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FAULT TOLERANT CONTROL FOR NONLINEAR SYSTEMS DESCRIBED BY TAKAGI-SUGENO MODELS

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ABSTRACT: *In this paper the problem of active fault tolerant control (FTC) in noisy systems is studied. The proposed FTC strategy is based on the known of the fault estimate and the error between the faulty system state and a reference system state. A proportional integral observer is used in order to estimate the state and the actuator faults. The obtained results are then extended to nonlinear systems described by nonlinear Takagi-Sugeno models. The problem of conception of the proportional integral observer and the FTC strategy is formulated in linear matrix inequalities (LMI) which can be solved easily. Simulation examples are given to illustrate the proposed method for the linear and nonlinear systems.*

KEYWORDS: *fault estimation, active fault tolerant control, proportional integral observer, multiple models, Takagi-Sugeno models, actuator faults.*

1 Introduction

A state observer is a dynamical system allowing the state reconstruction from the system model and the knowledge of its inputs and outputs (D.G. Luenberger, 1971). For linear models, state estimation methods are very efficient (C. Edwards, 2004). However for many real systems, the linearity hypothesis cannot be assumed. Indeed, the unceasing demand in terms of reliability and performance of systems has led to the use of nonlinear models to represent systems. Therefore obtained models are very complex and the task of model-based fault diagnosis becomes more difficult to achieve. In that case, the synthesis of a nonlinear observer allows the reconstruction of the system state. Many different approaches have been developed for dealing with that problem. Among them, let us cite sliding mode observers (C. Edwards and S.K. Spurgeon, 2000), the Thau-Luenberger observers (F.E. Thau, 1973) and the so-called multiple observers for nonlinear systems described by Takagi-Sugeno models (P. Bergsten, *et al.*, 2002).
Recently, Takagi-Sugeno Fuzzy systems have been the subject of many researches by virtue of their approximation capabilities. They can represent exactly a nonlinear model (K. Tanaka, *et al.*, 1998), (M. Witczak *et al.*, 2008). They are constructed by a set of linear models blended together with nonlinear functions holding the convex-sum property. In the case of Takagi-Sugeno Fuzzy systems, state estimation is based on the design of a nonlinear observer (multiple observer) using the same nonlinear weighting functions as the Takagi-Sugeno model.
In most cases, processes are subjected to disturbances which have as origin the noises due to its environment and the model uncertainties. Moreover, sensors and/or actuators can be corrupted by different faults or failures. Many works are dealing with state estimation for systems with unknown inputs or parameter uncertainties. (S.H. Wang, *et al.*, 1975) propose an observer able to entirely reconstruct the state of a linear system in the presence of unknown inputs and in (L. M. Lyubchik and Y. T. Kostenko, 1993), to estimate the state, a model inversion method is used. Using the Walcott and Zak structure observer (B. L. Walcott and S. H. Zak, 1988), Edwards *et al.* (C. Edwards and S.K. Spurgeon, 2000),(C. Edwards and S.K. Spurgeon, 1994) have also designed a convergent observer using the Lyapunov approach.
In the context of nonlinear systems described by Takagi-Sugeno models, some works tried to reconstruct the system state in spite of the unknown input existence. This reconstruction is assured via the elimination of unknown inputs (Y. Guan and M. Saif, 1991). Other works choose to estimate, simultaneously, the unknown inputs and system state (A. Akhenak *et al.*, 2009), (D. Ichalal *et al.*, 2009), (A. Khedher *et al.*, 2008), (A. Khedher *et al.*, 2010), (R. Orjuela *et al.*, 2009). Unknown input observers can be used to estimate actuator faults provided they are assumed to be considered as unknown inputs. This estimation can be obtained using of a proportional in-

tegral observer (A. Khedher *et al.*, 2008). That kind of observers gives some robustness property of the state estimation with respect to the system uncertainties and perturbations (S. Beale and B. Shafai, 1989), (R. Orjuela *et al.*, 2008).
Faults affecting systems have harmful effects on the normal behaviour of the process and their estimation can be used to conceive a control strategy able to minimize their effects (fault tolerant control (FTC)). A control loop can be considered fault tolerant if there exist adaptation strategies of the control law included in the closed-loop that introduce redundancy in actuators (M. Witczak *et al.*, 2008). Fault Tolerant Control (FTC) is, relatively, a new idea in the research literature (M. Blanke *et al.*, 2003) which allows to have a control loop that fulfils its objectives when faults appear.
There are two main groups of Control strategies: the active and the passive techniques. The passive techniques are control laws that take into account the fault appearance as a system perturbation. This kind of control is described in (M. Blanke *et al.*, 2003), (J. Chen *et al.*, 1998), (Y. Liang *et al.*, 2000), (Z. Qu *et al.*, 2001), (F. Liao *et al.*, 2002), (Z. Qu *et al.*, 2003). The active fault tolerant control techniques consist on adapting the control law using the information given by the FDI block (M. Blanke *et al.*, 2003),(Y. Zhang *et al.*, 2003).
In this paper, an active FTC strategy is proposed. A similar FTC strategy is proposed in (M. Witczak *et al.*, 2008) for the class of discrete systems. In (M. Witczak *et al.*, 2008) Witczak *et al.* use the error between the faulty and the reference systems. In real cases the faulty system state is unknown. The main contribution in this work is to propose a solution to this problem replacing the faulty system state by its estimate. State estimation is made using a proportional integral observer to estimate faults. Once the fault is estimated, the FTC controller is implemented as a state feedback controller. In this work the observer design and the control implementation can be made simultaneously. First, the propose approach is developed in the context of linear systems and then it is extended to Takagi-Sugeno fuzzy systems.
The paper is organised as follows. Section 2 presents the proposed method in the case of linear systems. The extension of this method for Takagi-Sugeno (T-S) systems is the subject of section 3. Numerical examples which show the performance of the proposed approach are presented in the two sections.

2 The linear system case

The objective of this part is to conceive an actuator fault tolerant control for linear systems case

2.1 Problem formulation

Consider the linear model described by:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1a)$$
$$y(t) = Cx(t) \quad (1b)$$

where $x(t) \in R^n$ represents the system state, $y(t) \in R^m$ is the measured output, $u(t) \in R^r$ is the system input. A, B and C are known constant matrices with appropriate dimensions.
Consider the linear model affected by actuator faults and a measurement noise described by:

$$\dot{x}_f(t) = Ax_f(t) + Bu_f(t) + Ef(t) \quad (2a)$$
$$y_f(t) = Cx_f(t) + Dw(t) \quad (2b)$$

where $x_f(t) \in R^n$ represents the system state, $y_f(t) \in R^m$ is the measured output, $u_f(t) \in R^r$ is the system input, $f(t)$ represents the fault which is assumed to be bounded and $w(t)$ is the measurement noise. E and D are respectively the fault and the noise distribution matrices which are assumed to be known.
The structure of the chosen proportional integral observer is as follows:

$$\dot{\hat{x}}_f(t) = A\hat{x}_f(t) + Bu_f(t) + E\hat{f}(t) + K(\tilde{y}_f(t)) \quad (3a)$$
$$\dot{\hat{f}}(t) = L(\tilde{y}(t)) \quad (3b)$$
$$\hat{y}_f(t) = C\hat{x}_f(t) \quad (3c)$$

where $\hat{x}_f(t)$ is the estimated state, $\hat{f}(t)$ represents the estimated fault, $\hat{y}_f(t)$ is the estimated output, K is the proportional observer gain and L is its integral gain which must be computed. $\tilde{y}_f(t) = y_f(t) - \hat{y}_f(t)$.
The system input $u_f(t)$ is conceived by being inspired of the strategy proposed in (M. Witczak *et al.*, 2008) and described by the following expression :

$$u_f(t) = -S\hat{f}(t) + N(x(t) - \hat{x}_f(t)) + u(t) \quad (4)$$

where S and N are two constant matrices with appropriate dimensions. The objective is to find the matrices S and N which permit to the state x_f to converge to x.
Let us define $\tilde{x}(t)$ the error between the states $x(t)$ and $x_f(t)$, $\tilde{x}_f(t)$ the estimation error of the state x_f and $\tilde{f}(t)$ the fault estimation error :

$$\tilde{x}(t) = x(t) - x_f(t) \quad (5)$$
$$\tilde{x}_f(t) = x_f(t) - \hat{x}_f(t) \quad (6)$$
$$\tilde{f}(t) = f(t) - \hat{f}(t) \quad (7)$$

The dynamics of $\tilde{x}(t)$ is given by:

$$\dot{\tilde{x}}(t) = \dot{x}(t) - \dot{x}_f(t)$$
$$= (A - BN)\tilde{x}(t) + BS\hat{f}(t) - BN\tilde{x}_f(t) - Ef(t) \quad (8)$$

Choosing S so that $E = BS$, the dynamics of $\tilde{x}(t)(t)$ becomes :

$$\dot{\tilde{x}}(t) = (A - BN)\tilde{x}(t) - BN\tilde{x}_f(t) - E\tilde{f}(t) \quad (9)$$

The dynamics of $\tilde{x}_f(t)$ is expessed as follow :

$$\begin{aligned}\dot{\tilde{x}}_f(t) &= \dot{x}_f(t) - \dot{\hat{x}}_f(t) \\ &= (A - KC)\tilde{x}_f(t) + E\tilde{f}(t) - KDw(t) \quad (10)\end{aligned}$$

and the dynamics of the fault estimation error is described by:

$$\begin{aligned}\dot{\tilde{f}}(t) &= \dot{f}(t) - \dot{\hat{f}}(t) \\ &= \dot{f}(t) - LC\tilde{x}_f(t) - LDw(t) \quad (11)\end{aligned}$$

In order to simplify the notations, the time index (t) will be omitted henceforth.
The following vectors are introduced:

$$\varphi = \begin{bmatrix} \tilde{x}^T & \tilde{x}_f^T & \tilde{f}^T \end{bmatrix}^T \text{ and } \psi = \begin{bmatrix} w^T & \dot{f}^T \end{bmatrix}^T \quad (12)$$

From the equations (9), (10) and (11), one can obtain:

$$\dot{\varphi} = A_0\varphi + B_0\psi \quad (13)$$

with :

$$A_0 = \begin{bmatrix} A - BN & -BN & -E \\ 0 & A - KC & E \\ 0 & -LC & 0 \end{bmatrix} \text{ and } B_0 = \begin{bmatrix} 0 & 0 \\ -KD & 0 \\ -LD & I \end{bmatrix} \quad (14)$$

In order to analyse the convergence of the generalized estimation error $\varphi(t)$, let us consider the following quadratic Lyapunov candidate function $V(t)$:

$$V(t) = \varphi^T(t)P\varphi(t) \quad (15)$$

where P denotes a positive definite matrix.
The problem of robust state and fault estimation is reduced to find the gains K and L of the observer to ensure an asymptotic convergence of $\tilde{x}_f$ and $\tilde{f}$ toward zero when $\psi(t) = 0$ and to ensure a bounded error when $\psi(t) \neq 0$. The problem of the design of the input $u_f(t)$ is reduced to find the matrix N to ensure the convergence of $\tilde{x}(t)$ to zero. φ converges to zero if $\dot{V} < 0$. $\dot{V} < 0$ if $A_0^T P + PA_0 < 0$.
The matrices A_0 and B_0 can be expressed as:

$$A_0 = \begin{bmatrix} A - BN & E_1 \\ 0 & \tilde{A} - \tilde{K}\tilde{C} \end{bmatrix} \text{ and } B_0 = \begin{bmatrix} 0 \\ \tilde{I} - \tilde{K}\tilde{D} \end{bmatrix} \quad (16)$$

with :

$$\tilde{A} = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}, \ \tilde{K} = \begin{bmatrix} K \\ L \end{bmatrix}, \ \tilde{I} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \quad (17)$$

$$\tilde{C} = \begin{bmatrix} C & 0 \end{bmatrix}, \ \tilde{D} = \begin{bmatrix} D & 0 \end{bmatrix}, \ E_1 = \begin{bmatrix} -BN & -E \end{bmatrix} \quad (18)$$

Assuming that P has the block diagonal form $P = diag(P_1, P_2)$, it can be observed from the structure of A_0 that the eigenvalues of the matrix A_0 are the union of those of $A - BM$ and $\tilde{A} - \tilde{K}\tilde{C}$. This clearly indicates that the design of the control $u_f(t)$ and the P. I. observer can be carried out independently (separation principle). Thus, it is clear from the expression of P that φ converges to zero iff there exist matrices $P_1 > 0$ and $P_2 > 0$ such that following inequalities are satisfied:

$$(A - BN)^T P_1 + P_1(A - BN) < 0 \quad (19)$$
$$(\tilde{A} - \tilde{K}\tilde{C})^T P_2 + P_2(\tilde{A} - \tilde{K}\tilde{C}) < 0 \quad (20)$$

By multiplying (19) from left and right by P_1^{-1} one obtain :

$$P_1^{-1}(A - BN)^T + (A - BN)P_1^{-1} < 0 \quad (21)$$

Substituing $W = P_1^{-1}$, the equation (21) becomes :

$$W(A - BN)^T + (A - BN)W < 0 \quad (22)$$

φ converge to zero if there exist two definite and positive matrices W and P_2 satisfying (20) and (22).
The inequalities (20) and (22) are not linear, substituting $X = NW$, and $Y = P_2\tilde{K}$, their become :

$$WA^T + AW - X^T B^T - BX < 0 \quad (23)$$
$$\tilde{A}^T P_2 + P_2\tilde{A} - Y\tilde{C} - \tilde{C}^T Y^T < 0 \quad (24)$$

The resolution of the linear matrices inequalities (LMI) (23) and (24) permits to find the matrices W, P_2, X and Y. The matrices N and $\tilde{K}$ are computed using the following equations :

$$N = XW^{-1} \quad (25)$$
$$\tilde{K} = P_2^{-1}Y \quad (26)$$

2.2 Example

Consider the linear systems described by the equations (1) and (2) with $C = I$ and:

$$A = \begin{bmatrix} -0.3 & -3 & -0.5 & 0.1 \\ -0.7 & -5 & 2 & 4 \\ 2 & -0.5 & -5 & -0.9 \\ -0.7 & -2 & 1 & -0.9 \end{bmatrix}, \ E = \begin{bmatrix} 1 & 2 \\ 5 & 1 \\ 4 & -1 \\ 1 & 2 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.5 & 0.5 \\ 0.2 & 0.2 \\ 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 2 \\ -1 & -2 \end{bmatrix}$$

The system input $u(t) = \begin{bmatrix} u_1(t)^T & u_2(t)^T \end{bmatrix}^T$ with: $u_1(t)$ is a telegraph type signal varying between zero and one, $u_2(t) = 0.3 + 0.1\sin(\pi t)$
The fault $f(t) = \begin{bmatrix} f_1(t)^T & f_2(t)^T \end{bmatrix}^T$ with :

$$f_1 = \begin{cases} 0, & t \leq 4sec \\ 0.1 * sin(\pi t), & t > 4sec \end{cases}$$

and $f_2 = \begin{cases} 0, & t \leq 1.5sec \\ 0.4, & t > 1.5sec \end{cases}$

The computation of the matrices K, L and N gives :

$$L = \begin{bmatrix} 0.140 & 6.863 & 4.682 & 0.007 \\ 3.069 & 3.192 & -2.167 & 5.699 \end{bmatrix}$$
$$N = \begin{bmatrix} 0.056 & -1.063 & 1.754 & -0.731 \\ 0.750 & -0.865 & -1.210 & -0.284 \end{bmatrix}$$
$$K = \begin{bmatrix} 2.590 & 0.564 & -0.239 & 0.637 \\ -3.635 & -1.160 & 1.083 & 0.138 \\ 1.562 & 2.585 & -1.357 & -0.498 \\ 0.415 & 2.551 & -0.283 & 3.234 \end{bmatrix}$$

The simulation results are shown in the figures (1) to (3) :

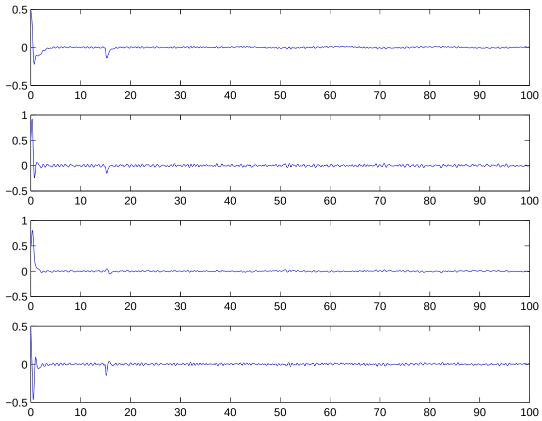

Figure 1: Error between x and x_f

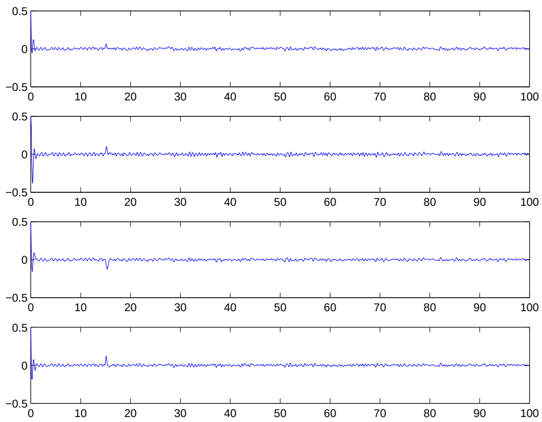

Figure 2: Estimation error of x_f

The input $u_f(t)$ is computed using the equation (4), this input permits to the system (2) to have the same behaviour with the system (1). This input is shown in figure (4).
The conceived observer allows to estimate the state x_f and the control $u_f(t)$ is a fault tolerant control applied to the system (2).

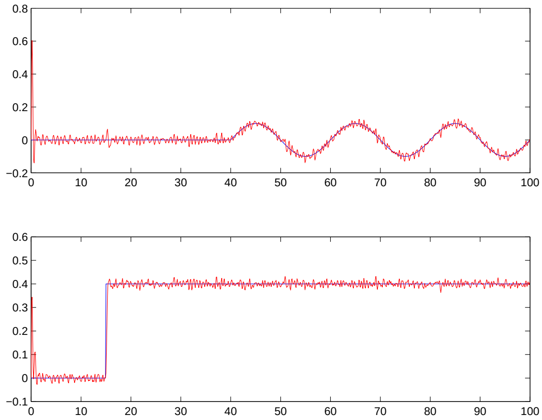

Figure 3: Faults and their estimations

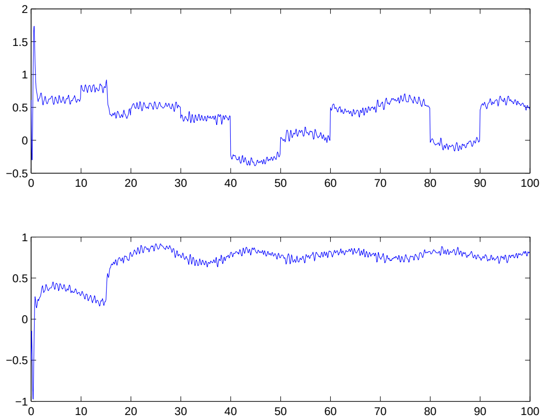

Figure 4: fault tolerant control u_f

2.3 Conclusion

In this part the problem of fault estimation and fault tolerant control strategy is studied in the case of linear system. A method which permits simultaneously the fault estimation and the conception of the fault tolerant control is proposed. This control is computed using the fault estimate and the error between the state of a system affected by a fault and a reference system state. In the next section the proposed method will be extended to nonlinear systems described with multiple models.

3 Extension to multiple models representation

Multiple model approach is an appropriate tool for modelling complex systems using a mathematical model which can be used for analysis, controller and observer design. The basis of the multiple model approach is the decomposition of the operating space of

the system into a finite number of operating zones. Hence, the dynamic behaviour of the system inside each operating zone can be modelled using a simple submodel, for example a linear model. The relative contribution of each submodel is quantified with the help of a weighting function. Finally, the approximation of the system behaviour is performed by associating the submodels and by taking into consideration their respective contributions. Note that a large class of nonlinear systems can accurately be modelled using multiple models.
The choice of the structure used to associate the submodels constitutes a key point in the multiple modelling frameworks. Indeed, the submodels can be aggregated using various structures (D. Filev, 1991). Classically, the association of submodels is performed in the dynamic equation of the multiple model using a common state vector. This model, known as Takagi-Sugeno multiple model, has been initially proposed, in a fuzzy modelling framework, by Takagi and Sugeno (T. Takagi and M. Sugeno, 1985) and in a multiple model modelling framework by Johansen and Foss (T.A. Johansen and A.B. Foss, 1992). This model has been largely considered for analysis, modelling, control and state estimation of nonlinear systems.

3.1 On the multiple model representation

The structure of a Takagi-Sugeno model is :

$$\dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) \left(A_i x(t) + B_i u(t)\right) \quad (27a)$$

$$y(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) C_i x(t) \quad (27b)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^r$ control vector, $y(t) \in R^m$ vector of measures and A_i, B_i and C_i are known constant matrices with appropriate dimensions.
The membership functions $\mu_i(\xi(t))$ assure a progressive passage between the local models. These have the following proprieties:

$$\sum_{i=1}^{M} \mu_i(\xi(t)) = 1, \forall t \quad (28)$$

$$\text{and } \ 0 \leq \mu_i(\xi(t)) \leq 1, \ \ \forall i = 1...M, \forall t \quad (29)$$

The variable of decision $\xi(t)$ is accessible in real time and it depends of measurable variables like system inputs or outputs.
Let's remark that state matrices of this kind of multiple models are built by the made of a level-headed sum, with variable weight of different matrices of local models. One can also make a similarity between multiple models and systems with variables parameters in time.

If, in the equation which defines the output, we impose that $C_1 = C_2 = ... = C_M = C$, the output of the multiple model (27) is reduced to : $y(t) = Cx(t)$ and the multiple model becomes:

$$\dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \quad (30a)$$

$$y(t) = Cx(t) \quad (30b)$$

In this part the method proposed for linear systems will be extended to nonlinear systems described by multiple models.

3.2 Problem formulation

A non linear system described by multiple model can be expressed as follow:

$$\dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) A_i x(t) + Bu(t) \quad (31a)$$

$$y(t) = Cx(t) \quad (31b)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^r$ is the input vector, $y(t) \in R^m$ the output vector and A_i, B_i and C are known constant matrices with appropriate dimensions. The scalar M represents the number of local models.
Consider the following nonlinear Takagi-Sugeno model affected by actuator faults and measurement noise:

$$\dot{x}_f(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) A_i x_f(t) + Bu_f(t) + Ef(t) \quad (32a)$$

$$y_f(t) = Cx_f(t) + Dw(t) \quad (32b)$$

where $x_f(t) \in R^n$ is the state vector, $u_f(t) \in R^r$ is the input vector, $y_f(t) \in R^m$ the output vector. $f(t)$ represents the fault which is assumed to be bounded and $w(t)$ is the measurement noise. E and D are respectively the fault and the noise distribution matrices which are assumed to be known.
The structure of the proportional integral observer is chosen as follows:

$$\dot{\hat{x}}_f(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i \hat{x}_f(t) + K_i(\tilde{y}(t))) + Bu_f(t) + E\hat{f}(t) \quad (33a)$$

$$\dot{\hat{f}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(L_i \tilde{y}(t)) \quad (33b)$$

$$\hat{y}_f(t) = C\hat{x}_f(t) \quad (33c)$$

where $\hat{x}_f(t)$ is the estimated system state, $\hat{f}(t)$ represents the estimated fault, $\hat{y}_f(t)$ is the estimated output, K_i are the local models proportional observer gains and L_i are their integral gains to be computed. $\tilde{y}(t) = y_f(t) - \hat{y}_f(t)$.

The system input $u_f(t)$ is conceived on the base of the strategy described by the following expression :

$$u_f(t) = -S\hat{f}(t) + u(t) \quad (34)$$

where S is a constant matrix with appropriate dimensions. The dynamics of the errors defined in (5), (6) and (7) can be written as follow:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))A_i\tilde{x}(t) + BS\hat{f}(t) - Ef(t) \quad (35)$$

Choosing S so that $E = BS$, the dynamics of $\tilde{x}(t)$ becomes :

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))A_i\tilde{x}(t) - E\tilde{f}(t) \quad (36)$$

The dynamic of $\tilde{x}_f(t)$ can be written in the case of multiple models :

$$\dot{\tilde{x}}_f(t) = (\sum_{i=1}^{M} \mu_i(\xi(t))(A_i - K_iC)\tilde{x}_f(t) - K_iDw(t)) + E\tilde{f}(t) \quad (37)$$

The dynamic of the fault error estimation can be written :

$$\dot{\tilde{f}}(t) = \dot{f}(t) - \sum_{i=1}^{M} \mu_i(\xi(t))(L_iC\tilde{x}_f(t) + L_iDw(t)) \quad (38)$$

In order to simplify the notations, the time index (t) will be omitted henceforth.
The equations (36), (37) and (38) can be rewritten :

$$\dot{\varphi} = A_m\varphi + B_m\psi \quad (39)$$

with φ and ψ are given by the equation (12) and :

$$A_m = \sum_{i=1}^{M} \mu_i(\xi(t))A_{mi} \text{ and } B_m = \sum_{i=1}^{M} \mu_i(\xi(t))B_{mi} \quad (40)$$

where :

$$A_{mi} = \begin{bmatrix} A_i & 0 & -E \\ 0 & A_i - K_iC & E \\ 0 & -L_iC & 0 \end{bmatrix} \text{ and } B_{mi} = \begin{bmatrix} 0 & 0 \\ -K_iD & 0 \\ -L_iD & I \end{bmatrix} \quad (41)$$

Considering the Lyapunov function given in (15) the errors converge to zero if $\dot{V} < 0$. $\dot{V} < 0$ if $A_{mi}^T P + PA_{mi} < 0, \quad \forall i \in \{1, ..., M\}$.
The matrices A_{mi} and B_{mi} can be rewritten :

$$A_{mi} = \begin{bmatrix} A_i & E_1 \\ 0 & \tilde{A}_i - \tilde{K}_i\tilde{C} \end{bmatrix} \text{ and } B_{mi} = \begin{bmatrix} 0 \\ \tilde{I} - \tilde{K}_i\tilde{D} \end{bmatrix} \quad (42)$$

with :

$$\tilde{A}_i = \begin{bmatrix} A_i & E \\ 0 & 0 \end{bmatrix}, \tilde{K}_i = \begin{bmatrix} K_i \\ L_i \end{bmatrix}, \tilde{I} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \quad (43)$$

$$\tilde{C} = \begin{bmatrix} C & 0 \end{bmatrix}, \ \tilde{D} = \begin{bmatrix} D & 0 \end{bmatrix}, \ E_1 = \begin{bmatrix} 0 & -E \end{bmatrix} \quad (44)$$

Assuming that P has the block diagonal form $P = diag(P_1, P_2)$, φ converges to zero iff there exist matrices $P_1 > 0$ and $P_2 > 0$ such that following inequality is satisfied:

$$\begin{bmatrix} A_i^T P_1 + P_1A_i & E_1P_2 + P_1E_1 \\ P_2E_1^T + E_1^T P_1 & (\tilde{A}_i - \tilde{K}_i\tilde{C})^T P_2 + P_2(\tilde{A}_i - \tilde{K}_i\tilde{C}) \end{bmatrix} < 0 \quad (45)$$

Substituting $V_i = P_2\tilde{K}_i$, (45) becomes:

$$\begin{bmatrix} A_i^T P_1 + P_1A_i & E_1P_2 + P_1E_1 \\ P_2E_1^T + E_1^T P_1 & \tilde{A}^T P_2 + P_2\tilde{A} - V_i\tilde{C} - \tilde{C}^T V_i^T \end{bmatrix} < 0 \quad (46)$$

The resolution of the linear matrix inequality (LMI) (47) permits to find the matrices P_1, P_2 and V_i. The matrices $\tilde{K}_i$ are computed using $\tilde{K}_i = P_2^{-1}V_i$.
Summarizing the following theorem can be proposed:
Theorem: *The system (39) describing the evolution of the errors $\tilde{x}(t)$, $\tilde{x}_f(t)$ and $\tilde{f}(t)$ is stable if there exist symmetric definite positive matrices P_1 et P_2 and matrices V_i, $i \in \{1...M\}$ so that the following LMI are verified :*

$$\begin{bmatrix} A_{ai}^T P_1 + P_1A_{ai} & E_1P_2 + P_1E_1 \\ P_2E_1^T + E_1^T P_1 & \tilde{A}^T P_2 + P_2\tilde{A} - V_i\tilde{C} - \tilde{C}^T V_i^T \end{bmatrix} < 0 \quad (47)$$

The observer gains are obtained by: $\tilde{K}_i = P_2^{-1}V_i$.

3.3 Illustrative example

Let us consider the multiple model (31), made up of two local models and involving four states and four outputs with $C = I$, $\xi(t) = u(t)$ and:

$$A_1 = \begin{bmatrix} -0.3 & -3 & -0.5 & 0.1 \\ -0.7 & -5 & 2 & 4 \\ 2 & -0.5 & -5 & -0.9 \\ -0.7 & -2 & 1 & -0.9 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 2 \\ -1 & -2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.2 & -3 & -0.6 & 0.3 \\ -0.6 & -4 & 1 & -0.6 \\ 3 & -0.9 & -7 & -0.22 \\ -0.5 & -1 & -2 & -0.8 \end{bmatrix}, D = \begin{bmatrix} 0.5 & 0.5 \\ 0.2 & 0.2 \\ 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}$$

Consider the non linear system affected by an actuator fault and described by the equation (32) with:

$$E = \begin{bmatrix} 1 & 5 & 4 & 1 \\ 2 & 1 & -1 & 2 \end{bmatrix}^T$$

The chosen weighting functions depend on the two inputs of the system. They have been created on the basis of Gaussian membership functions. Figure 5 shows their time-evolution showing that the system is clearly nonlinear since μ_1 and μ_2 are not constant

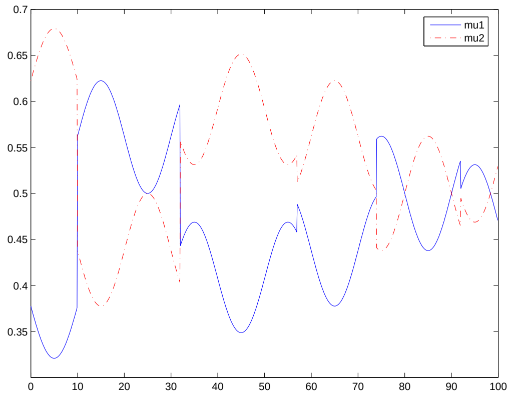

Figure 5: Weighting functions

functions. The system input and the faults are used for the linear example. The computation of the matrices K_1, L_1, K_2 and L_2 gives :

$$L_1 = \begin{bmatrix} -0.362 & 8.727 & 6.036 & -0.823 \\ 4.736 & 4.751 & -3.795 & 8.575 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -0.475 & 8.308 & 6.643 & 1.225 \\ 4.951 & 1.660 & -3.470 & 8.753 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} 3.958 & 1.106 & -0.601 & 1.055 \\ -3.830 & 0.703 & 1.766 & 0.026 \\ 1.590 & 3.225 & 0.510 & -1.028 \\ 1.335 & 3.025 & -0.750 & 5.637 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 4.057 & 0.901 & -0.166 & 1.165 \\ -3.503 & 1.718 & 2.229 & 0.623 \\ 2.053 & 1.344 & -1.495 & -0.541 \\ 1.615 & -1.160 & -3.587 & 5.730 \end{bmatrix}$$

Simulation results are shown in figures (6) to (8). The proposed observer allows well the state and fault estimation. Even in the case of nonlinear system described by multiple models the proposed method permit to conceive a fault tolerant control strategy. The control conceived is applied to a system affected by an actuator fault. Fault estimation is very important because the fault estimate is used to compute the fault tolerant control strategy. This control is shown in the figure (9).

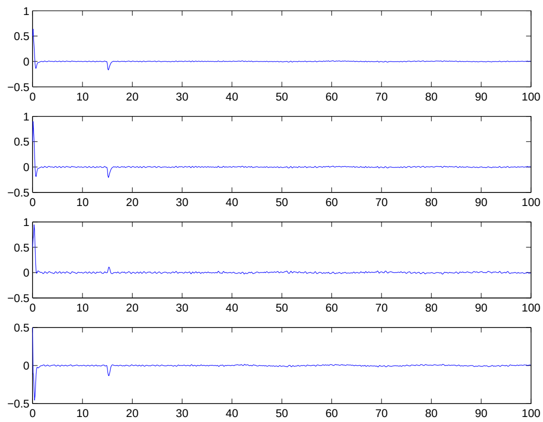

Figure 6: Error between x and x_f

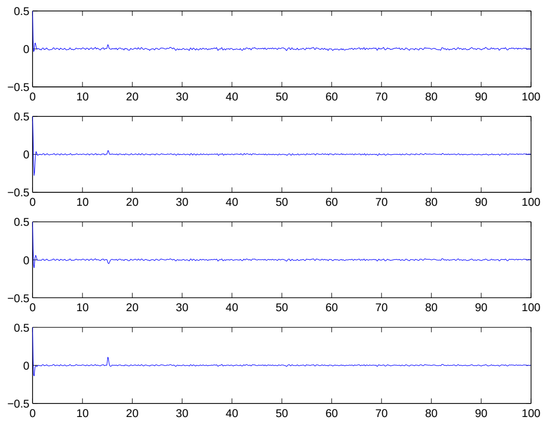

Figure 7: Estimation error of x_f

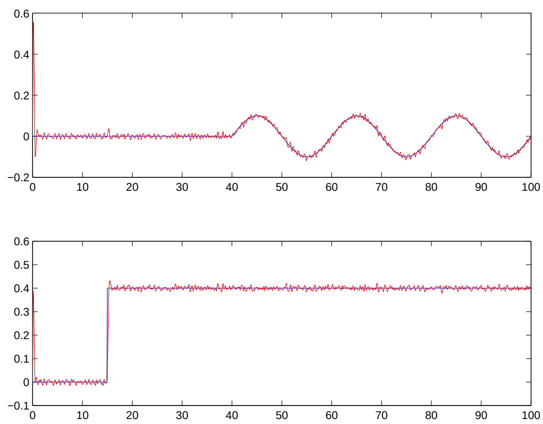

Figure 8: Faults and their estimations

4 Conclusion

In this work, an active FTC strategy was proposed. First, this approach was developed in the case of linear systems and then it was extended to Takagi-Sugeno fuzzy systems. The main contribution of the proposed approach is in the use of the proportional integral observer to estimate faults. Once the fault is estimated, the FTC controller is implemented as a state feedback controller. This controller is designed such that it can stabilize the faulty plant using Lyapunov theory and LMIs. The observer design and the control implementation can be made simultaneously. Illustrative examples both for linear and non-linear systems described by T-S fuzzy models are provided that show the effectiveness of the proposed Proportional integral observer and the FTC approach. Fur-

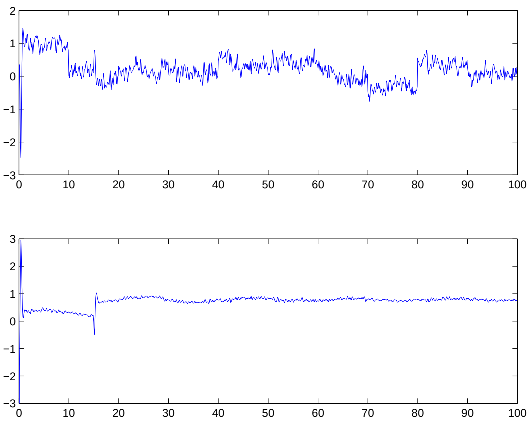

Figure 9: Fault tolerant control u_f

ther research will be oriented towards implementing an adaptive FTC strategy in the case of systems affected by sensors faults.

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