ANALYTICAL REDUNDANCY FOR SYSTEMS WITH UNKNOWN INPUTS - APPLICATION TO FAULTS DETECTION

José RAGOT, Didier MAQUIN, Frédéric KRATZ

Centre de Recherche en Automatique de Nancy
BP 40 - Rue du doyen Marcel Roubault
54 501 Vandoeuvre les Nancy - France
Tel : (33) 83 50 30 80 Fax (33) 83 50 30 96

ABSTRACT

A well known method for residual or redundancy generation is extended to systems with unknown inputs. It is based on the singular form of the system equations which allows the generation of the substate of the system which is insensitive to the unknown inputs. This generation is achieved by a two-steps algorithm including, first, the elimination of the unknown inputs and second, the generation of the redundancy equations. Using these results for fault detection and isolation, we present a simulated example to illustrate our method.

KEYWORDS

Redundancy equations, unknown inputs, faults detection, singular systems.

INTRODUCTION

The basic principle of fault detection is the comparison of the actual behavior of a process with a reference behavior. Fault detection and isolation (FDI) schemes may be thought of as consisting of two stages : residual generation and analysis of these residuals (detection of events, classification of events into faults and normal modifications of the process and the isolation of the faulty components). Generally, the residual generation is issued from the knowledge available upon the process for example analytical models, knowledge base, sensors and actuators signals, ... One of the most popular approaches is based on the analytical redundancy (using the mathematical models of the physical system) which leads to express a set of invariants : the residuals of the process model. These residuals represent the inconsistency between the actual plant variables and the mathematical model. They are computed from the plant observations and with the model of this plant ; they are ideally zero but become non-zero if the actual system differs from the ideal one (this may be due to sensor or actuator faults, modelling errors, non exact parameters of the model, ...) Then, the fault detection is achieved by testing the magnitude of these residuals. When only few sensors are available the analytical redundancy has to be generated on a "long" time interval. In this case, system observers are commonly used to estimate state time histories for processes with incomplete set of state measurements (Gertler 1991). However in a certain number of applications a complete knowledge of the system inputs is unavailable. Several researchers have investigated state observers when the inputs are unknown based on a closed-loop observer that can identify states and inputs simultaneously (Park 1988), with the use of a full-observer (Yang 1988), using an augmented model to estimate the state and the unknown inputs (Gleason 1990) or by designing a reduced-order unknown input observer (Kudva 1980), (Guan 1991). In 1989, the advantages of using the so-called Kronecker Canonical Form was pointed out by Frank as a basis for the mathematical derivation of the FDI procedure. These have also included, during the past few years, the works listed in the reference section : the geometric approach to design observer (Bhattacharyya 1978), the detection using a bank of unknown-input observers (Viswanadham 1987), the use of generalized inverse matrices (Miller 1982) and the design of disturbance decoupled observer via singular value decomposition (Fairmann 1984). Necessary and sufficient conditions for the existence of an observer with unknown inputs have been presented in Meditch (1974) and Kurek (1983). At the same time, singular systems, which can be considered as incomplete equation systems, have been studied (Dai 1989) with many applications in control and estimation. The connection between singular systems and systems with unknown inputs will be examined later.

Observers are used in two major applications : state reconstruction for state control and diagnosis of process. For the last purpose, the detection and isolation of sensors have received a lot of attention in the literature (Patton 1989), (Frank 1990). One of the most common approaches is based on the use of redundancies between the different variables characterizing the process under consideration. The generation of these redundancies has also received much attention, especially for linear systems (Staroswiecki 1989), (Ragot 1990 and 1992), (Nowakowski 1991) ; for systems with unknown inputs, some papers have been published (Chow 1984), (Gertler 1991), (Patton 1991) involving a robust residual generation based on analytical redundancy in terms of parity space. Recently, design methods for isolating additive failures from changes in the state transition matrix of a system have been examined (Ribbens 1991).

The aim of the paper is to derive some redundancy equations for fault detection and isolation. In order to eliminate some "unknown inputs" in the redundancy equations we use an annihilator matrix to left multiply the system equations that results in a singular system representation from which redundancy equations can be extracted. This is based on the approach proposed in (Frank 1989) with here an original computation procedure. By assigning different inputs or outputs as "unknown inputs" the fault isolation is achieved. The problem of the observer design and residual generation is the core of an FDI process. A systematic investigation of applying unknown input observer to the FDI process was carried out recently by Wünnenberg. Because the proposed analysis and design of the FDI observers rely heavily on the Kronecker canonical form transformation of a given pencil matrix, the FDI problem presentation and design procedure is made to appear more complicated than necessary (Hou 1991).

In the first section of this presentation, we shall give the principle of the unknown inputs elimination. We will proceed to a dimension reduction of the state equations, then we will explain the generation of the redundancy equations after eliminating the unknown inputs by using singular system representation. Finally, an example is given to show the different residuals which may be deduced from the redundancy equations.

PRINCIPLE OF UNKNOWN INPUTS ELIMINATION

Although the processes are usually continuous, the diagnosis calculations are generally performed on discrete data. Therefore, in this presentation we will only consider discretized models ; however it should be noted that the proposed algorithm also applies to a continuous model. The considered discrete systems are characterized by an n-dimensional state vector x, a r-dimensional input vector u, a m-dimensional output vector y and a s-dimensional input vector v of unknown components. We assume that the number of known output equations m is greater than the number of unknown inputs s, in order to be sure, a priori, that the system has at least one redundancy equation. The matrices describing the state evolution have appropriate dimensions. Systems are represented by the model :

$$x(k+1) = A\,x(k) + B\,u(k) + F\,v(k) \qquad (1a)$$

$$y(k) = C\,x(k) \qquad (1b)$$

In this form, $u(k)$ is a known input vector and the unknown input $v(k)$ directly appears in the dynamic part of the state equations. The B and F matrices are assumed to be of full column rank while the C matrix must be of full row rank. As it is well known, the representation (1a) may describe a wide variety of fault actuators. If each element of $v(k)$ corresponds to an actuator and if all performances of the actuators are wished to be observed, then F must be chosen equal to B ; if the j^{th} actuator fails, the effect of this failure can be improved by taking the j^{th} entry of $v(k)$ as the negative of the j^{th} entry of $u(k)$. If the j^{th} actuator has a bias of magnitude b, then the j^{th} entry of $v(k)$ is equal to b.

This model may include redundancy equations which essentially take two forms : direct redundancy when there are relations between outputs of sensors or temporal redundancy when the time relation between sensor outputs and actuator inputs are considered. Based on this redundancy, residuals are generated and evaluated in order to detect and locate actuator and sensor faults.

It should be noted that the system (1) may also be written in its input-output (ARX) form depending on the two inputs $u(z)$ and $v(z)$ respectively weighted by the transfer functions $G_u(z)$ and $G_v(z)$. The redundancy equations are obtained first by finding out a stable transfer function matrix $Q(z)$ left orthogonal to $G_v(z)$ and second by multiplying the ARX equation by $Q(z)$. This can be done in the stable factorization framework which is especially suitable for implementation on a computer (Viswanadham 1988) (Ding 1990).

In this paper, we propose using the state-space representation (1) of the system. In order to find which part of the output process y is insensitive to the unknown inputs v, we will try to eliminate these inputs by projection. We define the matrix :

$$R = \begin{pmatrix} E \\ N \end{pmatrix} \tag{2}$$

where E represents a left annihilator of F and N a left inverse of F. Expressed in equations these definitions mean :

$$R\,F = \begin{pmatrix} 0 \\ I_s \end{pmatrix} \tag{3}$$

where I_s is the s-dimension identity matrix. Multiplying equation (1a) by R gives :

$$E\,x(k+1) = E\,A\,x(k) + E\,B\,u(k) \tag{4a}$$
$$N\,x(k+1) = N\,A\,x(k) + N\,B\,u(k) + I_s\,v(k) \tag{4b}$$

Equation (4a) depends on the known input ; if its corresponding state is observable, then equation (4b) should be used to have an estimation of the unknown input v(k). Equations (4a) and (1b) describe, under a classical representation, a singular system. Then, a regular system with unknown inputs may be considered as a singular system with known inputs.

It is also interesting to note that the inverse transformation is possible. For that purpose the singular value decomposition of E is used :

$$E = U\,(\Sigma \quad 0)\,V^T \tag{5}$$

where U and V are two orthogonal matrices and Σ contains the eigenvalues of E. Using the change of variable $\bar{x} = V^T x$, the state equations are rewritten :

$$(\Sigma \quad 0)\,\bar{x}(k+1) = U^T E\,A\,V\,\bar{x}(k) + U^T E\,B\,u(k) \tag{6a}$$

$$y(k) = C\,V\,\bar{x}(k) \tag{6b}$$

If we subdivide $\bar{x}(k)$ into $\bar{x}_1(k)$ and $\bar{x}_2(k)$ according to the partition of the matrix $(\Sigma \quad 0)$, we obtain, with obvious definitions for $\bar{A}_1, \bar{A}_2$ and $\bar{B}$:

$$\bar{x}_1(k+1) = \bar{A}_1\,\bar{x}_1(k) + \bar{A}_2\,\bar{x}_2(k) + \bar{B}\,u(k) \tag{7a}$$

$$y(k) = \bar{C}_1\,\bar{x}_1(k) + \bar{C}_2\,\bar{x}_2(k) \tag{7b}$$

which looks like the structure of a system with partially known inputs. Then, a singular system may be considered as a regular system with unknown inputs.

For the redundancy extraction purpose we only consider equations (4a) and (1b) with simplified notation :

$$E\,x(k+1) = A\,x(k) + B\,u(k) \tag{8a}$$

$$y(k) = C\,x(k) \tag{8b}$$

with E of dimensions (n-s)xn, A of dimensions (n-s)xn, B of dimensions (n-s)xr and C of dimensions mxn. We assume that the number of state variables n is greater than the number of unknown inputs s in order to be sure that their elimination is possible. It is clear that the elimination of the state x(k) between the equations (8a) and (8b) gives the redundancy equations directly. Unfortunately, this is achieved through the calculation of the inverse of the singular pencil of matrices (q E - A) where q is the backward shift operator. We will see later how to overcome this difficulty.

DIMENSION REDUCTION OF THE STATE EQUATION

The objective of the present section is the dimension reduction of the state equations. Therefore, we try to isolate the measured part of the state vector by defining the partition $C = (C_1 \quad C_2)$ in which C_1 is the regular part of C and the (nxn) matrix P :

$$P = \begin{pmatrix} C_1^{-1} & -C_1^{-1}C_2 \\ 0 & I_{n-m} \end{pmatrix}$$

It can easily be verified that $CP = (I_m \quad 0)$. With the change of variable $x = P \begin{pmatrix} y \\ w \end{pmatrix}$, where the vector w is the unmeasured part of the state vector, the equation (8) are then expressed in the following form :

$$(E_1 \quad E_2) \begin{pmatrix} y(k+1) \\ w(k+1) \end{pmatrix} = A\,P \begin{pmatrix} y(k) \\ w(k) \end{pmatrix} + B\,u(k) \tag{9}$$

The matrix EP has been subdivided according to the partition of C, where E_1 and E_2 are respectively $(n\text{-}s)\times m$ and $(n\text{-}s)\times(n\text{-}m)$ matrices.

Two propositions are then established in order to reduce the dimension of the state equations.

Proposition 1

If the system (5) is observable, then rank $E_2 = n - m$.

Proof : assuming the observability of the system, rank $\begin{pmatrix} E \\ C \end{pmatrix} = n$ (see Dai 1989). As P is regular, we also have rank $\begin{pmatrix} EP \\ CP \end{pmatrix} = n$. Using the partitioning of EP and CP, it follows :

$$\text{rank} \begin{pmatrix} E_1 & E_2 \\ I_m & 0 \end{pmatrix} = n$$

As $\text{rank}(I_m) = m$, we can deduce the proposition. Moreover, E_2 is a full column rank matrix.

Proposition 2

A $(n\text{-}s)\times(n\text{-}s)$ regular Q matrix exists such that :

$$Q\,E_2 = Q \begin{pmatrix} E_{12} \\ E_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ I_{n-m} \end{pmatrix} \tag{10}$$

Proof : this proposition may be directly verified by using the matrix :

$$Q = \begin{pmatrix} I_{m-s} & -E_{12}E_{22}^{-1} \\ 0 & E_{22}^{-1} \end{pmatrix} \tag{11}$$

Let us now return to the simplification of the state equations. Multiplying left equation (9a) by the regular matrix Q leads to :

$$\begin{pmatrix} E_{11} & 0 \\ E_{21} & I \end{pmatrix} \begin{pmatrix} y(k+1) \\ w(k+1) \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} y(k) \\ w(k) \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} u(k) \tag{12}$$

where E_{ij}, A_{ij} and B_i are submatrices obtained by partitioning QE_1, QAP and QB. System (12) can then be rewritten under the form :

$$w(k+1) = A_{22}\, w(k) + \begin{pmatrix} A_{21} & -E_{21} & B_2 \end{pmatrix} \begin{pmatrix} y(k) \\ y(k+1) \\ u(k) \end{pmatrix} \tag{13a}$$

$$E_{11}\, y(k+1) - A_{11}\, y(k) - B_1\, u(k) = A_{12}\, w(k) \tag{13b}$$

Equation (13a) describes the dynamic evolution of the substate w(k) driven by the generalized input $u_r^T(k) = (y^T(k)\ y^T(k+1)\ u^T(k))$. We can also define a generalized output vector $y_r(k) = E_{11}\ y(k+1) - A_{11}\ y(k) - B_1\ u(k)$. Equation (13b), therefore represents a "measurement equation" depending on the state w(k) ; the presence of the variable y(k+1) at the time (k+1) is not constraining because it is only desired to generate redundancy equations. Thus system (13) can be considered as a standard dynamic system with the former definition of state, generalized input and generalized output. In order to simplify further notations, this system will be re-written with classical notation :

$$x_r(k+1) = A_r\, x_r(k) + B_r\, u_r(k) \tag{14a}$$

$$y_r(k) = C_r\, x_r(k) \tag{14b}$$

Note that this form applies even if $A_{12} = 0$. This case corresponds to the particular redundancy obtained from equation (13b) :

$$E_{11}\, y(k+1) - A_{11}\, y(k) - B_1\, u(k) = 0 \tag{15}$$

From equations (14), it is then possible to generate the redundancy equations between the inputs and the outputs. In order to avoid the matrix inversion, we suggest an iterative method which is based on a progressive elimination of the unknown state $x_r(k)$.

GENERATION OF REDUNDANCY EQUATIONS

Let us return to the general state equations (14) for which it is desired to extract the redundancy equations between the inputs and the outputs. There are several techniques which may be applied for that generation using, for example, the expansion in a power series of A_r (Faddev 1963) or the coprime factorization (Kailath 1980). We present here an algorithm based on a dimension reduction of the state equations. With C_{r1} as the greatest regular part of the matrix C_r, a simple permutation of the elements of x_r enables the breakdown of the state equation (14) into :

$$\begin{pmatrix} x_{r1}(k+1) \\ x_{r2}(k+1) \end{pmatrix} = \begin{pmatrix} A_{r11} & A_{r12} \\ A_{r21} & A_{r22} \end{pmatrix} \begin{pmatrix} x_{r1}(k) \\ x_{r2}(k) \end{pmatrix} + \begin{pmatrix} B_{r1} \\ B_{r2} \end{pmatrix} u_r(k) \tag{16a}$$

$$y_r(k) = C_{r1}\, x_{r1}(k) + C_{r2}\, x_{r2}(k) \tag{16b}$$

With the change of variables :

$$\bar{x}_1(k) = C_{r1}\, x_{r1}(k) + C_{r2}\, x_{r2}(k) \tag{17a}$$

$$\bar{x}_2(k) = x_{r2}(k) \tag{17b}$$

the state equations (16) are rewritten :

$$\begin{pmatrix} \bar{x}_1(k+1) \\ \bar{x}_2(k+1) \end{pmatrix} = \begin{pmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{pmatrix} \begin{pmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \end{pmatrix} + \begin{pmatrix} \bar{B}_1 \\ \bar{B}_2 \end{pmatrix} u_r(k) \tag{18a}$$

$$y(k) = \bar{x}_1(k) \tag{18b}$$

with the definitions :

$$\bar{A}_{11} = (C_{r1}\, A_{r11} + C_{r2}\, A_{r21})\, C_{r1}^{-1} \tag{19a}$$

$$\bar{A}_{12} = -\,\bar{A}_{11}\; C_{r2} + C_{r1}\, A_{12} + C_{r2}\, A_{r22} \tag{19b}$$

$$\bar{A}_{21} = A_{r21}\; C_{r1}^{-1} \tag{19c}$$

$$\bar{A}_{22} = A_{r22} - A_{r21}\, C_{r1}^{-1}\, C_{r2} \tag{19d}$$

$$\bar{B}_1 = C_{r1}\, B_{r1} + C_{r2}\, B_{r2} \tag{19e}$$

$$\bar{B}_2 = B_{r2} \tag{19f}$$

A more interesting presentation of the equations could be now obtained by eliminating the variable $\bar{x}_1(k)$ in the state equations which are re-written :

$$\bar{x}_2(k+1) = \bar{A}_{22}\; \bar{x}_2(k) \; + \; (\bar{A}_{21} \quad \bar{B}_2)\begin{pmatrix} y(k) \\ u_r(k) \end{pmatrix} \tag{20a}$$

$$z(k) = \bar{A}_{12}\ \bar{x}_2(k) \tag{20b}$$

with $z(k) = y(k+1) - \bar{A}_{11}\, y(k) - \bar{B}_1\, u_r(k)$ (20c)

This form highlights the generalized input ($y(k)$, $u_r(k)$) and the generalized measurement $z(k)$ which drive the evolution of the state variable $\bar{x}_2(k)$. Then, equations (20) are structurally the same as equations (14) ; therefore, the transformation used in equations (18) may be applied to equation (20). By this mean, we can eliminate unobservable variables. The procedure is repeated until the matrix A is reduced to a scalar or the matrix C becomes null.

EXAMPLE

The following example is to demonstrate the design of residuals by mean of the proposed method. The different codings of the residuals will also be discussed. We will consider the model system described by equations (1) (n=5, r=2, s=1 and m=4) in which the matrices are defined by :

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 2 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \quad F = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

The regular part of C may be constructed by selecting the columns 2, 3, 5 and 4 of the C matrix. This choice necessitates a rewriting of the state equation according to a permutation of the number of the state variables. Following the definition of E in equation (5) we can deduce the singular equation (6a) :

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{pmatrix} x(k+1) = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} x(k) + \begin{pmatrix} 0 & 0 \\ 2 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} u(k)$$

Using the proposition 2 and the equation (16), the reader should verify the following numerical result :

$$\begin{pmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix} \bar{x}(k+1) = \begin{pmatrix} 2 & 0 & -2 & 0 & -2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{pmatrix} \bar{x}(k) + \begin{pmatrix} 0 & -2 \\ 2 & 1 \\ 1 & 0 \\ 0 & -1 \end{pmatrix} u(k)$$

Thus, for that system, the state equations may be written :

$$\begin{aligned} \bar{x}_{11}(k+1) - 2\,\bar{x}_{13}(k+1) &= 2\,\bar{x}_{11}(k) - 2\,\bar{x}_{13}(k) - 2\,\bar{x}_2(k) - 2\,u_2(k) \\ \bar{x}_{12}(k+1) &= \bar{x}_{12}(k) + 2\,u_1(k) + u_2(k) \\ \bar{x}_{14}(k+1) &= \bar{x}_{13}(k) + u_1(k) \\ \bar{x}_{13}(k+1) - \bar{x}_2(k+1) &= \bar{x}_{13}(k) + u_2(k) \end{aligned}$$

where $\bar{x}_{11}, \bar{x}_{12}, \bar{x}_{13}$ and $\bar{x}_{14}$ are the elements of $\bar{x}_1$ which is completely measured (equation 10b). Here, the generation of the redundancy equation is straightforward because the unknown state variable $\bar{x}_2$ may be directly eliminated. We therefore obtain three redundancy equations, from which we can deduce the residuals under a matrix form (q is the time forward shift operator) :

$$\begin{pmatrix} r_1(k+1) \\ r_2(k+1) \\ r_3(k+1) \end{pmatrix} = \begin{pmatrix} q(q-2) & 0 & -2(q-1)^2 & 0 & 0 & 2(q-1) \\ 0 & q-1 & 0 & 0 & -2 & -1 \\ 0 & 0 & -1 & q & -1 & 0 \end{pmatrix} \begin{pmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \\ y_4(k) \\ u_1(k) \\ u_2(k) \end{pmatrix}$$

and more succinctly :

$$r(k+1) = (\, M_y \;\; M_u \,) \begin{pmatrix} y(k) \\ u(k) \end{pmatrix}$$

where r, y and u state for the vectors of residuals, output and input and M_y and M_u are convenient matrices.

A significant non-zero value of these residuals indicates the presence of failures in the inputs and/or outputs measurements. Note that the residuals are a mix of input and output variables ; therefore the isolation of failures is difficult. One way of enhancing the residuals involves generating structured residuals which are sensitive to a particular fault (Gertler 1991). The main advantage of using structured residuals is the resulting simplification of the diagnostic analysis. A satisfactory requirement for fault isolation is that the new residuals will have separated fault signatures. For example, assuming that the actuators are functioning correctly, in order that the residuals are unaffected by ns output faults, all occurrences of the ns concerned variables have to be eliminated ; this can be done by using the classical pivotal technique in the incidence matrix M_y. For the previous example, with 3 residual equations and 4 outputs, the maximum number of outputs that may be eliminated is 2. That means that a residual may be sensitive to 2 outputs and consequently the perfect isolation of sensor faults may not be achieved. If we are concerned with the actuator faults (assuming that the sensors are functioning correctly) then it is possible to have a complete isolation of the faults. With 3 residuals and 2 outputs it is possible to obtain, by linear combination, residuals with only one input variable. For example, considering the submatrix M_u, it is possible to define a matrix W :

$$W = \begin{pmatrix} 1 & 2(q-1) & -4(q-1) \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$$

such that :

$$W\ M_u = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 2(q-1) \end{pmatrix}$$

In these conditions, the new residuals e(k) = W r(k) are structured to be sensitive to specific faults. The second term $e_2(k)$ is affected by the failure of the first actuator, the third term $e_3(k)$ is affected by the failure of the second actuator while the first term $e_1(k)$ is not affected by actuator faults which could provide a test to prove that the sensors are functioning properly.

It is also possible to generate structured residuals to isolate faults on sensors and actuators. For example, let us suppose that faults on outputs 2 and 4 and on input 1 are wished to be isolated. If we denote M_d the regular matrix formed by columns 1, 2 and 5 of $(M_y\ \ M_u)$, the matrix W must be calculated such that the columns of the matrix W M_d be independent and contain only one non-zero element. This can be achieved by selecting :

$$W = \begin{pmatrix} \frac{1}{q(q-2)} & 0 & 0 \\ 0 & \frac{1}{q-1} & -\frac{2}{q-1} \\ 0 & 0 & -1 \end{pmatrix}$$

Hence the three new residuals e(k) are sensitive to only one of the specified faults. Notice that this perfect isolation of faults is not always possible because it depends on the rank of the matrix M_d.

Let us now consider the case of actuator faults detection using a bank of redundancy equations which are constructed to be directly sensitive to these faults (figure 1). To do this, we consider a process described by the state equation (21) :

$$x(k+1) = A\, x(k) + B\, u(k) \tag{21a}$$
$$y(k) = C\, x(k) \tag{21b}$$

We can rewrite the dynamical part :

$$x(k+1) = A\, x(k) + B_i\, u_i(k) + \bar{B}_i\, \bar{u}_i(k) \qquad i=1, ..., r \tag{22}$$

where B_i is the ith column of B and $\bar{B}_i$ is the nx(r-1) matrix obtained from B by deleting B_i. Let $u_i(k)$ be the i^{th} entry of u(k) and $\bar{u}_i(k)$ the (r-1) column vector obtained from u(k) by deleting $u_i(k)$. Now, we construct a set R_i of redundancy equations using the proposed elimination strategy by treating $\bar{u}_i(k)$ as the unknown inputs. By the nature of the construction, these redundancy equations are not sensitive to $\bar{u}_i(k)$ whereas variations and failures in $u_i(k)$ will affect the outputs y(k). We repeat the above procedure to construct r sets of unknown-input redundancy equations. By monitoring these sets of residuals, actuator failures can be isolated. Indeed, if all sensors are good and all the redundancy equations are verified apart from the i^{th} then the i^{th} actuator is faulty.

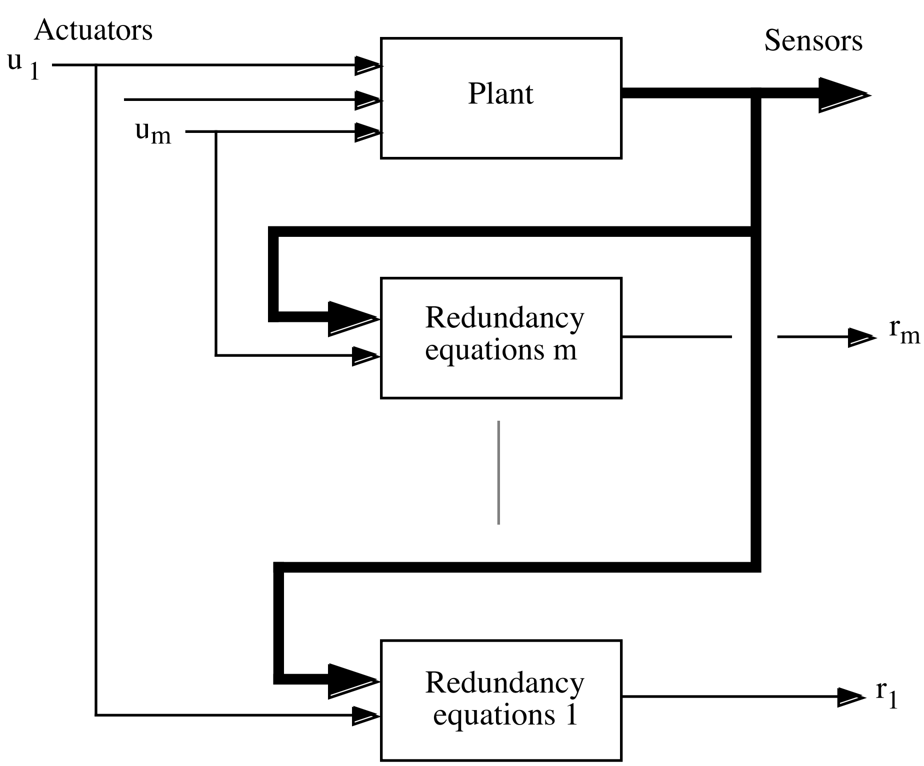

Figure 1 : actuator faults detection using unknown-input redundancy equations

Let us consider the previous example without explicit unknown input (i.e. F is a null vector). After eliminating the first input, we obtain the singular system :

$$\begin{pmatrix} 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \bar{x}(k+1) = \begin{pmatrix} 2 & 0 & -2 & 0 & -2 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{pmatrix} \bar{x}(k) + \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} u_2(k)$$

$$y(k) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \bar{x}(k)$$

The state equations are then reduced to :

$$y_1(k+1) - 2\, y_4(k+1) = 2\, y_1(k) - 2\, y_3(k) - 2\, \bar{x}_2(k)$$
$$y_2(k+1) - 2\, y_4(k+1) = y_2(k) - 2\, y_3(k) + u_2(k)$$
$$y_3(k+1) - y_4(k+1) = u_2(k)$$
$$y_4(k+1) - \bar{x}_2(k+1) = y_3(k)$$

The redundancy equations may be obtained by eliminating the state $\bar{x}_2(k)$ between the first and the last equations. It leads to the following set of residuals :

$$\begin{pmatrix} r_{11}(k+1) \\ r_{12}(k+1) \\ r_{13}(k+1) \end{pmatrix} = \begin{pmatrix} q(q-2) & 0 & 2(q-1) & 2q(1-q) & 0 \\ 0 & q-1 & 2 & -2q & -1 \\ 0 & 0 & q & -q & -1 \end{pmatrix} \begin{pmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \\ y_4(k) \\ u_2(k) \end{pmatrix}$$

The elimination of the second input leads to the following singular system :

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \bar{x}(k+1) = \begin{pmatrix} 2 & 0 & 0 & 0 & -2 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \bar{x}(k) + \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix} u_1(k)$$

$$y(k) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \bar{x}(k)$$

Then, this system is also described by the state equations :

$$y_1(k+1) = 2\, y_1(k) - 2\, \bar{x}_2(k) + 2\, u_1(k)$$
$$y_2(k+1) - y_3(k+1) = y_2(k) - y_3(k) + u_1(k)$$
$$y_4(k+1) = y_3(k) + u_1(k)$$
$$\bar{x}_2(k+1) = u_1(k)$$

As previously, the elimination of the state $\bar{x}_2(k)$ between the first and the last equations leads to the new following set of residuals :

$$\begin{pmatrix} r_{21}(k+1) \\ r_{22}(k+1) \\ r_{23}(k+1) \end{pmatrix} = \begin{pmatrix} q(q-2) & 0 & 0 & 0 & 2(1-q) \\ 0 & 1-q & q-1 & 0 & 1 \\ 0 & 0 & -1 & q & -1 \end{pmatrix} \begin{pmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \\ y_4(k) \\ u_1(k) \end{pmatrix}$$

Clearly, these two last sets of residuals are respectively de-coupled from the two inputs $u_1(k)$ and $u_2(k)$. Therefore, they can be used for detecting and identifying faulty actuators.

CONCLUSION

An attractive presentation of the well-known problem of residuals generation has been presented in the case of systems with unknown inputs. Using a transformation of the process model into a singular system, the redundancy equations are then formed involving simple calculus. The proposed method can be used for determining robust parity equations with regard to actuators failures.

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