UNSCENTED KALMAN FILTER FOR OPERATING MODE CHANGE DETECTION

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Abstract

Real-life processes are mostly characterized by several operating regimes or modes. Each operating mode corresponds to particular operating conditions and may require the use of a specific control strategy. Therefore, the task of identifying or determining continuously the current operating mode of the process is of a great value as it allows to implement the control strategy that is in accordance with the current operating mode of the process. The use of an unscented Kalman filter is considered here for the purpose of tracking the active operating mode of the process at every time instant and consequently detecting when a change occurs in the process operating mode.

Introduction

The modelling of processes or systems exhibiting several operating regimes or modes has always been a subject of interest in various research areas like economics [3], finances [5], climatology [10], and engineering sciences [2]. The development of such models comes from the extension of the underlying principles of the classical linear regression theory, leading to the birth of different generalizations known as switched regression models or switched models. The work of [1], among others, has paved the way to the first principles of this appealing modelling technique. Since then, many major contributions [6] has specified and strengthened this formalism which has the potential to be of a practical use as soon a process or system displays several operating modes and can eventually "switch", in a forced or natural manner, between these operating modes. The determination of the active operating mode for a process exhibiting several operating modes is a crucial task for the implementation of the appropriate control strategy in regard to the current operating mode. It is also a key to achieve safety and profitability objectives for the process by the means of appropriate control actions. When the changes in the process operating modes are triggered by known variables or conditions (for example a process operator pushing a button), the active operating mode determination is facilitated. In the other situation where the reason why the process switches from one operating mode to another is not well mastered, it has to be inferred by using the measured process variables. In this scenario, there is a need to provide a way to estimate at each time instant the active operating mode of the process so that a change in the operating modes can be easily detected when it occurs. In [4], the proposed procedure is based on the generation of analytical redundancy equations between the process input and output variables. The feasibility of the method is conditioned by the existence of the analytical redundancy equations.

In the rest of this paper, the focus is on a particular class of multiple operating regime processes or systems known as PWA (Piece-Wise Affine) systems [13] and their extension to dynamic systems PWARX (Piece-Wise Auto-Regressive eXogeneous). The presented problem is the task of determining at each time instant the active operating mode by using the measurements of the system's input and output variables. The problem is investigated in a supervised detection framework, i.e. assuming that the different operating modes have been previously listed and identified. This problem is explored for several years by researchers from the continuous systems community and simultaneously by those related to the area of hybrid systems [8]. The original point of the proposed approach is the use of a particular Kalman filter that is well suited for the joint estimation of the system's state and the current operating regime (equivalently the operating regime switching time instants).

Position of the problem

Introductory example. Consider the second order system described by the following equation:

$$\begin{cases} x_{1,k+1} = -a_{0,k}x_{2,k} + b_{0,k}u_k \\ x_{2,k+1} = x_{1,k} - a_{1,k}x_{2,k} + b_{1,k}u_k \\ \quad y_k = x_{2,k} \end{cases} \tag{1}$$

with the time-varying parameters being described by the equations:

$$\begin{cases} a_{0,k} = \mu_k a_{01} + (1-\mu_k)a_{02} \\ a_{1,k} = \mu_k a_{11} + (1-\mu_k)a_{12} \\ b_{0,k} = \mu_k b_{01} + (1-\mu_k)b_{02} \\ b_{1,k} = \mu_k b_{11} + (1-\mu_k)b_{12} \end{cases} \tag{2}$$

The variable μ_k is a time-varying variable that can depend on an exogenous variable. In the case of μ taking only the values 1 or 0, the model of equation (1) shows that the represented system has two operating modes M_1 and M_2 that are described by the equations:

$$M_1 \begin{cases} x_{1,k+1} = -a_{01}x_{2,k} + b_{01}u_k \\ x_{2,k+1} = x_{11} - a_{11}x_{2,k} + b_{11}u_k \\ \quad y_k = x_{2,k} \end{cases} \qquad M_2 \begin{cases} x_{1,k+1} = -a_{02}x_{2,k} + b_{02}u_k \\ x_{2,k+1} = x_{11} - a_{12}x_{2,k} + b_{12}u_k \\ \quad y_k = x_{2,k} \end{cases} \tag{3}$$

Knowing the input u and the output y, the diagnosis problem involves the determination at each time instant of the system's active operating mode, i.e. say which of the two models M_1 and M_2 is compatible with the input/output measurements. To answer this question, note first that the model (1) can be expressed independently of the unknown state variable x:

$$\begin{aligned} &\mu_{1,k+2}y_{k+2} + \mu_{1,k+1}a_{11}y_{k+1} + \mu_{1,k}a_{o1}y_k - \mu_{1,k+1}b_{11}u_{k+1} - \mu_{1,k}b_{01}u_k + \\ &\mu_{2,k+2}y_{k+2} + \mu_{2,k+1}a_{12}y_{k+1} + \mu_{2,k}a_{o2}y_k - \mu_{2,k+1}b_{12}u_{k+1} - \mu_{2,k}b_{02}u_k = 0 \end{aligned} \tag{4}$$

where, for the sake of simplicity in the notation, one has the following change of variable: $\mu_1 = \mu$ and $\mu_2 = 1-\mu$. In this form, an estimation of the active mode indicator is obtained by resolving equation (4) in regard to μ :

$$\begin{aligned} \mu_{k+1}\left((a_{11}-a_{10})y_{k+1} - (b_{11}-b_{12})u_k\right) + \mu_k\left((a_{01}-a_{02})y_k - (b_{01}-b_{02})u_k\right) = \\ -y_{k+2} - a_{12}y_{k+1} - a_{02}y_k + b_{12}u_{k+1} + b_{02}u_k \end{aligned} \tag{5}$$

The numerical integration of equation (4), with the initial conditions μ_0 and μ_1, can be used to obtain the successive values of the mode indicator μ_k. An alternative can be proposed to this computationally demanding solution. The sub-optimal solution is derived by formulating the following assumption on the time window $[k : k+2]$:

$$\mu_k = \mu_{k+1} = \mu_{k+2} \tag{6}$$

To justify this assumption, one can consider the situation where the operating regime changes occur at low frequency and therefore the system stays in an operating mode for a period that far exceeds its main time constant.

From equations (5) and (6), one has:

$$\begin{cases} \mu_k = \dfrac{\varepsilon_2}{\varepsilon_2 - \varepsilon_1}, \quad \text{if } \varepsilon_2 - \varepsilon_1 \neq 0 \\ \varepsilon_1 = y_{k+2} + a_{11}y_{k+1} + a_{01}y_k - b_{11}u_{k+1} - b_{01}u_k \\ \varepsilon_2 = y_{k+2} + a_{12}y_{k+1} + a_{02}y_k - b_{12}u_{k+1} - b_{02}u_k \end{cases} \tag{7}$$

An on-line estimate of the value of the mode indicator can then be obtained. From equation (7)), one can notice that if the mode M_1 is active then ε_1 is equal to zero, which gives to μ the value 1. Inversely, if ε_2 takes the value zero then μ is equal to zero. Thus, the proposed estimator allows the determination, at each time instant, at every moment, of the system's active mode. In fact, the presence of measurement errors on the input and output variables might make the estimated values of μ vary around 0 or 1 instead of being exactly 0 or 1. The estimation procedure is also ineffective if the two errors ε_1 and ε_2 are both close to zero. It will then be necessary to set a threshold so as to decide the active mode.The technique that has been proposed can hardly be generalized to systems with order higher than two because the elimination of the state on the one hand and the extraction of the mode change indicator on the other hand are not easily achievable. Therefore, it is more convenient to rely on an estimation method of these two quantities through an observer. Here, the usage of a Kalman filter has been adopted.

System's Model. A switched system is generally described by:

$$x_{k+1} = f_{\sigma_k}(x_k, u_k) \tag{8}$$

where $\sigma : \mathbb{R}^+ \rightarrow \mathcal{I} = 1, 2 \ldots, N$ is a piece-wise constant function triggering the change from one operating mode to another, $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$ respectively stand for the system's state and input variables, $f_i(.,.)$ are vector fields used to represent the operating modes of the system.

The model of equation (8) represents a system with N operating modes, where each mode is described by a model, which is assumed to be known, and the change from one mode to another is governed by external (or internal) conditions that are not known *a priori*. In practice, these conditions may be modelled by a variable that causes the change in the operating mode when its value satisfies certain conditions. This approach of system modelling can be compared with the multi-model approach model that aggregates the models representing the operating modes with weighting or transition functions. The difference between the case study considered here and the multi-model one is precisely the fact that the variable that causes the transition from one mode to another is not assumed to be known *a priori*. The proposed approach is directly applicable to the case of non-linear models.

To take into account the presence of modelling errors or disturbances, the system's model, for the operating mode i, is written as:

$$\begin{cases} x_k = f_i(x_{k-1}, u_{k-1}) + q_{k-1} \\ y_k = h(x_k) + r_k \end{cases} \tag{9}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^p$, $y_k \in \mathbb{R}^m$, $q_k \in \mathbb{R}^n$, $r_k \in \mathbb{R}^m$ with the statistical assumptions of normal distribution for the process and measurement noises: $q_k \sim \mathcal{N}(0, Q_k)$ and $r_k \sim \mathcal{N}(0, R_k)$. To describe all the operating modes, let us introduce a parameter that reflects the change from one operating mode to another. This parameter can have an inner dynamical behaviour reflecting the rate at which the change of operating mode is performed. Here, this dynamic is modelled as a random walk process depending on a variable $s_k \in \mathbb{R}^{n_s}$ and the model of equation (9) is then modified in the form:

$$\begin{cases} x_k = f(x_{k-1}, u_{k-1}, \mu_{i,k-1}) + q_{k-1} \\ \mu_{i,k} = \mu_{i,k-1} + s_{k-1}, \quad s_k \sim \mathcal{N}(0, S_k) \\ y_k = h(x_k) + r_k \end{cases} \tag{10}$$

Problem statement. The problem statement is the following one: assuming that input/output measurements of the system are available and knowing the system's model (10), provide an estimation of the generalized state (x and μ) at each time instant.

Principle of the estimation of the operating mode changes

As mentioned above, the joint estimation of the system's state and switching parameters must be obtained by the means of the input/output measurements. Given the structure of the system's model, this estimation cannot be achieved through a linear filter due to the linkage between the system's state and switching parameters. The estimation methods that can be considered to tackle this task belong to the family of iterative estimation methods or model equations linearisation methods. It is well known that functions linearisation in the *EKF* (Extended Kalman Filter) leads to the conservation of the Gaussian nature of the estimation errors. However, the resulting Gaussian distribution, which is an approximation of the true distribution, does not necessarily have the same first and second order statistical moments as the true distribution. The *UKF* filter proposed in [7] uses a Gaussian approximation of the *a posteriori* distribution. The propagation of this distribution then simply requires the calculation of the mean and covariance, which is carried out through a a deterministic sampling technique known as unscented transformation.

The improvement brought by the UKF in comparison to the EKF is that the statistical moments are exactly calculated [**??**]. The UKF relies on the use of a finite number of sample points $(2n+1)$, n being the size of the system's state vector, selected around the mean value of the distribution to be propagated. These points known as sigma-points in the literature have the property to capture the mean and the variance of the distribution.

The unscented Kalman filter (UKF). Without limiting the generality of the foregoing, the model of equations (4), (5) and (6) can be rewritten as:

$$\begin{cases} X_k = F(X_{k-1}, u_{k-1}) + \eta_{k-1} \\ y_k = h(X_k) + r_k \end{cases} \tag{11}$$

This equation describes the dynamics of the augmented state vector:

$$X_k = \begin{pmatrix} x_k & \mu_{1,k} & \dots & \mu_{N,k} \end{pmatrix}^T, \quad \eta_k = \begin{pmatrix} q_k & s_{1,k} & \dots & s_{N,k} \end{pmatrix}^T \tag{12}$$

One can then apply to the model (11) the previously presented estimation procedure with the following assumptions: $\eta_k \sim \mathcal{N}(0, Q_k)$ and $r_k \sim \mathcal{N}(0, R_k)$.

• Prediction of the state mean m_k^- ant its covariance matrix P_k^- :

$$\begin{cases} X_{k-1} = \begin{pmatrix} m_{k-1} & \dots & m_{k-1} \end{pmatrix} + \sqrt{c}\begin{pmatrix} 0 & \sqrt{P_{k-1}} & -\sqrt{P_{k-1}} \end{pmatrix} \\ \hat{X}_k = f(X_{k-1}) \\ m_k^- = \hat{X}_k w_m \\ P_k^- = \hat{X}_k W \hat{X}_k^T + Q_{k-1} \end{cases} \tag{13}$$

• Update of the output mean and its covariance matrix:

$$\begin{cases} X_k^- = \begin{pmatrix} m_k^- & \dots & m_k^- \end{pmatrix} + \sqrt{c}\begin{pmatrix} 0 & \sqrt{P_k^-} & -\sqrt{P_k^-} \end{pmatrix} \\ Y_k^- = h(X_k^-) \\ \mu_k = Y_k^- w_m \\ S_k = Y_k^- W Y_k^{-T} + R_k \\ C_k = X_k^- W Y_k^{-T} \end{cases} \tag{14}$$

• Computation of the filter gain

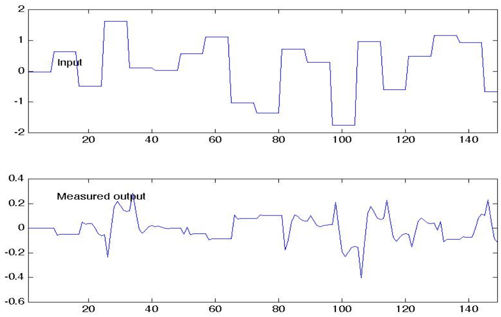

Figure 1: System's input and output

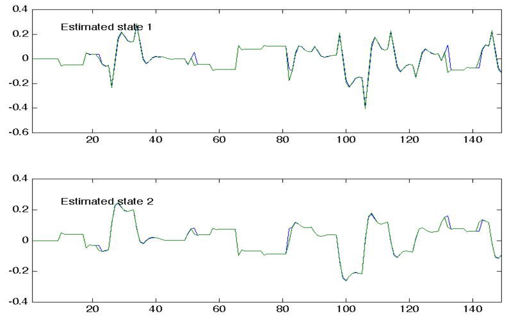

Figure 2: State estimation

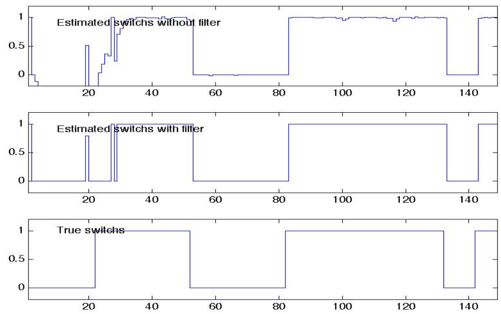

Figure 3: Estimation of the active mode

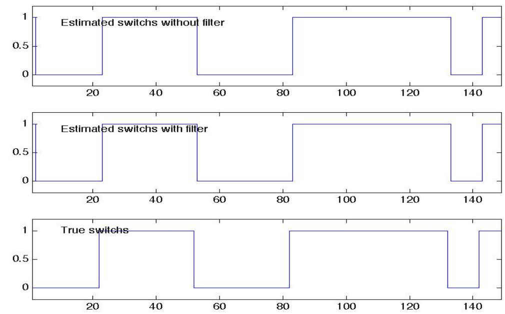

Figure 4: Estimation of the active mode

$$K_k = C_k S_k^{-1} \tag{15}$$

• Update of the state mean and its covariance matrix:

$$\begin{cases} m_k = m_k^- + K_k(y_k - \mu_k) \\ P_k = P_k^- - K_k S_k K_k^T \end{cases} \tag{16}$$

Numerical results. Consider a second order system represented by equation (17):

$$\begin{cases} x_{k+1} = a_k x_k + b u_k \\ \quad a_k = \mu_k a_1 + (1 - \mu_k) a_2 \\ \quad y_k = x_k \end{cases} \tag{17}$$

Simulation results (with operating mode changes at the time instants 20, 50, 80, 130 and 140) are presented in figures 1 through 3. The figures successively show the time evolution of:

- The system's input u and its output y. Note that the system's input and output are corrupted by measurement noise.
- The state estimation provided by the *UKF*.
- The estimation of the indicator of operating mode change μ, the filtered value of the indicator μ, and the true mode indicator.

The results illustrated by figure 4 have been obtained with the same system (equation (17)), input sequence and noise, except the only difference that the model of a_k is rewritten as $a_k = \tanh(\mu_k a_1 + (1 - \mu_k) a_2)$. For this example, It can be noticed that there is a perfect match between the estimated active mode and the true one.

Conclusion

The proposed method in this paper addressed the issue of the determination of the active operating mode of a system that may exhibit several operating modes or regimes. This problem is of a great importance for monitoring procedures where there is a mandatory need to have on-line information on the current behaviour of the system. From a practical point of view, the various operating modes can be healthy operating modes as well as a mix between healthy and faulty ones. In the first situation, the goal of the monitoring procedure is to determine the active operating mode in order to eventually adapt the control strategy. In the later situation, the detection of a change from a healthy operating mode to a faulty one helps to redefine the control strategy in order to compensate the effects of faults on the system if possible or to completely shut down the system if required.

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