Process input estimation with a multiple model. Application to communication

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Goals

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- To design a nonlinear observer with unknown inputs
- To be able to estimate the unknown inputs

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Difficulties

- Design of the multiple model (out of purpose)
- Satisfaction of structural constraints
- Resolution of bilinear matrix inequalities

Brief recall of the linear case (1)

Linear dynamical model

$$\begin{cases} x(t+1) = Ax(t) + Bu(t) + R\bar{u}(t) \\ y(t) = Cx(t) \end{cases}$$

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State estimation error

$$e(t) = x(t) - \hat{x}(t)$$

$$e(t) = (I + EC)x(t) - z(t)$$

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State estimation error dynamics

$$e(t+1) = Ne(t) + (PA - NP - LC)x(t) + (PB - G)u(t) + (PR - KF)\bar{u}(t)) + EF\bar{u}(t+1)$$

Brief recall of the linear case (2)

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Autonomous conditions

$$PA - NP - LC = 0$$

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Asymptotic convergence

N stable

Structure description

$$\begin{cases} x(t+1) = \displaystyle\sum_{i=1}^{M} \mu_i(\xi(t)) \left(A_i x(t) + B_i u(t) + R_i \bar{u}(t)\right) \\ y(t) = C x(t) + F \bar{u}(t) \end{cases}$$

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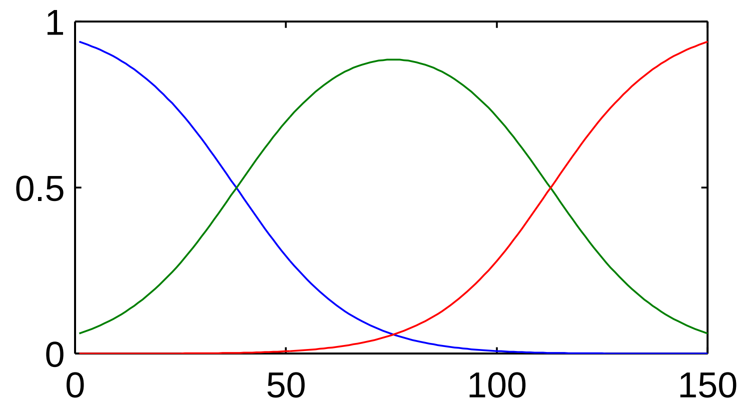
with $$\begin{cases} \xi(t) = \{u(t), x(t), y(t)\} \\ \sum_{i=1}^{M} \mu_i(\xi(t)) = 1, \quad 0 \leq \mu_i(\xi(t)) \leq 1 \end{cases}$$



Design of a multiple observer (1)

Multiple model

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$$\begin{cases} z(t+1) = \sum_{i=1}^{M} \mu_i(\xi(t)) \big(N_i z(t) + G_i u(t) + L_i y(t)\big) \\ \hat{x}(t) = z(t) - E y(t) \end{cases}$$

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Asymptotic convergence of the estimation error

$$e(t+1) = \sum_{i=1}^{M} \mu_i(\xi(t)) N_i e(t)$$

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$$e(t+1) = \sum_{i=1}^{M} \mu_i(\xi(t)) N_i e(t)$$

if

$$\begin{cases} P &= I + EC \\ N_i P &= P A_i - L_i C \\ K_i &= N_i E + L_i \\ P R_i &= K_i F \\ G_i &= P B_i \\ E F &= 0 \end{cases}$$

and

$$N = \sum_{i=1}^{M} \mu_i(\xi(t)) N_i \quad \text{stable}$$

Design of a multiple observer (3)

Asymptotic convergence of the estimation error

(A_i, C) observable pairs, F with full column rank and $\forall i, \in \{1, ..., M\}$:

$$\begin{aligned}
N_i^T X N_i - X &< 0 \\
N_i &= PA_i - K_i C \\
P &= I + EC \\
PR_i &= K_i F \\
EF &= 0 \\
L_i &= K_i - N_i E \\
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\end{aligned}$$

where $X \in \mathbb{R}^{n \times n}$ is symmetric and positive definite.

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Solution of the system of equation (1)

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Solution

$$E = I - F F^-$$

F^- generalized inverse of F

Solution of the system of equation (2)

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Solution

$$N_i^T X N_i - X = (PA_i - K_i C)^T X (PA_i - K_i C) < 0$$

$$PR_i = K_i F$$

Bilinear matrix inequalities w.r.t. to K_i and X s.t. equality constraints.

Solution of the system of equation (3)

Matrix inequalities

$$\begin{cases} (PA_i - K_i)^T X (PA_i - K_i) < 0 \\ PR_i = K_i F \end{cases}$$

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Change of variables

$$W_i = XK_i$$

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Schur complement

$$\begin{cases} \begin{pmatrix} X & A_i^T PX - C^T W_i \\ XPA_i - W_i C & X \end{pmatrix} > 0 \\ XPR_i = W_i F \end{cases}$$

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linear matrix inequalities w.r.t. X and W_i

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\end{aligned}
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Sequence of calculus

$E \longrightarrow P \longrightarrow G_i$ and $(X, W_i) \longrightarrow K_i = X^{-1} W_i \longrightarrow N_i \longrightarrow L_i$

Unknown input estimation (1)

Model of the system

$$\begin{cases} x(t+1) = \sum_{i=1}^{M} \mu_i(\xi(t)) \left(A_i x(t) + B_i u(t) + R_i \bar{u}(t)\right) \\ y(t) = Cx(t) + F\bar{u}(t) \end{cases}$$

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Model applied on signal estimations

$$\begin{cases} \hat{x}(t+1) = \sum_{i=1}^{M} \mu_i(\xi(t)) \left(A_i \hat{x}(t) + B_i u(t) + R_i \hat{\bar{u}}(t)\right) \\ \hat{y}(t) = C \hat{x}(t) + F \hat{\bar{u}}(t) \end{cases}$$

where $\hat{\bar{u}}(t)$ is an estimation of the unknown input $\bar{u}(t)$

Unknown input estimation (2)

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Unknown input estimation

$$\hat{\hat{u}}(t) = (W^T W)^{-1} W^T \begin{pmatrix} \hat{x}(t+1) - \sum_{i=1}^{M} \mu_i(\xi(t)) \left(A_i \hat{x}(t) + B_i u(t)\right) \\ y(t) - C\hat{x}(t) \end{pmatrix}$$

with $W = \begin{pmatrix} \sum_{i=1}^{M} \mu_i \left(\xi(t)\right) R_i \\ F \end{pmatrix}$

Multiple model

$$\begin{cases} x(t+1) = \sum_{i=1}^{2} \mu_i(\xi(t))\big(A_i x(t) + R_i \bar{u}(t)\big) \\ y(t) = Cx(t) + F\bar{u}(t) \end{cases}$$

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with

$$\begin{cases} \xi(t) = y(t) \\ \mu_1(\xi(t)) = \frac{1}{2}(1 - \tanh(\xi(t))) \\ \mu_2(\xi(t)) = 1 - \mu_1(\xi(t)) \end{cases}$$

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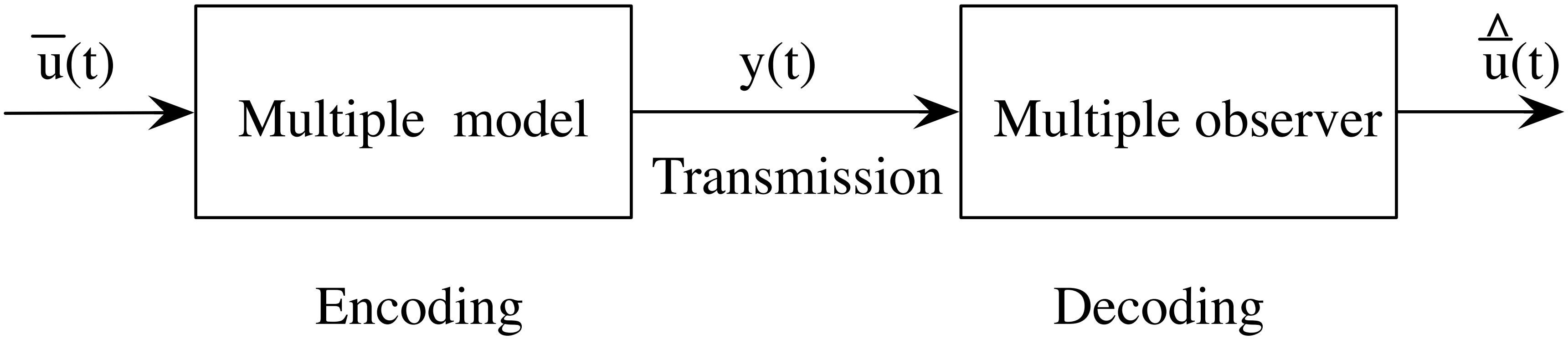
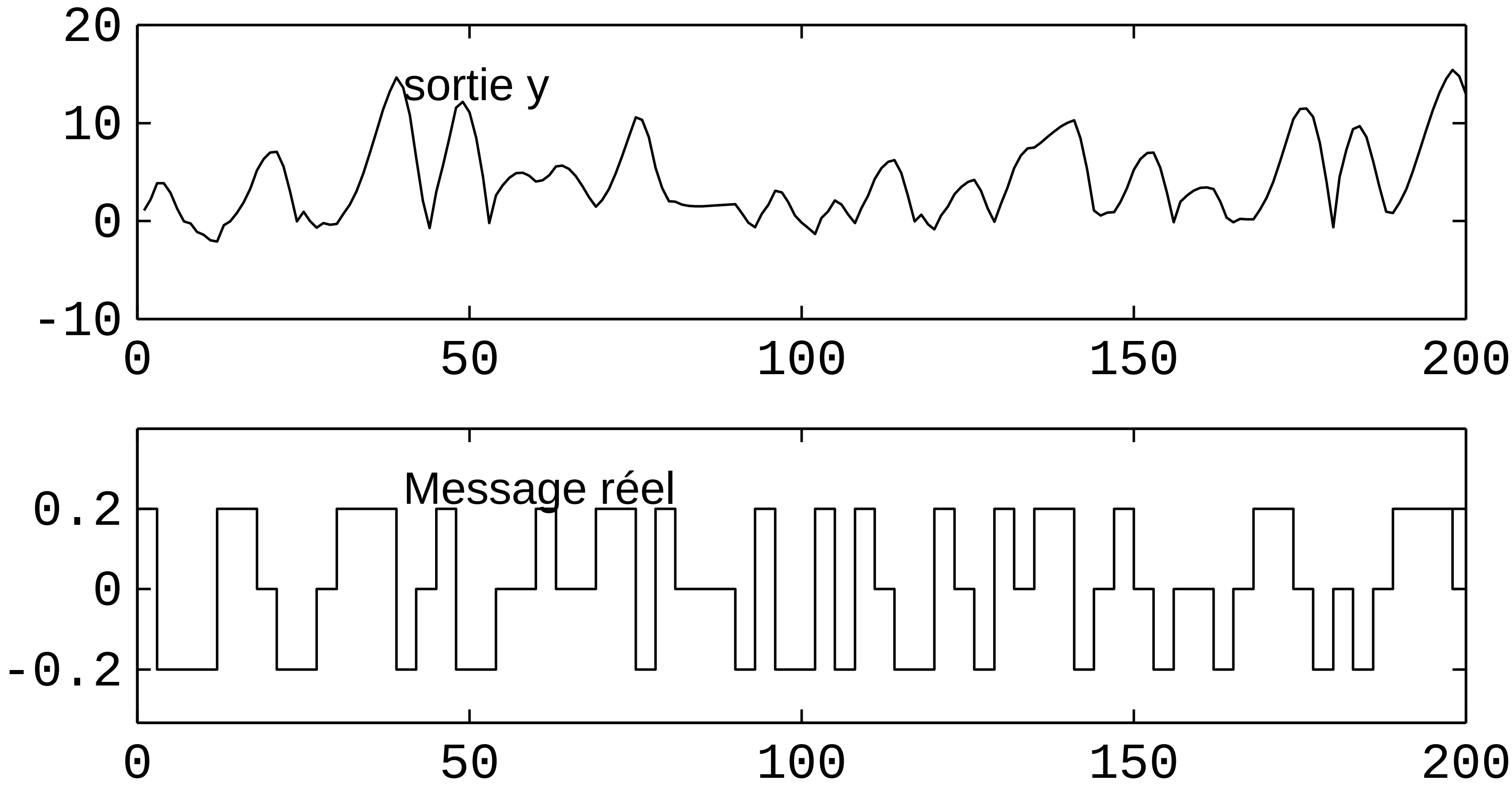
$$A_1 = \begin{bmatrix} -1.1 & 0.5 \\ 0.3 & 0.7 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.8 & -0.1 \\ 1 & 1.1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}, \quad F = 5$$

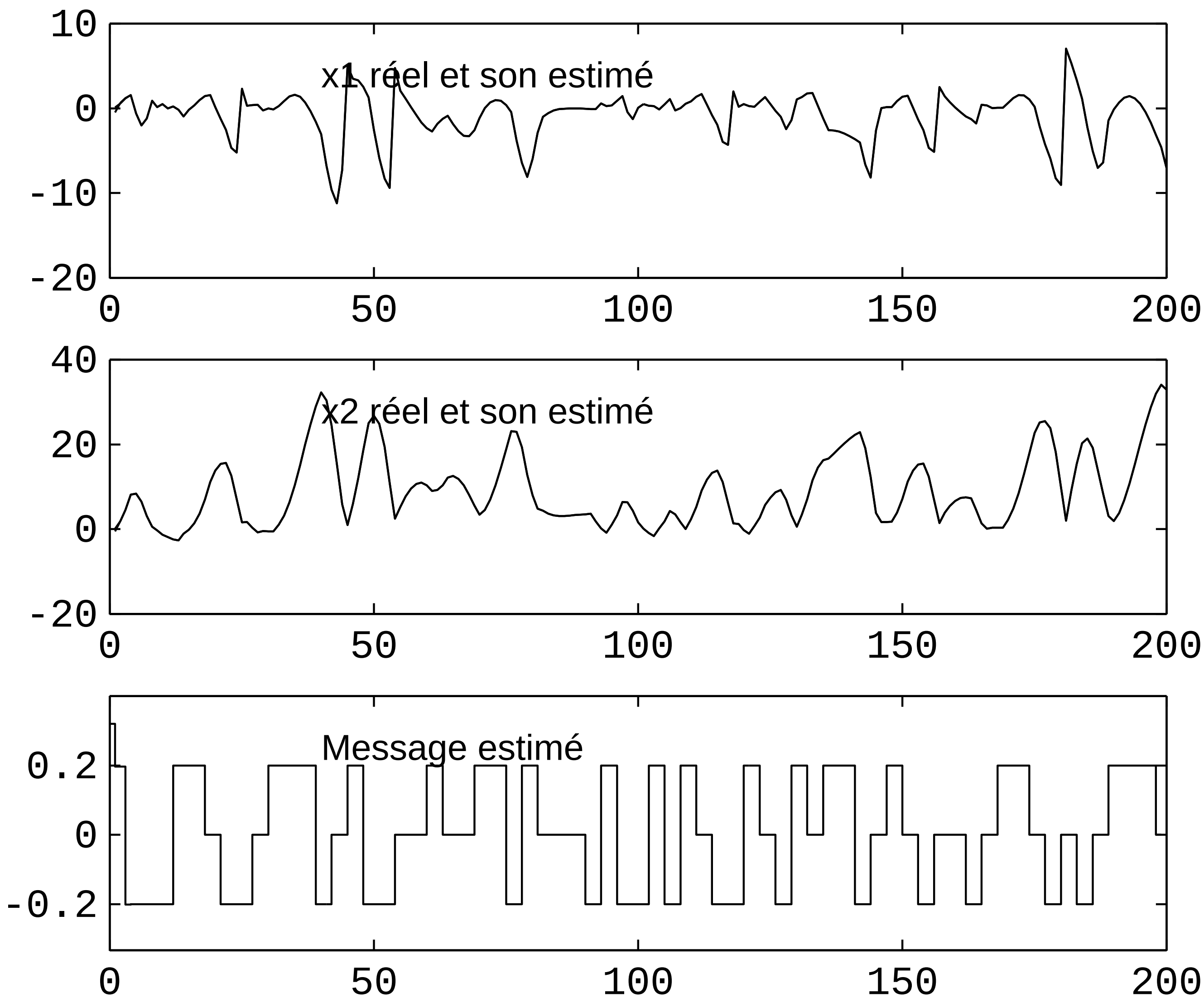
Encoding – decoding



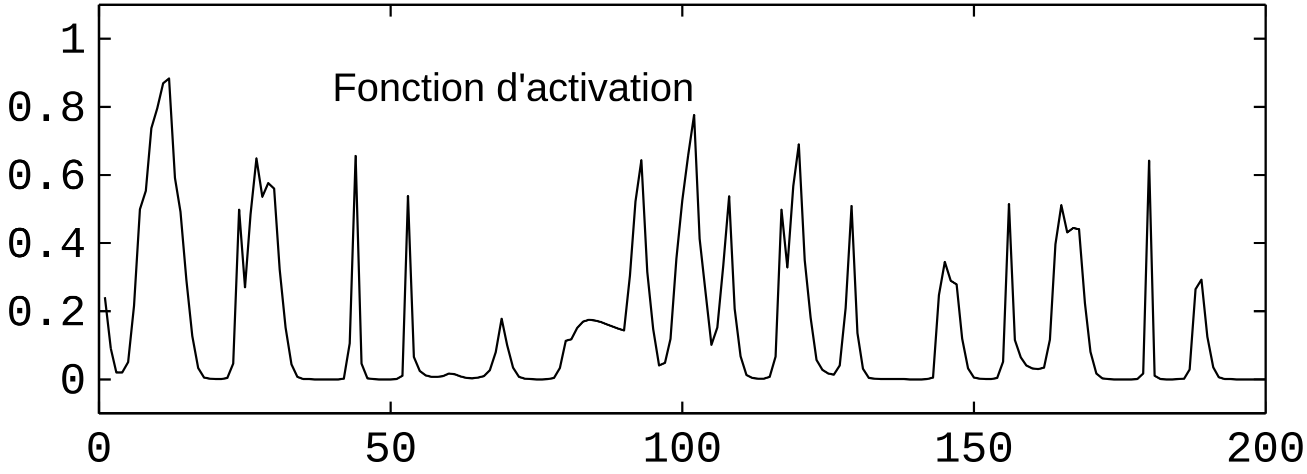
Transmitted and original signals



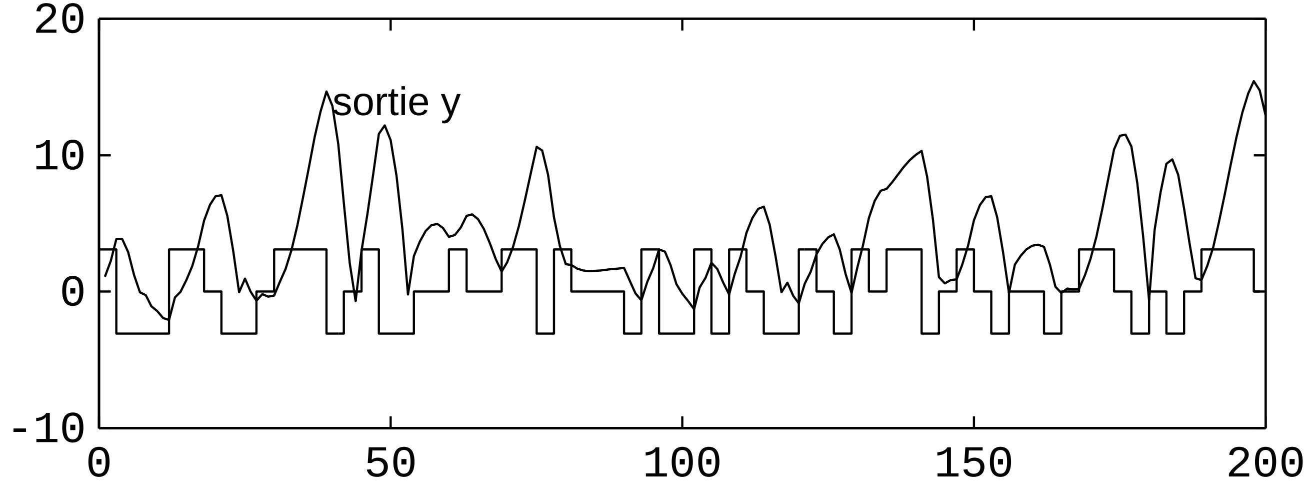
Actual and estimated states and decoded message



Activation functions



Output and message



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- Capacity to transpose the classical “results” (linear case) to nonlinear systems
- General range of the method suggested related to the capacity of representation of the multiple model
- Promising application in the context of secure communications