OBSERVABILITY, REDUNDANCY, RELIABILITY AND INTEGRATED DESIGN OF MEASUREMENT SYSTEMS

Marie LUONG[*], Didier MAQUIN[*], Chanh Trung HUYNH[**], José RAGOT[*]

[*] C.R.A.N. (C.N.R.S. - URA 821)
2, Avenue de la Forêt de Haye, B.P. 3
54516 VANDOEUVRE cedex
Phone: 83 59 59 59 Fax: 83 59 56 44

[**] Elf Aquitaine
26, avenue des lilas
64018 PAU cedex
Phone: 59 83 62 18 Fax: 59 83 57 35

Abstract. The problem of sensor placement is considered. The quality of diagnosis depends on the number and location of measurements. We first deal with this dependence by analysing the observability. The concepts of degree of redundancy is introduced. The criterion of reliability is useful for designer to enhance the reliability of the measurement system. When some variables are required for the control of the process, the reliability of the measurement system may be computed taking into account the reliability of all sensors. In order to improve the diagnosis, we propose an original and computational tool for designer to conceive the automatic and optimal sensor placement. Our objective is to design the measurement system meeting the following constraints: observability of variables required for the controlling and the maintenance, fulfilment of different degrees of redundancy imposed for some variables and fulfilment of criterion of sensor cost or/and reliability.

Key Words: Degree of redundancy, observability, reliability, canonical matrix, cycle analysis.

1. INTRODUCTION

It is important for process control improvement to integrate the design of the measurement system while conceiving the process itself. The main problem to define a measurement system concerns the selection of variables to be measured; especially the state estimation of the process is influenced by this choice. Indeed, estimation techniques are only applicable for observable systems; one of the fundamental steps then consists in isolating observable parts of the process. Publications on observability concepts are numerous; (Vaclavek 1969) is one of the firsts to undertake the linear system study. (Mah et al., 1976) and (Gomolka et al., 1992) develop a technique based on the graph theory. One can also quote the works performed for bilinear systems in the domain of data reconciliation (Kretsovalis et al., 1988; Crowe 1989; Maquin et al., 1989 and Ragot et al., 1991).

Some synthesis works deal with this topic; indeed, in order to carry out a diagnosis, a minimum number of information provided from the sensors of the process is required. If the amount of information is not sufficient, a number of supplementary sensors is proposed so as to complete "poor" parts in information of the process (Vaclavek 1969; Ragot et al., 1986; Darouach et al., 1986 and Maquin et al., 1987). The conception of measurement system for bilinear models has also been published (Ragot et al., 1992). Thus, the observability concept has provided one of the criteria for the improvement of the measurement systems. More recently, optimization study for the global reliability of the measurement system taking into account simultaneously observability and sensor reliability has been studied (Turbatte et al., 1993).

Our purpose is focused on the conception of sensor placement by considering the reliability of sensors. Based on the concepts of observability, redundancy and reliability, this communication presents an automatic sensor placement technique allowing to minimize the global cost of the instrumentation. More precisely, given the list of variables required for the control, the maintenance and the safety of the process (in the following, we only use the word control instead), we seek the optimal sensor placement that meets the following objectives: 1) guaranteeing the observability of variable required for the control 2) guaranteeing degrees of redundancy of some variables 3) ensuring minimal cost of the instrumentation.

The proposed algorithm is based on the analysis of cycles of the graph associated with a process. It presents an interest as much for its facility of implementation as for its rapidity with regard to matrix methods (Darouach et al., 1986 and Maquin et al., 1987). Having concern for the simplicity, we limit this communication to the linear system study. It is organized as follows: the section 2 is relative to the process description; the section 3 presents the analysis of the process according to the concepts of observability, redundancy and reliability; using the analysis results of section 3, the fourth section proposes an algorithm to solve the problem of sensor placement and presents some numerical results.

2. DESCRIPTION OF A PROCESS

A process can be described by equations linking different variables; in a linear system, these equations may be represented by a graph made up of m arcs and n nodes. The complete set of equations of the process can be written as:

$$M X = 0 \qquad (1)$$

where M represents the (n.m) node incidence matrix of rank n, and X the (m.1) vector of variables of the system. Considering L, the set of variables of the process, it is divided into two following subsets: L_1 is the subset of variables required for the control and L_2 the subset of variables non required for the control. In the case of a process equipped with sensors, measured and unmeasured variables can be distinguished into the two lists X_m and $X_{\bar{m}}$.

As an example, let us consider the process network of figure 1; it can be described by the node incidence matrix M (table 1), where the first row contains the numbers of variables, and the first column the numbers of equations (nodes). The subsets of variables required and non required for the control respectively are:

$$L_1 = \{1, 4, 6, 9, 10\} \qquad L_2 = \{2, 3, 5, 7, 8\}$$

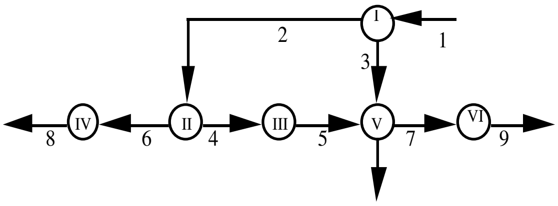

Fig. 1. A process network

	1	2	3	4	5	6	7	8	9	10
I	1	-1	-1	.	.	.	.	.	.	.
II	.	1	.	-1	.	-1	.	.	.	.
III	.	.	.	1	-1	.	.	.	.	.
IV	.	.	.	.	.	1	.	-1	.	.
V	.	.	1	.	1	.	-1	.	.	-1
VI	.	.	.	.	.	.	1	.	-1	.

Table 1. Node incidence matrix M

From this process network, the construction of a cyclic graph is possible by connecting arcs corresponding to its feeds (entries) and its products (exits) to a so-called environment node. This graph is interesting because it allows all the necessary cycles for the implementation of algorithms that we will present to be determined (Berge 1989).

The cycle matrix

It is important to note that a process graph is independent of its measurement system. Especially, cycles are the same whether the process is supplied with sensors or not. The research of cycles depends on the determination of a spanning tree of the graph. The literature proposes several methods among which the method of Gauss-Jordan is retained. This one uses linear equation combinations (rows) and variable exchanges (columns) in order to obtain a canonical form of the node incidence M (n.m) called the canonical node incidence matrix or the fundamental cutset matrix (Maquin et al., 1987; Gomolka et al., 1992):

$$M_C = [I_C \mid M_{CS}] \qquad (2)$$

where I_C is the identity matrix (n.n) whose columns correspond to the branches of the spanning tree, and M_{CS} the singular matrix (n.(m-n)) whose columns correspond to the chords. The fundamental cycle matrix C_F ((m-n). m) is defined by

$$C_F = [C_{FS} \mid I_F] \qquad (3)$$

$$(C_{FS})^T = - M_{CS} \qquad (4)$$

If we are solely interested in the occurrence of a variable in a cycle, the absolute value (of terms) of the matrix C_F is used and noted C_{FA}. The matrix containing all the cycles generated from the fundamental cycles will be noted C_T.

Theorem 1: a cycle containing k different chords results from the ring sum of k fundamental cycles containing these k chords.

Proof: If we consider the expressions (2) and (3), the variables of the identity submatrix I_F columns correspond to the chords. Then, k distinct fundamental cycles are composed of k distinct chords, therefore k different arcs. The ring sum of these k fundamental cycles is the same operation as the deletion of common arcs of these cycles. As these k chords are distinct, there are no possible deletion of these k chords.

Algorithm of cycle generation

Let us denote c = m-n, the number of fundamental cycles of a connected graph and C_c^k the binomial number of k elements among c. According to the theorem 1, each of the c fundamental cycles contains a chord. Cycles formed by the addition of k distinct fundamental cycles contain k chords. The number of these cycles is C_c^k . To obtain all the cycles, we use the recursive procedure:

-by using the fundamental cycle matrix, we generate cycles containing 2 chords,

-then, we generate cycles containing 3 chords from cycles containing 2 chords and fundamental cycles,

-more generally, we generate cycles containing k chords from cycles containing (k-1) chords and fundamental cycles.

The total number of cycles is given by Nbc:

$$Nbc = C_c^1 + C_c^2 + ...+ C_c^c = \sum_{i=1}^{c} C_c^i \qquad (5)$$

This algorithm will be useful both for the analysis of a process equipped with sensors and for the conception of a measurement system.

3. ANALYSIS OF AN INSTRUMENTED PROCESS

In this section, we introduce the concepts of observability, degree of redundancy of a variable and reliability.

3.1. Matrix of all the cycles of a graph

For an instrumented process, we define four lists whether a variable is measured or not and according to its membership to L_1 and L_2:

X_{1m}: measured variables required for the control,
$X_{1\bar{m}}$: unmeasured variables required for the control,
X_{2m}: measured variables non required for the control,
$X_{2\bar{m}}$: unmeasured variables non required for the control.

These four lists allow the columns of the matrix M to be rearranged as follows:

<---------measured-------->		<-------unmeasured-------->	
X_{1m}	X_{2m}	$X_{1\bar{m}}$	$X_{2\bar{m}}$
M_1	M_2	M_3	M_4

Table 2. Node incidence matrix M

Then, this matrix is restructured. Using Gauss-Jordan elimination, we first construct the greatest identity matrix I_{C2} with unmeasured variables. Then, if the rank of this matrix is not equal to n, we reiterate the operation with measured variables, which generates the matrix I_{C1}. The matrix M_5 concerns the remaining unmeasured variables. The submatrices M_6, M_7 and M_8 are constituted of remaining measured variables. Thus, we obtain the canonical matrix M_C with the following form:

me	$\bar{m}$	$\bar{m}\bar{e}$	$m\bar{e}$	me
I_{C1}	0	0	0	M_8
0	I_{C2}	M_5	M_6	M_7

Table 3. Canonical matrix M_C

For each list L_i (i = 1, 2), we define:
X_{ime}: variables corresponding to the identity submatrix I_{C1} and the submatrices M_8 and M_7
$X_{i\bar{m}}$: variables corresponding to the identity submatrix I_{C2}
$X_{i\bar{m}\bar{e}}$: variables corresponding to the submatrix M_5
$X_{im\bar{e}}$: variables corresponding to the submatrix M_6

The cycle matrix C_{FA} is easily obtained, using the relationships (3) and (4).

me	$\bar{m}$	$\bar{m}\bar{e}$	$m\bar{e}$	me
0	$\lvert M_5^T \rvert$	I_{F1}	0	0
0	$\lvert M_6^T \rvert$	0	I_{F2}	0
$\lvert M_8^T \rvert$	$\lvert M_7^T \rvert$	0	0	I_{F3}

Table 4. Fundamental cycle matrix

For the process of the figure 1 where X_m = {3, 4, 5, 8, 9 10} and $X_{\bar{m}}$ = {1, 2, 6, 7}, the cycle matrix, where measured variables are in bold type, is presented in the table 5 (the fundamental cycle matrix corresponds to the four first rows of this matrix).

4	9	1	6	2	7	8	10	3	5
.	.	1	1	1	.	1	.	.	.
.	1	.	.	.	1	.	1	.	.
.	1	1	.	.	1	.	.	1	.
1	1	1	.	1	1	.	.	.	1
.	1	1	1	1	1	1	1	.	.
.	1	.	1	1	1	1	.	1	.
1	1	.	1	.	1	1	.	.	1
.	.	1	.	.	.	.	1	1	.
1	.	1	.	1	.	.	1	.	1
1	.	.	.	1	.	.	.	1	1
.	.	.	1	1	.	1	1	1	.
1	.	.	1	.	.	1	1	.	1
1	.	1	1	.	.	1	.	1	1
1	1	.	.	1	1	.	1	1	1
1	1	1	1	.	1	1	1	1	1

Table 5. Cycle matrix C_T

3.2. Observability concepts

The observability analysis consists in classifying variables according to four categories. A deducible and measured variable (redundant) is a measured variable whose value can still be deduced from measurements of the other variables of an equation when the sensor measuring this variable fails. A measured and non deducible variable is a measured variable whose value cannot be deduced from other variables of an equation when the sensor measuring this variable fails. An unmeasured and deducible variable is an unmeasured variable whose value can be obtained from other variables of an equation. The equation in which a deducible variable occurs is a deduction equation. An unmeasured non deducible variable is an unmeasured variable appearing only in equations comprising two unmeasured variables at least.

Measured variables that are deducible or not and unmeasured deducible variables are observable variables. The canonical matrix structure of the table 3 leads to the variable classification by researching redundancy and deduction equations. However, we are going to show that the use of cycles facilitates this classification.

Rules of observability

The observability analysis is based on the study of cycles, using the four next rules:

Rule I: a measured variable is deducible if, and only if, it only belongs to cycles where at least two variables are measured.

Rule II: a measured variable is not deducible if, and only if, it at least belongs to a cycle where it is the only measured variable.

Rule III: an unmeasured variable is deducible if, and only if, it only belongs to cycles comprising one measured variable at least.

Rule IV: an unmeasured variable is not deducible if, and only if, it at least belongs to a cycle where no variable is measured.

Observability algorithm

Using these four rules, we propose the following algorithm for the observability study:

1) computing the cycle matrix C_T

2) computing cycles comprising from 0 to 2 measured variables among cycles of C_T, (the knowledge of cycles with more than two measurements is not necessary according to the rule I)

3) detecting cycles which do not contain measured variables: unmeasured variables belonging to these cycles are unmeasured and non deducible variables (rule IV). The other unmeasured variables of the system are unmeasured but deducible variables.

4) detecting cycles containing only one measured variable: measured variables belonging to these cycles are measured and non deducible variables (rule II). The other measured variables of the system are deducible measured variables.

The cycle matrix comprising a maximum number of two measured variables is obtained from the cycle matrix C_T (table 5) and is presented in the table 6:

4	9	1	6	2	7	8	10	3	5
.	.	1	1	1	.	1	.	.	.
.	1	.	.	.	1	.	1	.	.
.	1	1	.	.	1	.	.	1	.
.	.	1	.	.	.	.	1	1	.

Table 6. Matrix of cycles comprising two measured variables at the most per cycle.

After having analysed this matrix, we can classify the variables in the following lists according to their observability state:

$X_{\bar{m}\bar{e}} = \emptyset$ $\qquad$ $X_{\bar{m}e} = \{1, 2, 6, 7\}$

$X_{m\bar{e}} = \{8\}$ $\qquad$ $X_{me} = \{3, 4, 5, 9, 10\}$

3.3. Degree of redundancy

Definition 1: a redundant variable of degree k is an observable variable whose value remains deducible in case of simultaneous failures of any k sensors.

Especially, a minimal observable variable is a variable of zero redundancy degree; a variable belonging to a redundancy equation is redundant of degree 1 at least.

Definition 2: an unmeasured variable is pseudo-redundant (the difference with a redundant variable is that a pseudo-redundant variable is not measured) of degree 1 if it at least belongs to an equation where all the other variables are redundant or pseudo-redundant of degree 1.

By applying the next rules (V and VI), one can more generally deduce the redundancy degree of a measured variable, and similarly the pseudo-redundancy degree of an unmeasured variable:

Rule V: a measured variable is redundant of degree k if, and only if, it only belongs to cycles where at least (k+1) variables are measured.

Rule VI: an unmeasured variable is pseudo-redundant of degree k if, and only if, it only belongs to cycles where at least (k+1) variables are measured.

The redundancy degree of a variable is found by computing the minimum number of measured variables contained in cycles (among these of C_T) to which this variable belongs. If this number is equal to k, then the redundancy degree is (k-1).

Again, consider the process of the figure 1. From the cycle matrix (table 5), we obtain the following results:

- the measured variable 8 that belongs to the only cycle (the first) containing one measurement is a minimal observable variable.
- the unmeasured variables 1, 2 and 6 belong to the only cycle where one variable is measured. They are pseudo-redundant variables of degree 0 or minimal observable variables.
- the measured variables 3, 9 and 10 are redundant of degree 1 because they belong to cycles whose minimum number of measured variables is two.
- the measured variables 4 and 5 are redundant of degree 2 because they belong to cycles whose minimum number of measured variables is three.

According to the preceding results, values of the variables 3, 9 and 10 are available even if any one sensor fails. Values of the variables 4 and 5 remain always available in case of simultaneous failures of any two sensors.

In practice, the obtaining of the redundancy degrees of variables is interesting since that allows the security of the process functioning to be guaranteed.

3.4. Reliability of a measurement system

The measurement system function is to provide necessary information for the control of the process. Then, this function is not fulfilled if the measurement of a required variable for the control is no longer available. The reliability of an instrumentation system is defined as the probability that information required for the control are available through measurements or deduction during the time interval [0, t]. Then, it is advisable to compute the number of sensor failures conserving the observability of the variables required for the control (list L_1).

Assumptions

H0: sensors are considered irreparable.

H1: there is no breakdown of common cause.

H2: the failure rate λ of a sensor is independent of the time, which allows the reliability of a sensor to be defined analytically by using for example, the law of Poisson:

$$r(t) = \exp(-\lambda\, t) \qquad (6)$$

H3: initially, the system is observable.

By definition, the reliability of a sensor i is the probability $r_i(t)$ that no failure occurs during the time interval [0, t]. More generally, the reliability of a system constituted of p sensors with respective reliabilities $r_i(t)$ may be expressed by:

$$R(t) = f(r_1(t), r_2(t), ..., r_p(t)) \qquad (7)$$

We also define the MTTF function (Mean Time To Failure) by:

$$MTTF = \int_0^{\infty} R(t)dt \qquad (8)$$

The calculation of the MTTF may or may not take into account the redundancy equations. If we do not consider the model, the failure of a sensor measuring a value required for the control cannot be admitted. The reliability $R_0(t)$ is the probability that all sensors well function.

If the model is taken into account, some sensor failures can be tolerated. Indeed, one might estimate the value of a variable whose sensor has failed through redundant equations owing to the model. Practically, it is necessary to determine:

- the maximum number v of admissible failed sensors conserving the observability of the variables required for the control.
- the probability $R_i(t)$ so that the variables required for the control are available when i sensors (i =1,..,v) fail.

Finally, the reliability of the measurement system is expressed by:

$$R(t) = \sum_{i=0}^{v} R_i(t) \qquad (9)$$

It is obvious from (9) that the reliability increases if analytic relationships of the process are taken into account. It is shown that there is a relationship linking the observability, the degree of redundancy and the reliability.

The computation of the reliability necessitates to enumerate the cases where the failure of sensors can be tolerated. The value of a variable whose sensor fails is no longer known if, among all cycles to which this variable belongs, there is at least one cycle which do not contain measured variables (Rule IV). The availability of the value of a variable therefore depends on the number of measured variables in the cycle possessing the less ones; the more this cycle has measured variables, the more the measurement system can tolerate sensor breakdowns.

The computation of the maximal number v of admissible failed sensor is based on the following theorem.

Theorem 2: in a connected graph of (n+1) nodes, the maximal number of deducible variables through others is n; these variables are carried by the branches of the considered spanning tree.

Proof: considering the expression (2) and the canonical matrix (table 3) whose regular part variables are carried by the branches of the spanning tree. It is obvious that these variables are deducible through the knowledge of variable values of the singular part of the matrix.

3.5. Classification of variables required for the control

The canonical matrix structure (table 3) allows redundancy and deduction equations of variables required for the control to be determined (subset L_1). Indeed, variables are classified in the following classes (i=1,2):

X_{ime}: deducible measured variables of L_i
$X_{i\bar{m}}$: unmeasured variables of L_i
$X_{i\bar{m}e}$: unmeasured deducible variables of L_i
$X_{i\bar{m}\bar{e}}$: unmeasured non deducible variables of L_i
$X_{im\bar{e}}$: measured non deducible variables of L_i

The unmeasured deducible variables of $X_{i\bar{m}}$ belong to the rows of the submatrix I_{C2}. These rows correspond to null rows of M_5. From the canonical matrix (table 3), variables of the list $X_{i\bar{m}e}$ are obtained by rearranging M_5 to make its null rows appear. As the system is initially observable (according to *H3*), non deducible variables of $X_{1\bar{m}\bar{e}}$ do not exist. Since we are interested in the observability of variables of the list L_1, the equations containing variables of the list L_2 as well as the columns corresponding to variables of the lists $X_{2\bar{m}e}$, $X_{2\bar{m}\bar{e}}$ are suppressed. We obtain the following reduced matrix:

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
	X_{1me}	X_{2me}	$X_{1\bar{m}e}$	$X_{1m\bar{e}}$	$X_{2m\bar{e}}$	X_{1me}	X_{2me}
1	I_{C11}					M_{81}	
2		I_{C12}					
3			I_{C211}	M_{61}		M_{71}	

Table 8. Reduced canonical matrix M_C

Finally, columns of bloc (g) are reordered to obtain zero submatrix in the zone (g1):

(a)	(b)	(c)	(d)	(e)	(f)	(ga)	(gb)
X_{1me}	X_{2me}	$X_{1\bar{m}e}$	$X_{1m\bar{e}}$	$X_{2m\bar{e}}$	X_{1me}	X_{2me}	X_{2me}
I_{C11}					M_{811}		
	I_{C12}						M_{812}
		I_{C211}	M_{61}		M_{711}		M_{712}

Table 9. Reduced canonical matrix M_C

If we consider the theorem 2 and the "horizontal bands" (1) and (3) of the table 8, the knowledge of the values of variables corresponding to the columns of (d), (e), (f), (ga) and (gb) allows the deduction of necessary variables (columns of (a) and (c)) in case of simultaneous failure of sensors measuring variables corresponding to the columns of (a) and (b). The number v is equal to (m_a+m_b) where m_a et m_b are respective numbers of columns of (a) and (b). If the submatrix M_{712} of dimension $(n_3.m_{gb})$ is a null matrix (n_3 is the total number of lines of the matrix I_{C211}), the equations (2) of the table 8 will no longer be useful for the deduction of necessary variables. Then, the number v is equal to $(m_a+m_b+m_{gb})$.

3.6. Reliability under constraint

The problem consists in determining the number of failures conserving the observability of variables required for the control (list L_1). If equations of the process are not considered, the probability that all sensors function is equal to:

$$R_0(t) = r(t)^p \qquad (10)$$

On the other hand, if we take into account these equations, the probability that a sensor fails while (p-1) others (p > 1) well function is: $(1-r(t))r(t)^{p-1}$. If α_1 is the number of possibilities to consider a failed sensor while ensuring the observability of necessary variable, the probability that the system is operational while any one sensor fails is

$$R_1(t) = \alpha_1\ (1-r(t))r(t)^{p-1} \qquad (11)$$

More generally, we have:

$$R_i(t) = \alpha_i\ (1 - r(t))^i\, r(t)^{p-i} \qquad (12)$$

where the coefficient α_i is the total number of configurations that admit a number i of failed sensors while ensuring the observability of variables required for the control of the process.

The problem consists in computing $R_i(t)$. We therefore compute the coefficients α_i for i = 0,...,v. According to the definition of the coefficient α_i, cycles comprising only variables non required for the control (list L_2) have not to be counted. Furthermore, we eliminate cycles whose number of measured variables is higher than v. These cycles are not useful for the proposed analysis in the case where the number of simultaneous failures is less or equal to v. We finally obtain a so-called C_{Tv} cycle matrix whose analysis comprises three steps:

step 1: extracting among cycles of the matrix C_{Tv} those whose number of measured variables varies from 1 to i (cycles with k measured variables such that k is higher than i will have one measured variable at least when i sensors fail simultaneously, which is not interesting for the research of non admitted combinations),
step 2: forming C_p^i combinations of i failed sensors among p sensors. For each combination, variables measured by failed sensors are considered as unmeasured ones. We then calculate the number of measured variables for each cycle. According to the observability rule III, if all cycles include a measured variable at least, the considered combination is admissible,
step 3: deducing α_i which is equal to the number of admissible combinations, then $R_i(t)$.

Consider the example of the figure 1 and the following lists:

$L_1 = \{1, 4, 6, 9, 10\}$ $\quad$ $L_2 = \{2, 3, 5, 7, 8\}$
$X_m = \{3, 4, 5, 8, 9, 10\}$ $\quad$ $X_{\bar{m}} = \{1, 2, 6, 7\}$

The greatest admissible number of failed sensors is equal to 2. Cycles of the graph are presented in the table 6. The matrix C_{T2} whose cycles contain 2 measured variables at the most, is given in the table 10:

4	**9**	1	6	2	7	**8**	**10**	**3**	**5**	**N**
.	.	1	1	1	.	**1**	.	.	.	1
.	**1**	.	.	.	1	.	**1**	.	.	2
.	**1**	1	.	.	1	.	.	**1**	.	2
.	.	1	.	.	.	.	**1**	**1**	.	2

Table 10. Matrix of cycles C_{T2}.

Let us note λ the common failure rate of all sensors. The coefficient α_0 is equal to 1. Among six possibilities to consider a sensor failing, only the possibility to consider the failure of the sensor measuring the variable 8 is not admitted (according to the cycle 1). Thus, α_1 is equal to 5. Combinations including the variable 8 are not retained. Besides, next combinations are not accepted either: (9,10), (9,3), (10, 3) according to the last three cycles. We have 7 admissible combinations for α_2. Therefore, the function of reliability is:

$$R(t) = e^{-6\lambda t} + 5(1-e^{-\lambda t})e^{-5\lambda t} + 7(1-e^{-\lambda t})^2 e^{-4\lambda t}$$

Then, the value of the MTTF is $0.45/\lambda$

4. DESIGN OF MEASUREMENT SYSTEM

4.1. Position of the problem and objective

We are provided with the subset of variables required for the control of the process (list L_1). A weight, that is proportional to the cost of sensor, is assigned to each variable. Among the variables of the subset L_1, some are important for the functioning security; these are "high availability" variables. This demand may be ensured by analytic redundancy. According to the previous rules and especially the rule V, we can obtain a redundancy degree k for a high availability variable solely if the minimum number of measurable variables of all cycles to which this variable belongs is greater or equal to k+1. Necessary variables (subset L_1) are assumed measurable. Therefore, the purpose is the selection of variables to be measured with regard to the criterion of the minimum sensor cost so as to ensure:
- the minimal observability of variables required for the control (L_1),
- redundancy degrees of variables of the sets Ld_k.

Having concern for the simplicity and without loss of generality, the value of k is limited to 1.

4.2. Algorithm

It comprises three steps: the first consists to create the matrix of all cycles, the second to select an optimal set of variables to be measured with respect to the criterion of cost and minimal observability of necessary variables (list L_1), the third to choose supplementary variables to be measured in order to ensure redundancy degree 1 of variables of the set Ld_1 while minimizing the corresponding sensor cost and taking into account measured variables given by the second step.

step 1: *useful cycle matrix*
From the lists L_1 and L_2, the procedure of researching a spanning tree is the same as that of section 3. The measured and unmeasured variables might be replaced by those of the lists L_1 and L_2 respectively. Then, we deduce the fundamental cycle matrix C_{FA} and the cycle matrix C_T.
step 2: *minimal observability of variables required for the control.*
According to the rule V, we need to measure a variable at least per cycle containing a variable of L_1. We simplify this research by eliminating, among cycles of C_T, those containing only variables of the subset L_2. The resulting matrix C_1 of dimension $(n_{c1}.m_{c1})$ is obtained. During this step, we first compute all possible sets of variables to be measured L_{int}, then select one, L_{s1} which satisfies the cost criterion.
step 3: *redundancy of variables of the set Ld_1.*
According to the rule V, we have to measure two variables at least per cycle containing a variable of Ld_1. First, all possible sets of variables to be measured are computed. Before selecting a list satisfying the cost criterion, we take into account the proposed measured variables of the precedent list L_{s1}. Then, we select the list L_{s2} fulfilling the cost criterion.
Finally, the set of variables to be measured fulfilling the objective is the union of the lists L_{s1} and L_{s2}.

4.3. Results

Consider the process network of the figure 1 with $L_1 = \{1, 4, 6, 9, 10\}$ and $L_2 = \{2, 3, 5, 7, 8\}$, we wish to define the measurement system with a redundancy degree 1 for variables of the list $Ld_1 = \{1, 9\}$. To each variable is associated a weight representing the cost of the sensor (table 11).

Variable	*Cost*	Variable	*Cost*
1	*3*	6	*4*
2	*3*	7	*8*
3	*4*	8	*7*
4	*1*	9	*2*
5	*9*	10	*5*

Table 11. Costs associated with variables.

step 1: the cycle matrix is the same as that presented in the table 6. It only includes cycles containing at least a variable of the list L_1; let us note it by C_1.
step 2: the whole possible measurement lists for the selection (lists of intersection L_{int}) is not presented here due to their sizes. Taking into account the constraint of minimum cost, the following variables are to be measured to guarantee the minimal observability of the list L_1: $L_{s1} = \{1, 2, 4, 9\}$
step 3: the variables 1, 2, 4 and 9 are considered as measured ones, the matrix C_{11} contains cycle number 2, 8, 11 and 12 of the matrix C_1 (because they contain less than two measured variables). Besides, we have suppressed columns of C_1 corresponding to variables 1 and 9 (list Ld_1). The resultant matrix C_{11} is the following.

4	6	**2**	7	8	10	3	5
.	.	**.**	1	.	1	.	.
.	.	**.**	.	.	1	1	.
.	1	**1**	.	1	1	1	.
1	1	**.**	.	1	1	.	1

Table 12. Cycle matrix C_{11}

We need to measure one variable of each list of the lists (corresponding to a cycle of C_{11}): {7,10}, {10,3}, {6,2,8,10,3}, {4,6,8,10,5}. Finally, the supplementary variable to be measured is the variable 10.

In conclusion, the variables to be measured that meet the objective while minimizing the sensor cost are {1, 2, 4, 9, 10}. The corresponding total cost is 14. The reader may verify that cycles (of the matrix C_1) containing variables 1 and 9 comprise at least two measured variables. We can calculate the reliability of this measurement system with the help of the algorithm of the section 3. The evolution of the reliability is represented by R(t). The calculation is carried out with an identical failure rate for each sensor being equal to $1.25\ E^{-4}$. The calculation of MTTF is in fact represented by the area of the surface limited between the curve R(t) and the time axis. In the figure 3, the curve in solid line represents the reliability R(t) of the system with the measured variable list {1, 2, 4, 9, 10}. As a comparison, the curve in dashed line represents the reliability of the system obtained with the measured variable list {1, 2, 4, 9}, that only ensures the minimal observability of variables required for the control.

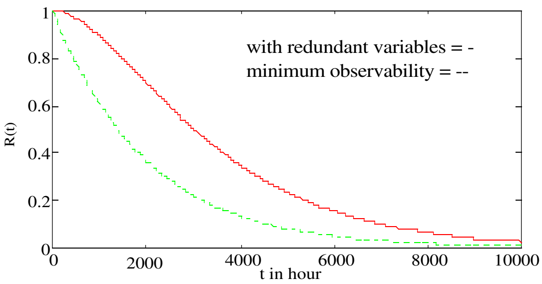

Fig. 3. Evolution of the reliability.

For the measurement system ensuring only the minimal observability of variables required for the control, we obtain a value of MTTF being equal to $0.25/1.25\ E^{4}$. The measurement system satisfying the objective gives a better value of MTTF with $0.45/1.25.E^{4}$. We observe that the system ensuring the redundancy degree 1 of variables 1 and 9 is more reliable than that not providing redundancy equations.

If the cost of sensors is not taken into account, several configurations may satisfy the objectives of minimal observability and degrees of redundancy but present a less important reliability with a same number of sensors. Let us consider a measurement system (number 1) involving the measurements of the variables {1, 2, 4, 9, 10} and the second one (number 2) involving the measurements of variables {1, 4, 6, 9, 10}. It is easy to verify that the second measurement system meets the previous requirements. All the variables required for the control are observable and variables 1 and 9 are redundant of degree 1. The figure 4 shows the reliability of the two measurement systems.

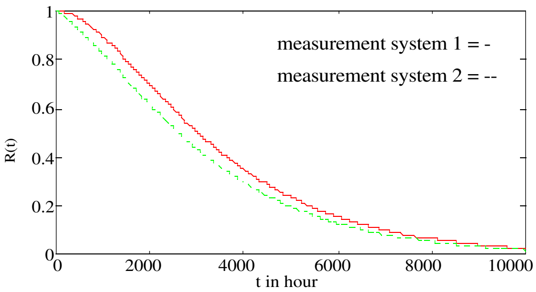

Fig. 4. Comparison of the evolution of the reliability

From this figure, it is obvious that the reliability of the measurement system 2 is inferior than that of measurement system 1 and so is the MTTF ($0.4/1.25.E^{4}$, for the measurement system 2) although the number of measurements is the same.

5. CONCLUSION

We have presented an analysis of the observability, the redundancy degree and the reliability; this technique is based on the analysis of cycles of the graph associated with the process. This analysis has been applied to the conception of the measurement system of a process. The proposed solution gives the optimum number of sensors and their location in order to: obtain (1) the minimal observability of variables required for the control, (2) redundancy degrees of variable of high degree of availability while minimizing the cost of measurements. In the case where several solutions provide a same minimum cost, we can use a complementary criterion taking into account the reliability. The use of cycles is all the more interesting for the conception of measurement system as it allows one to obtain a list of measurements easily, according to one or several given criteria. The solution of a multicriterion problem including precision of estimation, sensitivity to a modification of precision, observability, redundancy degree and reliability is a future direction of research.

6. REFERENCES

Berge C. (1989), "Graphs". Elsevier Science Publishers, Dunod, 413 p.

Crowe C. M. (1989), "Observability and redundancy of process data for steady-state reconciliation". Chemical Engineering Science, 4 (12), pp. 2909-2917.

Darouach M., Ragot J., Fayolle J., Maquin D. (1986), "Calcul hiérarchisé et décomposition des systèmes linéaires et bilinéaires appliqué à la validation des données." A.P.I.I., 20, pp. 405-432.

Gomolka W., Turbatte H. C., Maquin D., Ragot J. (1992), "Contribution à l'analyse de l'observabilité des systèmes linéaires décrits par des équations de bilan". Revue R.A.I.R.O. A.P.I.I., 26 (2), pp. 69-89.

Kretsovalis A., Mah R.S.H. (1988), "Observability and redundancy classification in generalized process networks. 1: theorems, 2: algorithms". Computers and Chemical Engineering, 12 (7), pp. 671-703.

Mah R. S. H., Stanley G. M., Downing D. M. (1976), "Reconciliation and rectification of process flow and inventory data". Industrial Engineering Chemistry Process Design Development, 15 (1), pp. 175-183.

Mah R. S. H. (1990), "Chemical Process Structures and Information Flows". Butterworths, 500 pages.

Maquin D., Fayolle J., Darouach M., Ragot J. (1987), "Le positionnement de capteurs dans les systèmes industriels de grande dimension". IMACS Congress, Villeneuve d'Asq, 1986. Published in Applied Modelling and Simulation of Technological Systems, Elsevier.

Maquin D., Darouach M., Ragot J. (1989), "Observability and data validation of bilinear systems". Congrès IFAC AIPAC' 89, Nancy, 3-5 juillet.

Ragot J., Darouach M., Maquin D., Bloch G. (1990), "Validation de données et diagnostic". Traité des nouvelles technologies, série diagnosis et maintenance, Hermès.

Ragot J., Maquin D., Darouach M. (1991), "Analysis of generalized bilinear systems - application to diagnostic". IMACS Symposium on Modelling an Control of Technological Systems.

Ragot J., Maquin D., Bloch G. (1992), "Sensor positioning for processes described by bilinear equations". Revue Diagnostic et Sûreté de Fonctionnement, 2 (2), pp. 115-132.

Schrick D. V. (1992), "Investigations of reliability for instrument fault detection state-estimator schemes". Revue Diagnostic et Sûreté de Fonctionnement, 1 (1), pp. 63-78.

Stanley G. M., Mah R. S. H. (1981), "Observability and redundancy classification in process networks." Chemical Engineering Science, 36, pp. 1941-1954.

Turbatte H. C., Maquin D., Huynh C. T., Ragot J. (1993), "Conception de l'architecture d'un système d'instrumentation - Application à un procédé pétrochimique". Proceedings of the International Conference on Fault Diagnostic, 1, pp. 406-415, Toulouse, France, avril 5-7.

Vaclavek V. (1969), "Studies on system engineering III. Optimal choice of the balance measurements in complicated chemical systems". Chemical Engineering Science, 24, pp. 947-955.

Villemeur A. (1988), "Sûreté de fonctionnement des systèmes industriels". Eyrolles, 795 p.