number $\bar{\xi}$ under the following LMI constraints in the variables P, K, Q and $\bar{\xi}$:

$$\begin{bmatrix} \Theta & P & PE_i \\ P & -Q & 0 \\ E_i^T P & 0 & -\bar{\xi} I \end{bmatrix} < 0, \ i = 1, \ldots, r$$

where :

$$\Theta = A_0^T P + PA_0 - C^T K^T - KC + \gamma^2 Q + I$$

The gain of the observer is computed by $L = P^{-1}K$. The resulting attenuation level is given by $\xi = \sqrt{\bar{\xi}}$. ■

Second approach relying on the perturbation attenuation

Difficulty

The weighting functions of the model depend on the actual state $x(t)$ when that of the observer depend on their estimates $\hat{x}(t)$.

Second approach relying on the perturbation attenuation

Difficulty

The weighting functions of the model depend on the actual state $x(t)$ when that of the observer depend on their estimates $\hat{x}(t)$.

An idea is to express the model using artificially weighting functions depending on that estimates.

Second approach relying on the perturbation attenuation

Difficulty

The weighting functions of the model depend on the actual state $x(t)$ when that of the observer depend on their estimates $\hat{x}(t)$.

An idea is to express the model using artificially weighting functions depending on that estimates.

Rewriting of the model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \left(\mu_i(\hat{x}(t)) \left(A_i x(t) + B_i u(t) \right) + \delta_i(t)(A_i x(t) + B_i u(t)) \right) \\ y(t) = Cx(t) \end{cases}$$

where :

$$\delta_i(t) = \mu_i(x(t)) - \mu_i(\hat{x}(t))$$

Rewriting of the model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \left(\mu_i(\hat{x}(t))\left(A_i x(t) + B_i u(t)\right) + \delta_i(t)(A_i x(t) + B_i u(t))\right) \\ y(t) = Cx(t) \end{cases}$$

where :

$$\delta_i(t) = \mu_i(x(t)) - \mu_i(\hat{x}(t)), \qquad -1 \leq \delta_i(t) \leq 1$$

Let us define :

$$\Delta A(t) = \sum_{i=1}^{r} \delta_i(t) A_i = \mathcal{A}\Sigma_A(t)E_A, \qquad \Delta B(t) = \sum_{i=1}^{r} \delta_i(t) B_i = \mathcal{B}\Sigma_B(t)E_B$$

where :

$$\Sigma_A(t) = \begin{bmatrix} \delta_1(t)I_n & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \delta_r(t)I_n \end{bmatrix}, \qquad \Sigma_B(t) = \begin{bmatrix} \delta_1(t)I_m & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \delta_r(t)I_m \end{bmatrix}$$

$$\Sigma_A^T(t)\Sigma_A(t) \leq I$$
$$\Sigma_B^T(t)\Sigma_B(t) \leq I$$

Second approach relying on the perturbation attenuation

Rewritten model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))\big((A_i + \Delta A(t))x(t) + (B_i + \Delta B(t))u(t)\big) \\ y(t) = Cx(t) \end{cases}$$

Second approach relying on the perturbation attenuation

Rewritten model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))\big((A_i + \Delta A(t))x(t) + (B_i + \Delta B(t))u(t)\big) \\ y(t) = Cx(t) \end{cases}$$

The original system with unmeasurable decision variables is transformed into an equivalent "uncertain" TS model with known decision variables.

The terms are not uncertain, but only unknown (unlike model uncertainties). This writing is used for observer design purpose only

Second approach relying on the perturbation attenuation

Rewritten model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))\big((A_i + \Delta A(t))x(t) + (B_i + \Delta B(t))u(t)\big) \\ y(t) = Cx(t) \end{cases}$$

The original system with unmeasurable decision variables is transformed into an equivalent "uncertain" TS model with known decision variables.

The terms are not uncertain, but only unknown (unlike model uncertainties). This writing is used for observer design purpose only

Proposed observer

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))\left(A_i\hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))\right) \\ \hat{y}(t) = C\hat{x}(t) \end{cases}$$

State estimation error

$$\dot{e} = \sum_{i=1}^{r} \mu_i(\hat{x}) \left((A_i - L_i C)e\right) + \Delta A x + \Delta B u$$

Second approach relying on the perturbation attenuation

State estimation error

$$\dot{e} = \sum_{i=1}^{r} \mu_i(\hat{x}) \left((A_i - L_i C) e \right) + \Delta A x + \Delta B u$$

Augmented system

$$e_a = \begin{bmatrix} e \\ x \end{bmatrix}, \qquad \begin{cases} \dot{e}_a = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\hat{x}) \mu_j(x) \left(\bar{A}_{ij} e_a + \bar{B}_{ij} u \right) \\ e = \begin{pmatrix} I & 0 \end{pmatrix} e_a \end{cases}$$

Second approach relying on the perturbation attenuation

Theorem 3

The system governing the augmented state estimation error is stable and the $\mathcal{L}_2$ gain of the transfer from $u(t)$ to the state estimation error is bounded by γ, if there exists two positive and symmetric matrices P_1 and P_2, matrices K_i, and positive scalars λ_1, λ_2 and $\bar{\gamma}$ such that the following LMIs hold, $\forall\ i, j \in \{1, .., r\}$:

$$\begin{bmatrix} \Psi_i & 0 & 0 & P_1\mathcal{A} & P_1\mathcal{B} \\ 0 & \Xi_j & P_2 B_j & 0 & 0 \\ 0 & B_j^T P_2 & -\bar{\gamma} I + \lambda_2 E_B^T E_B & 0 & 0 \\ \mathcal{A}^T P_1 & 0 & 0 & -\lambda_1 I & 0 \\ \mathcal{B}^T P_1 & 0 & 0 & 0 & -\lambda_2 I \end{bmatrix} < 0$$

where :

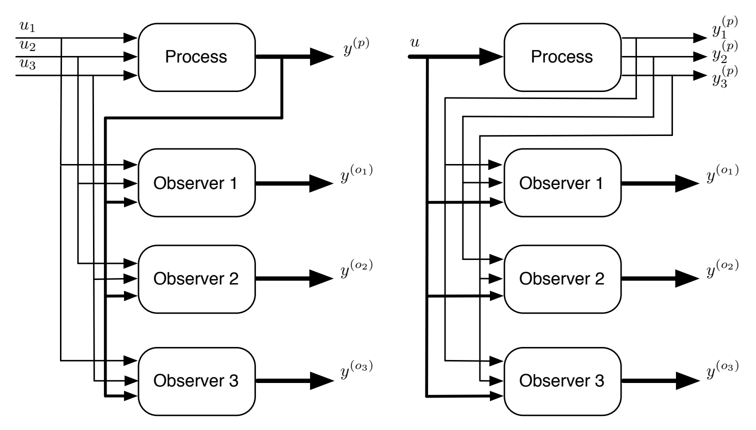
$$\Psi_i = A_i^T P_1 + P_1 A_i - K_i C - C^T K_i{}^T + I$$

$$\Xi_j = A_j^T P_2 + P_2 A_j + \lambda_1 E_A^T E_A$$

The gains of the observer are computed from $L_i = P_1^{-1} K_i$ and the resulting $\mathcal{L}_2$ gain from $u(t)$ to $e(t)$ is defined by $\gamma = \sqrt{\bar{\gamma}}$ ■

Outline of the talk

Classical approach for fault detection/isolation – Bank of observers



GOS for actuator fault detection (all u_i but one)

GOS for sensor fault detection (all y_i but one)

Partial decoupling of fault

Considered TS model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i d(t)\right) \\ y(t) = Cx(t) + Gd(t) \end{cases}$$

Partial decoupling of fault

Considered TS model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i d(t)\right) \\ y(t) = Cx(t) + Gd(t) \end{cases}$$

$d(t)$: unknown input vector (the dimension of d is less than that of y)

Partial decoupling of fault

Considered TS model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i d(t) \right) \\ y(t) = Cx(t) + Gd(t) \end{cases}$$

$d(t)$: unknown input vector (the dimension of d is less than that of y)

Rewriting of the model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left(A_i x(t) + B_i u(t) + E_i d(t) + \omega(t) \right) \\ y(t) = Cx(t) + Gd(t) \end{cases}$$

with : $\omega(t) = \sum_{i=1}^{r} \left(\mu_i(x(t)) - \mu_i(\hat{x}(t)) \right) \left(A_i x(t) + B_i u(t) + E_i d(t) \right)$

Partial decoupling of fault

Considered TS model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i d(t)\right) \\ y(t) = Cx(t) + Gd(t) \end{cases}$$

$d(t)$: unknown input vector (the dimension of d is less than that of y)

Rewriting of the model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left(A_i x(t) + B_i u(t) + E_i d(t) + \omega(t)\right) \\ y(t) = Cx(t) + Gd(t) \end{cases}$$

with : $\omega(t) = \sum_{i=1}^{r} \left(\mu_i(x(t)) - \mu_i(\hat{x}(t))\right) \left(A_i x(t) + B_i u(t) + E_i d(t)\right)$

Proposed observer

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left(N_i z(t) + G_i u(t) + L_i y(t)\right) \\ \hat{x}(t) = z(t) - Hy(t) \end{cases}$$

Partial decoupling of fault

Considered TS model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t))\,(A_i x(t) + B_i u(t) + E_i d(t)) \\ y(t) = Cx(t) + Gd(t) \end{cases}$$

$d(t)$: unknown input vector (the dimension of d is less than that of y)

Rewriting of the model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))\,(A_i x(t) + B_i u(t) + E_i d(t) + \omega(t)) \\ y(t) = Cx(t) + Gd(t) \end{cases}$$

with : $\omega(t) = \sum_{i=1}^{r} (\mu_i(x(t)) - \mu_i(\hat{x}(t)))\,(A_i x(t) + B_i u(t) + E_i d(t))$

Proposed observer

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))\,(N_i z(t) + G_i u(t) + L_i y(t)) \\ \hat{x}(t) = z(t) - Hy(t) \end{cases}$$

Partial decoupling of fault

Dynamics of the state estimation error

$$\begin{aligned}\dot{e}(t) &= \sum_{i=1}^{r} \mu_i(\hat{x}(t))((PA_i - N_i - K_iC)x(t) + (PB_i - G_i)u(t) \\ &\quad + (PE_i - K_iG)d(t) + P\omega(t) + N_ie(t)) + HG\dot{d}(t)\end{aligned}$$

with $P = I + HC$ and $K_i = N_iH + L_i$

Partial decoupling of fault

Dynamics of the state estimation error

$$\begin{aligned}\dot{e}(t) &= \sum_{i=1}^{r} \mu_i(\hat{x}(t))((PA_i - N_i - K_i C)x(t) + (PB_i - G_i)u(t) \\ &\quad +(PE_i - K_i G)d(t) + P\omega(t) + N_i e(t)) + HG\dot{d}(t)\end{aligned}$$

with $P = I + HC$ and $K_i = N_i H + L_i$

Structural conditions

$$\begin{aligned} HG &= 0 & N_i &= PA_i - K_i C \\ PB_i &= G_i & PE_i &= K_i G \\ L_i &= K_i - N_i H \end{aligned}$$

Partial decoupling of fault

Dynamics of the state estimation error

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))((PA_i - N_i - K_iC)x(t) + (PB_i - G_i)u(t)$$
$$+(PE_i - K_iG)d(t) + P\omega(t) + N_ie(t)) + HG\dot{d}(t)$$

with $P = I + HC$ and $K_i = N_iH + L_i$

Structural conditions

$$\begin{aligned} HG &= 0 & N_i &= PA_i - K_iC \\ PB_i &= G_i & PE_i &= K_iG \\ L_i &= K_i - N_iH \end{aligned}$$

Resulting dynamics of the state equation

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))\left(N_ie(t) + P\omega(t)\right) \quad \text{with} \quad |\omega(t)| \le \gamma |e(t)|$$

Theorem 4

An unknown input observer exists if there exists a symmetric positive definite matrix X, matrices M_i and S, and positive scalar λ such that the following LMIs hold, $\forall i = 1, ..., r$:

$$\begin{aligned} \begin{bmatrix} \Psi_i & (X+SC) \\ (X+SC)^T & -\lambda I \end{bmatrix} &< 0 \\ SG &= 0 \\ (X+SC)E_i &= M_i G \end{aligned}$$

where : $\Psi_i = A_i^T(X + C^T S^T) + (X + SC)A_i - C^T M_i^T - M_i C + \lambda\gamma^2 I$

The matrices defining the observer are computed according to :

$$\begin{aligned} H &= X^{-1}S \\ K_i &= X^{-1}M_i \\ N_i &= (I + HC)A_i - K_i C \\ L_i &= K_i - N_i H \\ G_i &= (I + HC)B_i \end{aligned}$$

Considered TS model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i d(t) + R_i \omega(t) \right) \\ y(t) = Cx(t) + Gd(t) + W\omega(t) \end{cases}$$

where $d(t)$ is the vector of unknown inputs and $\omega(t)$ a disturbance vector

Considered TS model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i d(t) + R_i \omega(t) \right) \\ y(t) = Cx(t) + Gd(t) + W\omega(t) \end{cases}$$

where $d(t)$ is the vector of unknown inputs and $\omega(t)$ a disturbance vector

Hypotheses

- **A1.** the system is stable
- **A2.** the signals $u(t)$, $d(t)$ and $\omega(t)$ are bounded
- **A3.** $\dot{d}(t) = 0$

Design of proportional integral observer

Considered TS model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i d(t) + R_i \omega(t)\right) \\ y(t) = C x(t) + G d(t) + W \omega(t) \end{cases}$$

where $d(t)$ is the vector of unknown inputs and $\omega(t)$ a disturbance vector

Hypotheses

- **A1.** the system is stable
- **A2.** the signals $u(t)$, $d(t)$ and $\omega(t)$ are bounded
- **A3.** $\dot{d}(t) = 0$

Proportional integral observer

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left(A_i \hat{x}(t) + B_i u(t) + E_i \hat{d}(t) + L_{Pi}(y(t) - \hat{y}(t))\right) \\ \dot{\hat{d}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left(L_{Ii}(y(t) - \hat{y}(t))\right) \\ \hat{y}(t) = C \hat{x}(t) + G \hat{d}(t) \end{cases}$$

Design of a proportional integral observer

Rewriting of the original model with an augmented state vector

$$x_a(t) = \begin{bmatrix} x(t) \\ d(t) \end{bmatrix},$$

Design of a proportional integral observer

Rewriting of the original model with an augmented state vector

$$x_a(t) = \begin{bmatrix} x(t) \\ d(t) \end{bmatrix}, \quad \begin{cases} \dot{x}_a(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left(\bar{A}_i x_a(t) + \bar{B}_i u(t) + \bar{\Gamma}_i \bar{\omega}(t)\right) \\ y(t) = \bar{C} x_a(t) + \bar{D} \bar{\omega}(t) \end{cases}$$

Rewriting of the original model with an augmented state vector

$$x_a(t) = \begin{bmatrix} x(t) \\ d(t) \end{bmatrix}, \quad \begin{cases} \dot{x}_a(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left(\bar{A}_i x_a(t) + \bar{B}_i u(t) + \bar{\Gamma}_i \bar{\omega}(t) \right) \\ y(t) = \bar{C} x_a(t) + \bar{D} \bar{\omega}(t) \end{cases}$$

Theorem 5

The proportional integral observer is obtained by solving, for $P = P^T > 0$, the following constrained optimization problem :

$$\min_{P, M_i} \quad \bar{\gamma} \quad s.t.$$

$$\begin{bmatrix} \bar{A}_i^T P + P \bar{A}_i - \bar{M}_i \bar{C} - \bar{C}^T \bar{M}_i^T + I & P \bar{\Gamma}_i - \bar{M}_i \bar{D} \\ \bar{\Gamma}_i^T P - \bar{D}^T \bar{M}_i^T & -\bar{\gamma} I \end{bmatrix} < 0$$

The gains of the observer are given by $\bar{L}_i = P^{-1} \bar{M}_i$ and the attenuation level is $\gamma = \sqrt{\bar{\gamma}}$

Example

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t))\left(A_i x(t) + B_i u(t) + E_i d(t) + R_i \omega(t)\right) \\ y(t) = Cx(t) + Gd(t) + W\omega(t) \end{cases}$$

Example

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i d(t) + R_i \omega(t) \right) \\ y(t) = Cx(t) + Gd(t) + W\omega(t) \end{cases}$$

with :

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{bmatrix}, A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 1 & 2 & -4 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 5 \\ 0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 3 \\ 1 \\ -7 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0 & 7 \\ 0 & 5 \\ 0 & 2 \end{bmatrix}, E_2 = \begin{bmatrix} 0 & 6 \\ 0 & 3 \\ 0 & 1 \end{bmatrix}, R_1 = R_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, W = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 5 & 0 \\ 1 & 0 \end{bmatrix}$$

Example

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i d(t) + R_i \omega(t)\right) \\ y(t) = Cx(t) + Gd(t) + W\omega(t) \end{cases}$$

with :

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{bmatrix},\ A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 1 & 2 & -4 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ 5 \\ 0.5 \end{bmatrix},\ B_2 = \begin{bmatrix} 3 \\ 1 \\ -7 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0 & 7 \\ 0 & 5 \\ 0 & 2 \end{bmatrix},\ E_2 = \begin{bmatrix} 0 & 6 \\ 0 & 3 \\ 0 & 1 \end{bmatrix},\ R_1 = R_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ W = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix},\ G = \begin{bmatrix} 5 & 0 \\ 1 & 0 \end{bmatrix}$$

Design of a proportional integral observer

Example

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t))\left(A_i x(t) + B_i u(t) + E_i d(t) + R_i \omega(t)\right) \\ y(t) = Cx(t) + Gd(t) + W\omega(t) \end{cases}$$

with :

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{bmatrix},\ A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 1 & 2 & -4 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ 5 \\ 0.5 \end{bmatrix},\ B_2 = \begin{bmatrix} 3 \\ 1 \\ -7 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0 & 7 \\ 0 & 5 \\ 0 & 2 \end{bmatrix},\ E_2 = \begin{bmatrix} 0 & 6 \\ 0 & 3 \\ 0 & 1 \end{bmatrix},\ R_1 = R_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ W = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix},\ G = \begin{bmatrix} 5 & 0 \\ 1 & 0 \end{bmatrix}$$

Example

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i d(t) + R_i \omega(t) \right) \\ y(t) = Cx(t) + Gd(t) + W\omega(t) \end{cases}$$

with :

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{bmatrix}, \; A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 1 & 2 & -4 \end{bmatrix}, \; B_1 = \begin{bmatrix} 1 \\ 5 \\ 0.5 \end{bmatrix}, \; B_2 = \begin{bmatrix} 3 \\ 1 \\ -7 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0 & 7 \\ 0 & 5 \\ 0 & 2 \end{bmatrix}, \; E_2 = \begin{bmatrix} 0 & 6 \\ 0 & 3 \\ 0 & 1 \end{bmatrix}, \; R_1 = R_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \; W = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \; G = \begin{bmatrix} 5 & 0 \\ 1 & 0 \end{bmatrix}$$

The weighting functions depend on the first entry $x_1(t)$ of the state vector $x(t)$:

$$\begin{cases} \mu_1(x(t)) = \frac{1 - \tanh(x_1(t))}{2} \\ \mu_2(x(t)) = 1 - \mu_1(x(t)) \end{cases}$$

Example cont'd

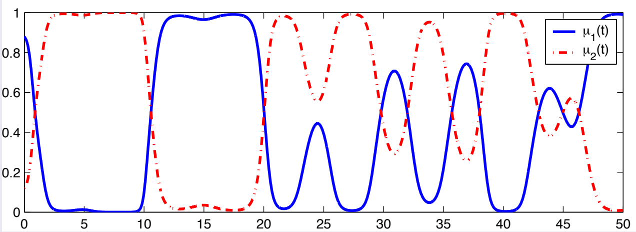


FIGURE: Time evolution of the weighting functions μ_1 and μ_2

- The perturbation $\omega(t)$ is a random bounded signal.
- The fourth derivatives of the two unknown inputs are null.

Example cont'd

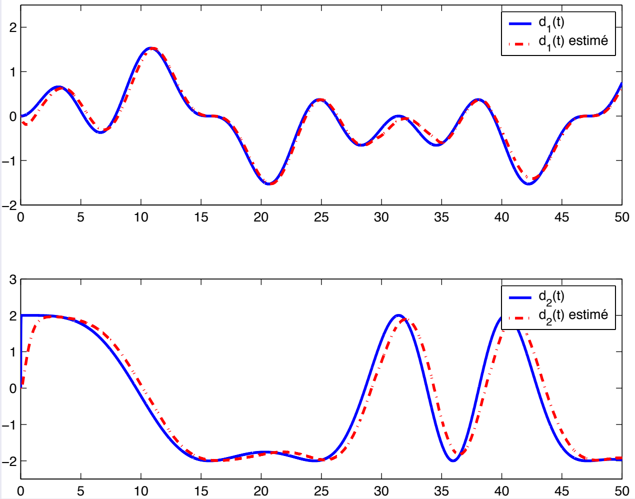


FIGURE: Unknown inputs and their estimates using a PI observer

Example cont'd

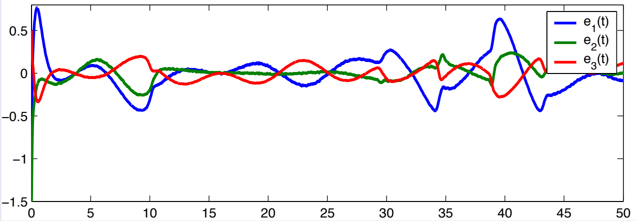


FIGURE: State estimation errors using a PI observer

Rewriting of the original model with an augmented state vector

$$x_a(t) = \begin{bmatrix} x(t) \\ d(t) \\ d_1(t) \\ \vdots \\ d_{q-1}(t) \end{bmatrix} \quad \text{with} \quad \begin{bmatrix} \dot{d}(t) \\ \dot{d}_1(t) \\ \vdots \\ \dot{d}_{q-1}(t) \end{bmatrix} = \begin{bmatrix} d_1(t) \\ d_2(t) \\ \vdots \\ d_q(t) \end{bmatrix}, \quad d^{(q)}(t) = 0$$

Rewriting of the original model with an augmented state vector

$$x_a(t) = \begin{bmatrix} x(t) \\ d(t) \\ d_1(t) \\ \vdots \\ d_{q-1}(t) \end{bmatrix} \quad \text{with} \quad \begin{bmatrix} \dot{d}(t) \\ \dot{d}_1(t) \\ \vdots \\ \dot{d}_{q-1}(t) \end{bmatrix} = \begin{bmatrix} d_1(t) \\ d_2(t) \\ \vdots \\ d_q(t) \end{bmatrix}, \quad d^{(q)}(t) = 0$$

$$\begin{cases} \dot{x}_a(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left(\bar{A}_i x_a(t) + \bar{B}_i u(t) + \bar{\Gamma}_i \bar{\omega}(t)\right) \\ y(t) = \bar{C} x_a(t) + \bar{D} \bar{\omega}(t) \end{cases}$$

Design of a proportional multiple integral observer

Rewriting of the original model with an augmented state vector

$$x_a(t) = \begin{bmatrix} x(t) \\ d(t) \\ d_1(t) \\ \vdots \\ d_{q-1}(t) \end{bmatrix} \quad \text{with} \quad \begin{bmatrix} \dot{d}(t) \\ \dot{d}_1(t) \\ \vdots \\ \dot{d}_{q-1}(t) \end{bmatrix} = \begin{bmatrix} d_1(t) \\ d_2(t) \\ \vdots \\ d_q(t) \end{bmatrix}, \quad d^{(q)}(t) = 0$$

$$\begin{cases} \dot{x}_a(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left(\bar{A}_i x_a(t) + \bar{B}_i u(t) + \bar{\Gamma}_i \bar{\omega}(t)\right) \\ y(t) = \bar{C} x_a(t) + \bar{D}\bar{\omega}(t) \end{cases}$$

Estimation of the q first derivatives of the unknown input

$$\begin{cases} \dot{\hat{d}}_j(t) = \sum_{i=1}^{r} \mu_i(z(t)) \left(\hat{d}_{j+1} + L_{Ii}^{j}(y(t) - \hat{y}(t))\right), j = 1, ..., q-1 \\ \dot{\hat{d}}(t) = \sum_{i=1}^{r} \mu_i(z(t)) \left(\hat{d}_1(t) + L_{Ii}(y(t) - \hat{y}(t))\right) \end{cases}$$

Example cont'd

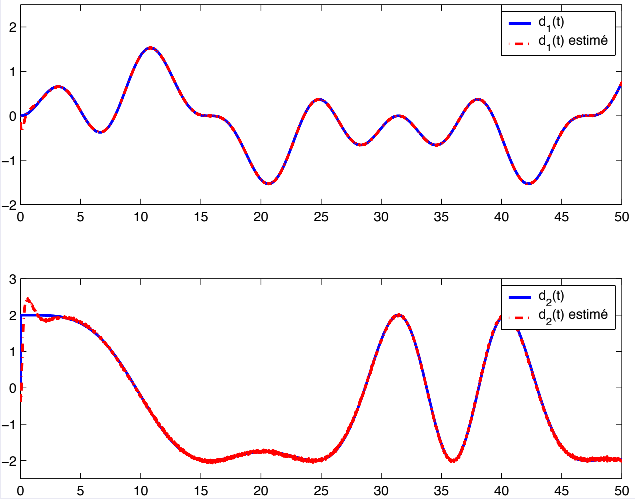


FIGURE: Unknown inputs and their estimates using a PMI observer

Example cont'd

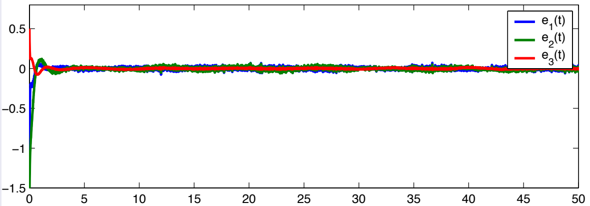


FIGURE: State estimation errors using a PI observer

Example cont'd

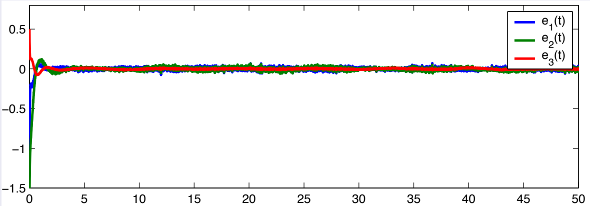


FIGURE: State estimation errors using a PI observer

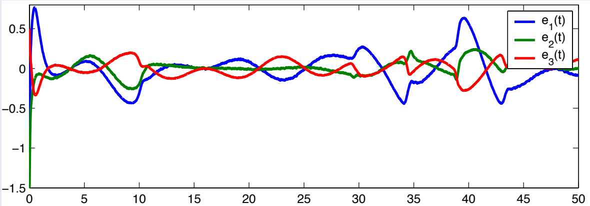


FIGURE: State estimation errors using a PI observer

Outline of the talk

Introduction

Introduction

- FDI has been (and remains) an active field of research over the past decades

Introduction

- FDI has been (and remains) an active field of research over the past decades
- A possible way to achieve FDI goals is to use bank of dynamic observers

Fault detection and isolation

Introduction

- FDI has been (and remains) an active field of research over the past decades
- A possible way to achieve FDI goals is to use bank of dynamic observers
- The previously proposed observers are well adapted for that goal

Introduction

- FDI has been (and remains) an active field of research over the past decades
- A possible way to achieve FDI goals is to use bank of dynamic observers
- The previously proposed observers are well adapted for that goal
- When using this kind approach, the output estimation errors are used as residuals

Introduction

- FDI has been (and remains) an active field of research over the past decades
- A possible way to achieve FDI goals is to use bank of dynamic observers
- The previously proposed observers are well adapted for that goal
- When using this kind approach, the output estimation errors are used as residuals

Very small illustrative example n° 1

System represented by a TS model with one input $u(t)$ and two outputs $y_i(t), i = 1, 2$:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) \right) \\ y(t) = Cx(t) + f(t) + w(t) \end{cases}$$

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{bmatrix}, A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 1 & 2 & -4 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.5 \\ 1 \\ 0.25 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$f(t)$ is a sensor fault vector and $w(t)$ a zero-mean noise vector.

First step

Design of an observer on the basis of $u(t)$ and the two noise-free outputs $y_i(t), i = 1, 2$ using the observer designed for attenuating the perturbation.

First step

Design of an observer on the basis of $u(t)$ and the two noise-free outputs $y_i(t), i = 1, 2$ using the observer designed for attenuating the perturbation.

Remember that, in that case, we have :

$$\dot{e} = \sum_{i=1}^{r} \mu_i(\hat{x}) \left((A_i - L_i C) e \right) + \Delta A x + \Delta B u$$

and it's useful to attenuate the effect of $u(t)$ on the state estimation error $e(t)$

$$\frac{\|e(t)\|_2}{\|u(t)\|_2} < \gamma, \quad \gamma > 0$$

Fault detection and isolation

First step

Design of an observer on the basis of $u(t)$ and the two noise-free outputs $y_i(t), i = 1, 2$ using the observer designed for attenuating the perturbation.

Remember that, in that case, we have :

$$\dot{e} = \sum_{i=1}^{r} \mu_i(\hat{x}) \left((A_i - L_i C) e \right) + \Delta A x + \Delta B u$$

and it's useful to attenuate the effect of $u(t)$ on the state estimation error $e(t)$

$$\frac{\|e(t)\|_2}{\|u(t)\|_2} < \gamma, \quad \gamma > 0$$

The state estimation error converges and the gain of the transfer from $u(t)$ to $e(t)$ is bounded by $\gamma = 0.0894$

Since the input $u(t)$ is bounded by 1, the state estimation error is bounded by $\gamma = 0.0894$ that may be considered as acceptable when considering the magnitude of the state.

Results

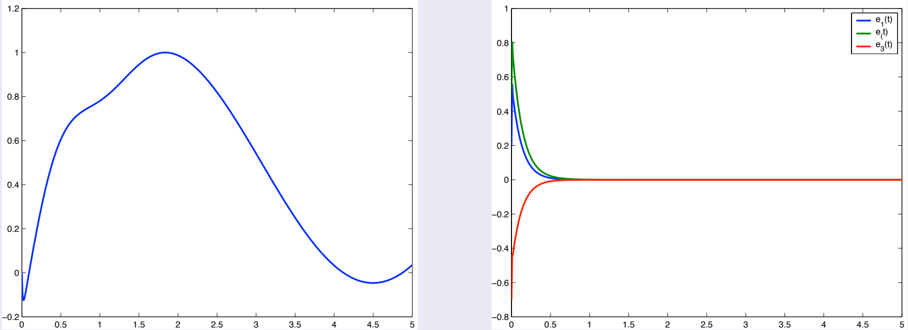


FIGURE: Input of the system (left) and state estimation errors (right)

Second step – Bank of observers

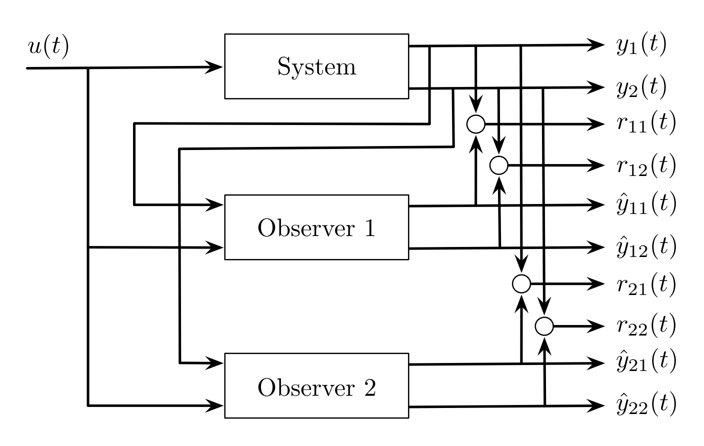


FIGURE: Generalized Observer Scheme (GOS) for sensor fault detection and isolation

Second step

Using the same procedure, two observers have been designed on the basis of noisy and faulty outputs.

Second step

Using the same procedure, two observers have been designed on the basis of noisy and faulty outputs.

The residual signals are defined as :

$$r_{ij}(t) = y_j(t) - \hat{y}_{ij}(t), \quad \forall\ i, j \in \{1, 2\}$$

where i represents the observer number and j the output number.

Second step

Using the same procedure, two observers have been designed on the basis of noisy and faulty outputs.

The residual signals are defined as :

$$r_{ij}(t) = y_j(t) - \hat{y}_{ij}(t), \quad \forall\ i, j \in \{1, 2\}$$

where i represents the observer number and j the output number.

The proposed scheme is such that :

- $r_{11}(t)$ is not sensitive to the fault $f_2(t)$
- $r_{22}(t)$ is not sensitive to the fault $f_1(t)$

Second step

Using the same procedure, two observers have been designed on the basis of noisy and faulty outputs.

The residual signals are defined as :

$$r_{ij}(t) = y_j(t) - \hat{y}_{ij}(t), \quad \forall\ i, j \in \{1, 2\}$$

where i represents the observer number and j the output number.

The proposed scheme is such that :

- $r_{11}(t)$ is not sensitive to the fault $f_2(t)$
- $r_{22}(t)$ is not sensitive to the fault $f_1(t)$

The measurements are corrupted by faults

$$f_1(t) = \begin{cases} 1, & 2 \leq t \leq 4 \\ 0, & \text{elsewhere} \end{cases} \qquad f_2(t) = \begin{cases} 1, & 6 \leq t \leq 8 \\ 0, & \text{elsewhere} \end{cases}$$

Residuals

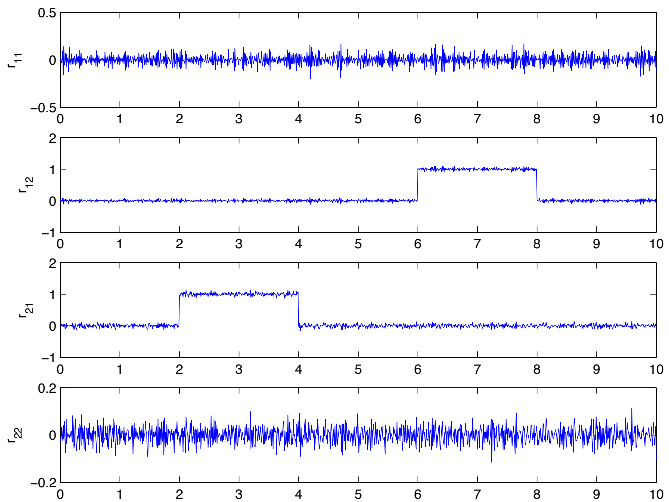


FIGURE: Residual signals

Small illustrative example n˚ 2

System represented by a TS model with unknown input :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t))\left(A_i x(t) + B_i u(t) + E_i d(t)\right) \\ y(t) = Cx(t) + Gd(t) \end{cases}$$

Small illustrative example n˚ 2

System represented by a TS model with unknown input :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i d(t) \right) \\ y(t) = Cx(t) + Gd(t) \end{cases}$$

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -4 \end{bmatrix}, A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 0.5 & 0.5 & -4 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0.3 \\ 0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.50 \\ 1 \\ 0.25 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, E_1 = \begin{bmatrix} 0.50 \\ -1 \\ 0.25 \end{bmatrix}, E_2 = \begin{bmatrix} -1 \\ 0.52 \\ 1 \end{bmatrix}, G = \begin{bmatrix} 0.3 \\ 0.9 \end{bmatrix}$$

Small illustrative example n˚ 2

System represented by a TS model with unknown input :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i d(t)\right) \\ y(t) = Cx(t) + Gd(t) \end{cases}$$

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -4 \end{bmatrix}, A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 0.5 & 0.5 & -4 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0.3 \\ 0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.50 \\ 1 \\ 0.25 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, E_1 = \begin{bmatrix} 0.50 \\ -1 \\ 0.25 \end{bmatrix}, E_2 = \begin{bmatrix} -1 \\ 0.52 \\ 1 \end{bmatrix}, G = \begin{bmatrix} 0.3 \\ 0.9 \end{bmatrix}$$

The considered unknown input is a piecewise constant function :

$$d(t) = \begin{cases} 0.5, & 4.5 \leq t \leq 11 \\ 0, & \text{elsewhere} \end{cases}$$

Small illustrative example n˚ 2

System represented by a TS model with unknown input :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i d(t)\right) \\ y(t) = Cx(t) + Gd(t) \end{cases}$$

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -4 \end{bmatrix}, A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 0.5 & 0.5 & -4 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0.3 \\ 0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.50 \\ 1 \\ 0.25 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, E_1 = \begin{bmatrix} 0.50 \\ -1 \\ 0.25 \end{bmatrix}, E_2 = \begin{bmatrix} -1 \\ 0.52 \\ 1 \end{bmatrix}, G = \begin{bmatrix} 0.3 \\ 0.9 \end{bmatrix}$$

The considered unknown input is a piecewise constant function :

$$d(t) = \begin{cases} 0.5, & 4.5 \leq t \leq 11 \\ 0, & \text{elsewhere} \end{cases}$$

Implementation of a PI observer

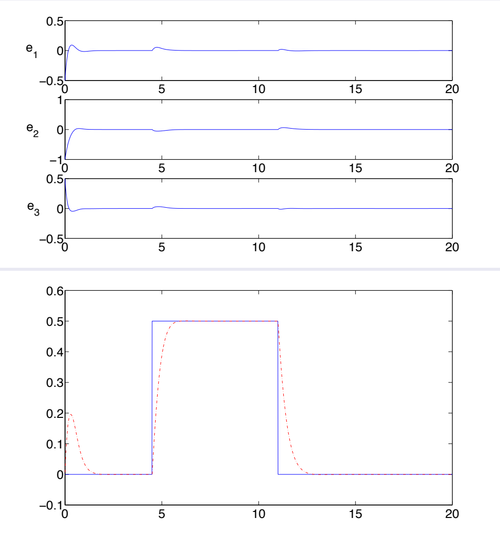


FIGURE: State estimation errors (up) ; Unknown input and its estimate (down)

Outline of the talk

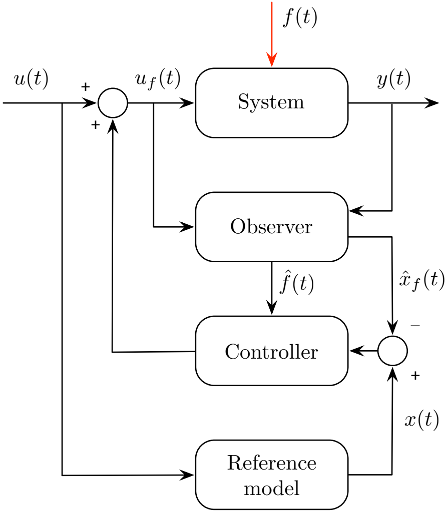


FIGURE: FTC by model reference approach

Reference model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(C_i x(t) + D_i u(t)\right) \end{cases}$$

Fault tolerant control

Reference model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t))\,(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(x(t))\,(C_i x(t) + D_i u(t)) \end{cases}$$

Actual (faulty) system

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{r} \mu_i(x_f(t))\,(A_i x_f(t) + B_i(u_f(t) + f(t))) \\ y_f(t) = \sum_{i=1}^{r} \mu_i(x_f(t))\,(C_i x_f(t) + D_i(u_f(t) + f(t))) \end{cases}$$

Reference model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(C_i x(t) + D_i u(t)\right) \end{cases}$$

Actual (faulty) system

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{r} \mu_i(x_f(t)) \left(A_i x_f(t) + B_i(u_f(t) + f(t))\right) \\ y_f(t) = \sum_{i=1}^{r} \mu_i(x_f(t)) \left(C_i x_f(t) + D_i(u_f(t) + f(t))\right) \end{cases}$$

Control law

$$u_f(t) = \sum_{i=1}^{r} \mu_i(x_f(t)) \left(-f(t) + K_{1i}(x(t) - x_f(t)(t)) + u(t)\right)$$

Fault tolerant control

Reference model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(C_i x(t) + D_i u(t)\right) \end{cases}$$

Actual (faulty) system

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{r} \mu_i(x_f(t)) \left(A_i x_f(t) + B_i(u_f(t) + f(t))\right) \\ y_f(t) = \sum_{i=1}^{r} \mu_i(x_f(t)) \left(C_i x_f(t) + D_i(u_f(t) + f(t))\right) \end{cases}$$

Control law

$$u_f(t) = \sum_{i=1}^{r} \mu_i(x_f(t)) \left(-f(t) + K_{1i}(x(t) - x_f(t)(t)) + u(t)\right)$$

Fault tolerant control

Reference model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(C_i x(t) + D_i u(t)\right) \end{cases}$$

Actual (faulty) system

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{r} \mu_i(x_f(t)) \left(A_i x_f(t) + B_i(u_f(t) + f(t))\right) \\ y_f(t) = \sum_{i=1}^{r} \mu_i(x_f(t)) \left(C_i x_f(t) + D_i(u_f(t) + f(t))\right) \end{cases}$$

Control law

$$u_f(t) = \sum_{i=1}^{r} \mu_i(\hat{x}_f(t)) \left(-\hat{f}(t) + K_{1i}(x(t) - \hat{x}_f(t)(t)) + u(t)\right)$$

Proportional integral observer

$$\begin{cases} \dot{\hat{x}}_f(t) & = & \sum_{i=1}^{r} \mu_i(\hat{x}_f(t)) \left(A_i \hat{x}_f(t) + B_i u_f(t) + L_i \hat{f}(t) + H_{1i}(y(t) - \hat{y}(t)) \right) \\ \dot{\hat{f}}(t) & = & \sum_{i=1}^{r} \mu_i(\hat{x}_f(t)) \left(H_{2i}(y_f(t) - \hat{y}(t)) \right) \\ \hat{y}_f(t) & = & \sum_{i=1}^{r} \mu_i(\hat{x}_f(t)) \left(C_i \hat{x}_f(t) + D_i(u_f(t) + \hat{f}(t)) \right) \end{cases}$$

Fault tolerant control

Proportional integral observer

$$\left\{\begin{array}{rcl} \dot{\hat{x}}_f(t) & = & \sum_{i=1}^{r} \mu_i(\hat{x}_f(t)) \left(A_i \hat{x}_f(t) + B_i u_f(t) + L_i \hat{f}(t) + H_{1i}(y(t) - \hat{y}(t)) \right) \\ \dot{\hat{f}}(t) & = & \sum_{i=1}^{r} \mu_i(\hat{x}_f(t)) \left(H_{2i}(y_f(t) - \hat{y}(t)) \right) \\ \hat{y}_f(t) & = & \sum_{i=1}^{r} \mu_i(\hat{x}_f(t)) \left(C_i \hat{x}_f(t) + D_i(u_f(t) + \hat{f}(t)) \right) \end{array}\right.$$

Estimation errors

$$\tilde{e}(t) = \begin{pmatrix} x(t) - x_f(t) \\ x_f(t) - \hat{x}_f(t) \\ f(t) - \hat{f}(t) \end{pmatrix}$$

$$\dot{\tilde{e}}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\hat{x}_f(t)) \mu_j(x_f(t)) \tilde{A}_{ij} \tilde{e}(t) + \Gamma \Delta(t)$$

Proportional integral observer

$$\begin{cases} \dot{\hat{x}}_f(t) & = & \sum_{i=1}^{r} \mu_i(\hat{x}_f(t)) \left(A_i \hat{x}_f(t) + B_i u_f(t) + L_i \hat{f}(t) + H_{1i}(y(t) - \hat{y}(t)) \right) \\ \dot{\hat{f}}(t) & = & \sum_{i=1}^{r} \mu_i(\hat{x}_f(t)) \left(H_{2i}(y_f(t) - \hat{y}(t)) \right) \\ \hat{y}_f(t) & = & \sum_{i=1}^{r} \mu_i(\hat{x}_f(t)) \left(C_i \hat{x}_f(t) + D_i(u_f(t) + \hat{f}(t)) \right) \end{cases}$$

Estimation errors

$$\tilde{e}(t) = \begin{pmatrix} x(t) - x_f(t) \\ x_f(t) - \hat{x}_f(t) \\ f(t) - \hat{f}(t) \end{pmatrix}$$

$$\dot{\tilde{e}}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\hat{x}_f(t)) \mu_j(x_f(t)) \tilde{A}_{ij} \tilde{e}(t) + \Gamma \Delta(t)$$

Example

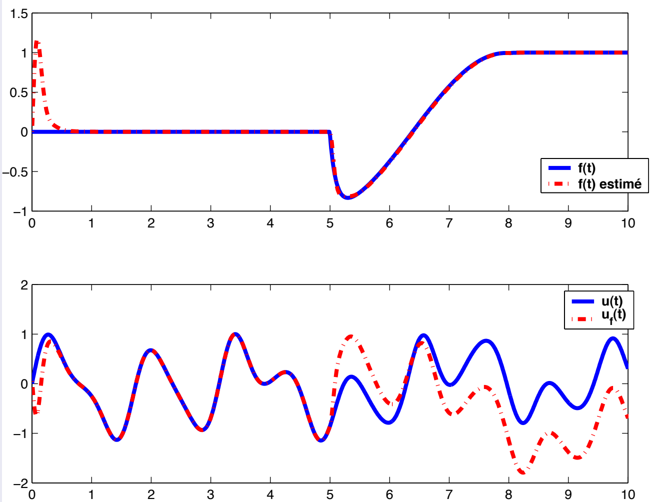


FIGURE: Fault and its estimate – Control signal (with and without tolerance)

Example

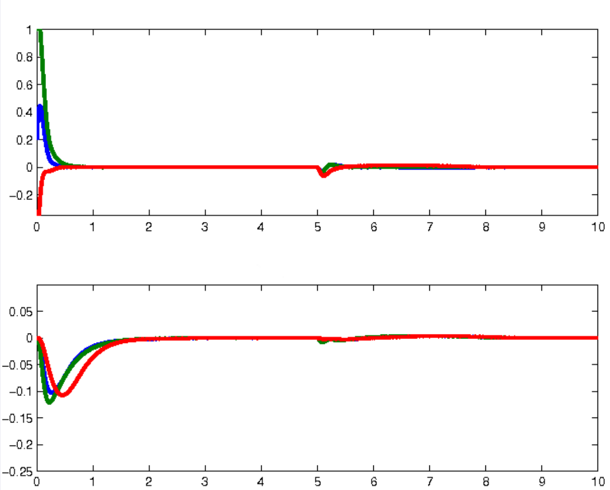


FIGURE: State estimation errors and tracking errors

Example

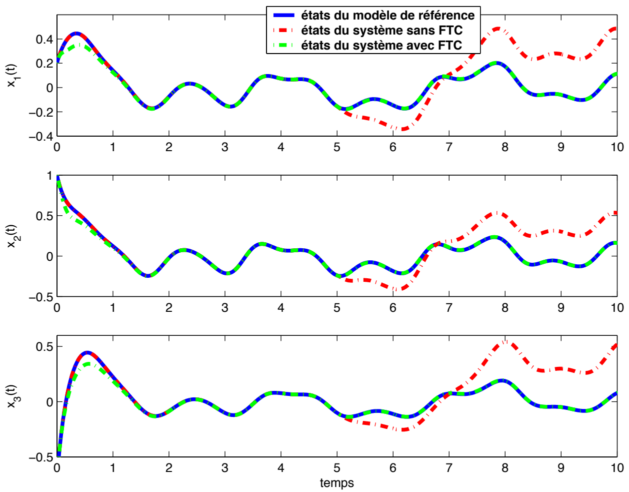


FIGURE: States of the reference model and system states with and without FTC

Outline of the talk

Conclusion

- Contribution to state estimation, fault detection and isolation and fault tolerant control of nonlinear systems
- Focus on TS models obtained by the nonlinear sector approach $\rightarrow$ unmeasurable decision variables
- Two different main approaches depending on the formulated hypotheses (Lipschitz or $\mathcal{H}_\infty$) for designing observers have been proposed
- Original proposition related to the rewriting of the model using weighting functions depending on estimated state variables
- Design of observers satisfying the requirements of FDI/FTC framework

Conclusion and prospects

Conclusion

- Contribution to state estimation, fault detection and isolation and fault tolerant control of nonlinear systems
- Focus on TS models obtained by the nonlinear sector approach $\longrightarrow$ unmeasurable decision variables
- Two different main approaches depending on the formulated hypotheses (Lipschitz or $\mathcal{H}_\infty$) for designing observers have been proposed
- Original proposition related to the rewriting of the model using weighting functions depending on estimated state variables
- Design of observers satisfying the requirements of FDI/FTC framework

Conclusion and prospects

Conclusion

- Contribution to state estimation, fault detection and isolation and fault tolerant control of nonlinear systems
- Focus on TS models obtained by the nonlinear sector approach $\rightarrow$ unmeasurable decision variables
- Two different main approaches depending on the formulated hypotheses (Lipschitz or $\mathcal{H}_\infty$) for designing observers have been proposed
- Original proposition related to the rewriting of the model using weighting functions depending on estimated state variables
- Design of observers satisfying the requirements of FDI/FTC framework

Conclusion

- Contribution to state estimation, fault detection and isolation and fault tolerant control of nonlinear systems
- Focus on TS models obtained by the nonlinear sector approach $\rightarrow$ unmeasurable decision variables
- Two different main approaches depending on the formulated hypotheses (Lipschitz or $\mathcal{H}_\infty$) for designing observers have been proposed
- Original proposition related to the rewriting of the model using weighting functions depending on estimated state variables
- Design of observers satisfying the requirements of FDI/FTC framework

Conclusion

- Contribution to state estimation, fault detection and isolation and fault tolerant control of nonlinear systems
- Focus on TS models obtained by the nonlinear sector approach $\rightarrow$ unmeasurable decision variables
- Two different main approaches depending on the formulated hypotheses (Lipschitz or $\mathcal{H}_\infty$) for designing observers have been proposed
- Original proposition related to the rewriting of the model using weighting functions depending on estimated state variables
- Design of observers satisfying the requirements of FDI/FTC framework

What I do not talk about

- Upstream study for obtaining the "best" TS model using nonlinear sector approach
- Discrete-time TS models (some results can be easily transposed)
- The nature of the Lyapunov function and the choice of more sophisticated candidate functions (reduction of the conservatism of the proposed solutions)
- The reduction of the number of LMI to solve
- And so on ...

What I do not talk about

- Upstream study for obtaining the “best” TS model using nonlinear sector approach
- Discrete-time TS models (some results can be easily transposed)
- The nature of the Lyapunov function and the choice of more sophisticated candidate functions (reduction of the conservatism of the proposed solutions)
- The reduction of the number of LMI to solve
- And so on ...

What I do not talk about

- Upstream study for obtaining the "best" TS model using nonlinear sector approach
- Discrete-time TS models (some results can be easily transposed)
- The nature of the Lyapunov function and the choice of more sophisticated candidate functions (reduction of the conservatism of the proposed solutions)
- The reduction of the number of LMI to solve
- And so on ...

What I do not talk about

- Upstream study for obtaining the "best" TS model using nonlinear sector approach
- Discrete-time TS models (some results can be easily transposed)
- The nature of the Lyapunov function and the choice of more sophisticated candidate functions (reduction of the conservatism of the proposed solutions)
- The reduction of the number of LMI to solve
- And so on ...

What I do not talk about

- Upstream study for obtaining the “best” TS model using nonlinear sector approach
- Discrete-time TS models (some results can be easily transposed)
- The nature of the Lyapunov function and the choice of more sophisticated candidate functions (reduction of the conservatism of the proposed solutions)
- The reduction of the number of LMI to solve
- And so on ...

Future works and ways of research

- Application of that method on real systems (wastewater treatment plant, aircraft and aerospace system - SIRASAS project, Robust and Innovative Strategies for Space and Aeronautical Systems Autonomy)

  Fondation de Recherche pour l'Aéronautique & l'Espace

- Observability analysis of TS models
- Reduction of the number of LMI (use of descriptor models)
- Simultaneous estimation of the state and weighting function parameters
- Fault diagnosis in closed-loop systems

Future works and ways of research

- Application of that method on real systems (wastewater treatment plant, aircraft and aerospace system - SIRASAS project, Robust and Innovative Strategies for Space and Aeronautical Systems Autonomy)

  Fondation de Recherche pour l'Aéronautique & l'Espace

- Observability analysis of TS models
- Reduction of the number of LMI (use of descriptor models)
- Simultaneous estimation of the state and weighting function parameters
- Fault diagnosis in closed-loop systems

Future works and ways of research

- Application of that method on real systems (wastewater treatment plant, aircraft and aerospace system - SIRASAS project, Robust and Innovative Strategies for Space and Aeronautical Systems Autonomy)

Fondation de Recherche pour l'Aéronautique & l'Espace

- Observability analysis of TS models
- Reduction of the number of LMI (use of descriptor models)
- Simultaneous estimation of the state and weighting function parameters
- Fault diagnosis in closed-loop systems

Future works and ways of research

- Application of that method on real systems (wastewater treatment plant, aircraft and aerospace system - SIRASAS project, Robust and Innovative Strategies for Space and Aeronautical Systems Autonomy)

  Fondation de Recherche pour l'Aéronautique & l'Espace

- Observability analysis of TS models
- Reduction of the number of LMI (use of descriptor models)
- Simultaneous estimation of the state and weighting function parameters
- Fault diagnosis in closed-loop systems

Conclusion and prospects

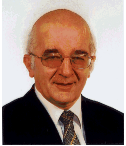
Future works and ways of research

- Application of that method on real systems (wastewater treatment plant, aircraft and aerospace system - SIRASAS project, Robust and Innovative Strategies for Space and Aeronautical Systems Autonomy)

  Fondation de Recherche pour l'Aéronautique & l'Espace

- Observability analysis of TS models
- Reduction of the number of LMI (use of descriptor models)
- Simultaneous estimation of the state and weighting function parameters
- Fault diagnosis in closed-loop systems

Aknowledgements



José Ragot
Professor in Automatic Control
National Polytechnic Institute of Nancy, France
National High School of Geology Engineering

Benoît Marx
Assistant-Professor in Automatic Control
National Polytechnic Institute of Nancy, France
National High School of Geology Engineering

Rodolfo Orjuela
Assistant-Professor in Automatic Control
University of Haute Alsace, Mulhouse, France

Dalil Ichalal
Lecturer in Automatic Control
Nancy-University, France

Get in touch

Didier Maquin

Professor in Automatic Control
National Polytechnic Institute of Nancy

High School of Electrical and Mechanical Engineering

Research Center for Automatic Control

didier.maquin@ensem.inpl-nancy.fr

More information ?

Personal : http://www.ensem.inpl-nancy.fr/Didier.Maquin/en/

Research Lab : http://www.cran.uhp-nancy.fr/anglais/


P
u
B

18th Mediterranean Conference on Control and Automation

June 23-25, 2010 – Marrakech, Morocco

http://www.med10.org

Mediterranean Control Association

IEEE Control System Society

General Chairs	:	Abdellah Benzaouia & Ahmed El Hajjaji
Program Chair	:	Dominique Sauter
Program Vice-Chairs	:	Joseba Quevedo & Abdel Aitouche

Submission deadline : January 15, 2010

SysTol'10

Conference on Control and Fault-Tolerant Systems

October 6-8, 2010 – Nice, France

http://www.systol10.org

Research Center for Automatic Control

IEEE Control System Society

General Chair	: Didier Maquin
Honorary Chair	: Ronald Patton
Program Chair	: Józef Korbicz
Program Vice-Chair	: Didier Theilliol

Submission deadline : May 15, 2010

Sixième Conférence Internationale Francophone d'Automatique

2-4 juin 2010 – Nancy, France

http://cifa2010.cran.uhp-nancy.fr/

Centre de Recherche en Automatique de Nancy

GdR Modélisation, Analyse et Conduite des Systèmes dynamiques

Président général	: Alain Richard
Comité de programme	: Jean-Pierre Richard & Didier Maquin