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State and unknown inputs estimation via a proportional integral observer with unknown inputs

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Résumé. This paper deals with the problem of fault detection and identification in noisy systems. A proportionnal integral observer with unknown inputs is used in order to estimate the state and the faults which are assumed as unknown inputs. The noise's effect on the system is also minimized. The obtained results can be extended to non-linear systems described by multiple model representations.

1 Introduction

The model's state estimation is an important field of research with numerous applications in control and diagnosis. Generally the whole system's state is not always measurable and the recourse to its estimation is a necessity. State estimation permits also to replace expensive sensors or those with a difficult maintenance.
An observer is generally a dynamical system wich permit the system's state reconstruction from the system's model and the measurments of its inputs and ouputs [Lu].
The state estimation methods of linear systems are efficient [Ed]. There exist a class of systems which are more complex and the hypothesis of linearity is not available. for these non linear systems, many studies are interested in the synthesis of non linear observers which permit the reconstruction of non measurable system's states. For example we can cite sliding mode observvres [ES] and the Thau-Luenberger observers [Th].
Approaches using Takagi-Sugeno model's (sometimes said multiple model) are the object of many works in varied contexts including the take into account of unknown inputs or parametrical uncertainties [AC,IM].
Various studies were published on the unknown inputs [SA,Ed,AC]. Some of them tried to reconstruct the system's state in spite of the unknown input existence. Those inputs can be faults or perturbations affecting the system. This reconstruction is assured via the partial ellimination of unknown inputs [SA]. Other works choose to estimate the unknown inputs and the faults in order to estimate the studied system's state[Ed,AC].

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The idea behind the multiple model is to represent a non-linear system in an interpolation's form of many local linear models. Each one of them is a linear time invariant (LTI) system valid around an operation point. The transition between models is done using the local model's activation functions [Ch,JF]. The Takagi-Sugeno structure is the most used in multiple model class representation for the system's design and analysis [MJ].
A non-linear model state estimation using this formalism is assured via a multiple observer which is an extension of linear system's observers [TS,PC,BP]. The state estimations can be used to generate fault indicators.

2 Linear system case

The objective of this part is to estimate a fault as an unknown input affecting a linear system via an unknown input proportional integral state observer.

2.1 Problem's formulation

Consider the linear model affected by an actuator fault and measurement noise. Its equation is given as following :

$$\dot{x}(t) = Ax(t) + Bu(t) + Ef(t) \tag{1}$$
$$y(t) = Cx(t) + Dw(t) \tag{2}$$

Where $x(t) \in R^n$ represents the system's state, $y(t) \in R^m$ is the measured output, $u(t) \in R^r$ is the system's input, $f(t)$ represents the fault and $w(t)$ is the measurement noise. A, B and C are a known coefficients matrices with appropriate dimensions. E and D are respectively the fault distribution and the noise matrices which are supposed to be known. The observer is choosen to be as following :

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + E\hat{f}(t) + K(y(t) - \hat{y}(t)) \tag{3}$$
$$\dot{\hat{f}}(t) = L(y(t) - \hat{y}(t)) \tag{4}$$
$$\hat{y}(t) = C\hat{x}(t) \tag{5}$$

Where $\hat{x}(t) \in R^n$ is the estimated system's state, $\hat{f}(t)$ represents the estimated fault, $\hat{y}(t) \in R^m$ is the estimated output, K is the proportional observer's gain and L is the integral gain to be computed. It is suposed that the fault affecting the system is bounded. The expressions of the state reconstruction error $\tilde{x}(t)$ and the fault reconstruction error are given by the equations (6) and (7) as following :

$$\tilde{x}(t) = x(t) - \hat{x}(t) \tag{6}$$
$$\tilde{f}(t) = f(t) - \hat{f}(t) \tag{7}$$

The dynamics of the state reconstruction error is given by the computation of $\dot{\tilde{x}}(t)$ which can be written :

$$\dot{\tilde{x}}(t) = \dot{x}(t) - \dot{\hat{x}}(t) = (A - KC)\tilde{x}(t) + E\tilde{f}(t) - KDw(t)$$

The fault error estimation is given as following :

$$\dot{\tilde{f}}(t) = \dot{f}(t) - \dot{\hat{f}}(t) = \dot{f}(t) - LC\tilde{x}(t) - LDw(t)$$

In order to simplify the notations, the time index (t) will be omitted henceforth. The following matrices are introduced :

$$\varphi = \begin{bmatrix} \tilde{x} \\ \tilde{f} \end{bmatrix}, \qquad \epsilon = \begin{bmatrix} w \\ \dot{f} \end{bmatrix} \tag{8}$$

From the equations (8) and (8), one can obtain :

$$\dot{\varphi} = A_0\varphi + B_0\epsilon \tag{9}$$

with :

$$A_0 = \begin{bmatrix} A - KC & E \\ -LC & 0 \end{bmatrix}, \qquad B_0 = \begin{bmatrix} -KD & 0 \\ -LD & I \end{bmatrix} \tag{10}$$

The matrix I is the identity matrix with appropriate dimensions. A Lyapunov function V is given as following :

$$V = \varphi^T P \varphi \tag{11}$$

Where P indicates a defined positive matrix.
The state reconstruction error $\tilde{x}(t)$ and the fault reconstruction error $\tilde{f}(t)$ tend towards zero if $\dot{V}(t) < 0$. In addition to that, the effect of the vector ϵ is minimal if the inequality (12) is solved with minimizing the positif scalar μ.

$$\dot{V}(t) + \varphi^T Q_\varphi \varphi - \mu^2 \epsilon^T Q_\epsilon \epsilon < 0 \tag{12}$$

The matrices Q_φ and Q_ϵ are two defined positive matrices. The inequality (12) can be written :

$$\psi^T \Omega \psi < 0 \tag{13}$$

with :

$$\psi = \begin{bmatrix} \varphi \\ \epsilon \end{bmatrix}, \qquad \Omega = \begin{bmatrix} A_0^T P + P A_0 + Q_\varphi & P B_0 \\ B_0^T P & -\mu^2 Q_\epsilon \end{bmatrix} \tag{14}$$

The inequality (12) is verified if $\Omega < 0$. The matrix A_0 can be written as following :

$$A_0 = \tilde{A} - \tilde{K}\tilde{C} \tag{15}$$

with :

$$\tilde{A} = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}, \tilde{K} = \begin{bmatrix} K \\ L \end{bmatrix}, \tilde{C} = \begin{bmatrix} C & 0 \end{bmatrix} \tag{16}$$

The matrix B_0 can be written on the following form :

$$B_0 = -\tilde{K}\tilde{D} + \tilde{I} \tag{17}$$

The matrices $\tilde{I}$ and $\tilde{D}$ are as following :

$$\tilde{I} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}, \qquad \tilde{D} = \begin{bmatrix} D & 0 \end{bmatrix} \tag{18}$$

Let's pose $G = P\tilde{K}$ and $m = \mu^2$. The matrix Ω can be written as :

$$\Omega = \begin{bmatrix} P\tilde{A} + \tilde{A}^T P - G\tilde{C} - \tilde{C}^T G^T + Q_\varphi & -G\tilde{D} + P\tilde{I} \\ \tilde{I}^T P - \tilde{D}^T G^T & -mQ_\epsilon \end{bmatrix} \tag{19}$$

The resolution of the inequality $\Omega < 0$ leads to find P and G. $\tilde{K}$ is determined via the resolution of $\tilde{K} = P^{-1}G$

2.2 Example

In order to validate the obtained results of the previous section, a linear system described by the matrices A, B, C and D is choosen as following :

$$A = \begin{bmatrix} -0.3 & -3 & -0.5 & 0.1 \\ -0.7 & -5 & 2 & 4 \\ 2 & -0.5 & -5 & -0.9 \\ -0.7 & -2 & 1 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 5 & 1 \\ 4 & -3 \\ 1 & 2 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.5 & 0.5 \\ 0.2 & 0.2 \\ 0 & 0.1 \end{bmatrix} \quad E = B$$

The system's input $u(t)$ is defined as following :

$$u(t) = \begin{bmatrix} u_1(t) & u_2(t) \end{bmatrix}^T$$

The signal $u_1(t)$ is a telegraph type signal varying between zero and one.
The signal $u_2(t)$ is computed by the following equation $u_2(t) = 0.3 + 0.1\sin(\pi t)$.
The fault $f(t)$ is written :

$$f(t) = \begin{bmatrix} f_1(t) & f_2(t) \end{bmatrix}^T$$

with :

$$f_1 = \begin{cases} 0, & t \leq 0.6sec \\ sin(0.5\pi t), & t > 0.6sec \end{cases} \quad , \quad f_2 = \begin{cases} 0, & t \leq 1sec \\ 0.4, & t > 1sec \end{cases} \tag{20}$$

The μ, K and L computation gives the following results : $\mu = 0.8367$,

$$K = \begin{bmatrix} 28.9634 & 4.2850 & 74.5131 \\ 0.9698 & 40.6182 & 22.2521 \\ 8.0409 & -75.1960 & -225.4935 \\ -0.5976 & 2.2673 & 38.2344 \end{bmatrix} \quad , \quad L = \begin{bmatrix} -39.6662 & 215.4581 & 75.1533 \\ 34.8900 & -22.8549 & 365.2186 \end{bmatrix}$$

The simulation results are shown in the figures (6) and (7). This method allows to estimate the unknown inputs well even in the case of non constant input.

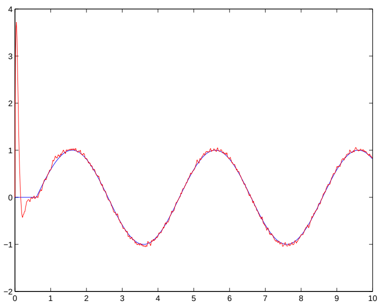

Fig. 1. fault f_1 and its estimate

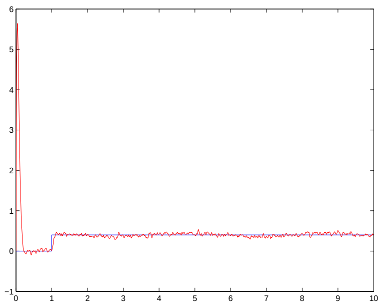

Fig. 2. fault f_2 and its estimate

3 Extension to multiple model representation

The objective of this part is to estimate a fault as an unknown input affecting a non-linear system represented by multiple model via an unknown input proportional integral state multiple observer.

3.1 Problem's formulation

The goal of this section is to extend the obtained results in paragraph 1 for the multiple model structure. Considering a non-linear system affected by an actuator fault, its equation is given as following :

$$\dot{x}(t) = \sum_{i=1}^{M} \mu_i(u(t))(A_i x(t) + B_i u(t) + E_i f(t)) \tag{21a}$$

$$y(t) = Cx(t) + Dw(t) \tag{21b}$$

Where $x(t) \in R^n$ represents the system's state, $y(t) \in R^m$ is the measured output, $u(t) \in R^r$ is the system's input, $f(t)$ represents the fault and $w(t)$ is the measurement noise. A_i, B_i and C are a known coefficients matrices with appropriate dimensions. E_i and D are respectively the fault distribution and the noise matrices which are supposed to be known. The scalar M represents the number of local models. The observer is choosen to be as following :

$$\dot{\hat{x}}(t) = \sum_{i=1}^{M} \mu_i(u(t))(A_i\hat{x}(t) + B_i u(t) + K_i(y(t) - \hat{y}(t)) + E_i\hat{f}(t)) \quad (22a)$$

$$\dot{\hat{f}}(t) = \sum_{i=1}^{M} \mu_i(u(t))L_i(y(t) - \hat{y}(t)) \quad (22b)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (22c)$$

Where $\hat{x}(t) \in R^n$ is the estimated system's state, $\hat{f}(t)$ represents the estimated fault, $\hat{y}(t) \in R^m$ is the estimated output, K_i are the local model's proportional observer's gains and L_i are the local model's integral gains to be computed. Like in paragraph 1, It is suposed that the fault affecting the system is bounded.
Using the same expression of $\tilde{x}(t)$ and $\tilde{f}(t)$ given by the equation (6) and (7), the dynamics of the state reconstruction error is given by the computation of $\dot{\tilde{x}}(t)$ which is written :

$$\dot{\tilde{x}}(t) = \dot{x}(t) - \dot{\hat{x}}(t) = \sum_{i=1}^{M} \mu_i(u(t))(A_i - K_iC\tilde{x}(t) + E_i\tilde{f}(t) + K_iDw(t)) \quad (23)$$

The fault estimation error can be written :

$$\dot{\tilde{f}}(t) = \dot{f}(t) - \dot{\hat{f}}(t) = \dot{f}(t) - \sum_{i=1}^{M} \mu_i(u(t))(L_iC\tilde{x}(t) - L_iDw(t)) \quad (24)$$

Like the linear case and in order to simplify the notations, the time index (t) will be omitted henceforth.
By the use of the defintions of φ and ϵ gives in (8), the equations (23) and (24) can be written :

$$\dot{\varphi} = A_m\varphi + B_m\epsilon \quad (25)$$

with :

$$A_m = \sum_{i=1}^{M} \mu_i(u)\tilde{A}_i, \quad \text{and} \qquad B_m = \sum_{i=1}^{M} \mu_i(u)\tilde{B}_i \quad (26)$$

where :

$$\tilde{A}_i = \begin{bmatrix} A_i - K_iC & E_i \\ -L_iC & 0 \end{bmatrix}, \qquad \tilde{B}_i = \begin{bmatrix} -K_iD & 0 \\ -L_iD & I \end{bmatrix} \quad (27)$$

The matrix I is the identity matrix with appropriate dimensions. By considering the same expression of th Lyapunov function V given in (11) the inequality (12) can be written as following :

$$\psi^T \Omega \psi < 0 \tag{28}$$

with :

$$\psi = \begin{bmatrix} \varphi \\ \epsilon \end{bmatrix}, \quad \Omega = \begin{bmatrix} A_m^T P + P A_m + Q_\varphi & P B_m \\ B_m^T P & -\mu^2 Q_\epsilon \end{bmatrix} \tag{29}$$

The inequality (28) is verified if $\Omega < 0$.
The matrix A_m can be put on the following form :

$$A_m = \tilde{A}_m - \tilde{K}_m \tilde{C} \tag{30}$$

The expressions of $\tilde{A}_m$, $\tilde{K}_m$ and $\tilde{C}$ are given as following :

$$\tilde{A}_m = \sum_{i=1}^{M} \mu_i(u(t)) \tilde{A}_{mi}, \quad \tilde{K}_m = \sum_{i=1}^{M} \mu_i(u(t)) \tilde{K}_{mi}, \quad \tilde{C} = \begin{bmatrix} C \; 0 \end{bmatrix} \tag{31}$$

with :

$$\tilde{K}_{mi} = \begin{bmatrix} K_i \\ L_i \end{bmatrix}, \quad \tilde{A}_{mi} = \begin{bmatrix} A_i & E_i \\ 0 & 0 \end{bmatrix} \tag{32}$$

In the same way, the matrix B_m can be formulated as following :

$$B_m = -\tilde{K}_m \tilde{D} + \tilde{I} \tag{33}$$

with :

$$\tilde{I} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}, \qquad \tilde{D} = \begin{bmatrix} D \; 0 \end{bmatrix} \tag{34}$$

Posing $G_m = P\tilde{K}_m$ and $m = \mu^2$,the matrix Ω can be put in the following form :

$$\Omega = \begin{bmatrix} P\tilde{A}_m + \tilde{A}_m^T P - G_m \tilde{C} - \tilde{C}^T G_m^T + Q_\varphi & -G_m \tilde{D} + P\tilde{I} \\ \tilde{I}^T P - \tilde{D}^T G_m^T & -m Q_\epsilon \end{bmatrix} \tag{35}$$

the following equations can be solved for $i = 1...M$:

$$\Omega_i < 0 \tag{36}$$

with :

$$\Omega_i = \begin{bmatrix} P\tilde{A}_{mi} + \tilde{A}_{mi}^T P - G_i \tilde{C} - \tilde{C}^T G_i^T + Q_\varphi & -G_i \tilde{D} + P\tilde{I} \\ \tilde{I}^T P - \tilde{D}^T G_i^T & -m Q_\epsilon \end{bmatrix} \tag{37}$$

and $G_i = P\tilde{K}_{mi}$.
The matrices P and G_i are computed via the resolution of the inequality $\Omega_i < 0$.
The matrix $\tilde{K}_{mi}$ is determined as following : $\tilde{K}_{mi} = P^{-1} G_i$.

3.2 Example

Considering the non-linear system described by a multiple model structure with two local models, four states and three outputs. Its structure is given by the following equation :

$$\dot{x}(t) = \sum_{i=1}^{2} \mu_i(u)(A_i x(t) + B_i u(t) + E_i f(t)) \tag{38a}$$

$$y(t) = Cx(t) + Dw(t) \tag{38b}$$

The system matrices are defined as below :

$$A_1 = \begin{bmatrix} -0.3 & -3 & -0.5 & 0.1 \\ -0.7 & -5 & 2 & 4 \\ 2 & -0.5 & -5 & -0.9 \\ -0.7 & -2 & 1 & -0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.2 & -3 & -0.6 & 0.3 \\ -0.6 & -4 & 1 & -0.6 \\ 3 & -0.9 & -7 & -0.22 \\ -0.5 & -1 & -2 & -0.8 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 2 \\ 5 & 1 \\ 4 & -3 \\ 1 & 2 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 4 & 6 \\ 0 & 0 \\ -4 & 2 \\ 7 & 6 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.5 & 0.5 \\ 0.2 & 0.2 \\ 0 & 0.1 \end{bmatrix},$$

$$E_1 = B_1, \quad E_2 = B_2$$

The system inputs are : $u(t) = \begin{bmatrix} u_1(t) \; u_2(t) \end{bmatrix}^T$.
The signal $u_1(t)$ is a telegraph type signal whose amplitude is included in $[0, 0.5]$.
The signal $u_2(t)$ is computed by the following equation $u_2(t) = 0.3 + 0.1 \sin(\pi t)$.
The fault$f(t)$ is written : $f(t) = \begin{bmatrix} f_1(t) \; f_2(t) \end{bmatrix}^T$ with :

$$f_1 = \begin{cases} 0, & t \leq 0.6sec \\ sin(0.5\pi t), & t > 0.6sec \end{cases}, \qquad f_2 = \begin{cases} 0, & t \leq 1sec \\ 0.4, & t > 1sec \end{cases} \tag{39}$$

Choosing $Q_\varphi = Q_\epsilon = I$, The μ, K_1, K_2, L_1 and L_2 computation gives the following results : $\mu = 4.4721$,

$$K_1 = \begin{bmatrix} 18.5771 & 13.6432 & 130.4662 \\ -6.5165 & 42.3315 & 67.5280 \\ 77.6976 & -81.7903 & -211.2738 \\ 0.1124 & -5.8306 & 43.0292 \end{bmatrix}, \quad K_2 = 10^5 * \begin{bmatrix} 0.0448 & -0.1260 & 0.0304 \\ 0.0172 & -0.0509 & 0.0177 \\ -0.4144 & 1.0144 & 0.0562 \\ 0.0411 & -0.1029 & -0.0005 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} -92.3284 & 208.4753 & 185.6276 \\ -22.2858 & -23.9227 & 363.7863 \end{bmatrix}, \quad L_2 = 10^5 * \begin{bmatrix} 0.2031 & -0.5101 & 0.0028 \\ 0.5230 & -1.2936 & -0.0427 \end{bmatrix}$$

The simulated results are shown in the figures (3,4,5,6 and 7). Even in the case of multiple model, the method presented allows to estimate well the system's state and the unknown inputs.

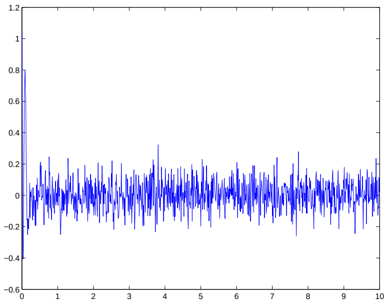

Fig. 3. first output reconstruction error

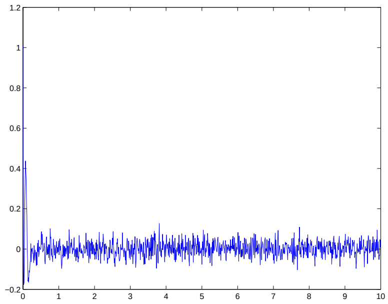

Fig. 4. second output reconstruction error

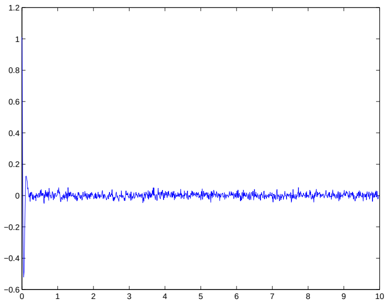

Fig. 5. third output reconstruction error

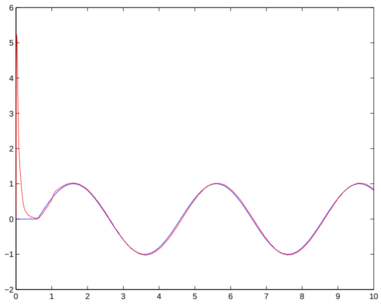

Fig. 6. fault f_1 and its estimate

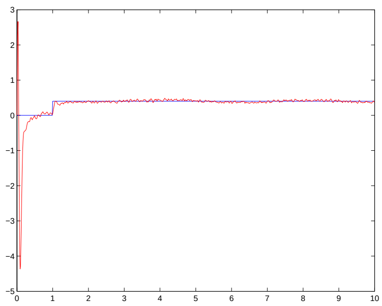

Fig. 7. fault f_2 and its estimate

4 Conclusion

This paper presents an estimation method of faults considered as unknown inputs in the linear and non linear case. The class of the considered non-linear systems are those described by multiple model structure. This method uses a proportionnal integral observer with unknown inputs and its advantage is the take into account of the non-null fault dynamics. The validation of the method is done on an academic example.

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