DESIGN OF OBSERVERS FOR TAKAGI-SUGENO DISCRETE-TIME SYSTEMS WITH UNMEASURABLE PREMISE VARIABLES

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Abstract: this paper deals with the problem of the state estimation of discrete time nonlinear systems described by Takagi-Sugeno structure with unmeasurable premise variables. The proposed observer design method is based on the use of the second method of Lyapunov and a quadratic function. Based on this procedure two methods are proposed. The convergence conditions of the observer are expressed in terms of Linear Matrix Inequalities (LMI). Finally, these two methods are compared with regard to their domains of validity.

Keywords: Multiple model approach; nonlinear observer; unknown premise variables; nonlinear discrete system; $\mathcal{L}_2$ optimization; linear matrix inequality.

1. INTRODUCTION

The nonlinear state estimation takes an increasingly important place in the automatic control. Several directions of research were followed, among them we can cite the works of Thau (Thau, 1973) who proposed sufficient conditions for asymptotic stability of the state estimation error. In (Bornard and Hammouri, 1991) and (Gauthier and Kupka, 1994), high gain observers are proposed, which are based on the search for a transformation of the nonlinear system toward the canonical form, this canonical form being used for the synthesis of the observer. However, the disadvantage of this method is the absence of a systematic procedure for finding the transformation functions of the system. Another more interesting approach consists in representing the nonlinear systems by a Takagi-Sugeno structure.

The Takagi-Sugeno structure is one of the privileged tools of representation for nonlinear systems. The particular form of these models, i.e models interconnected by nonlinear functions, makes it possible to exploit the tools and methods developed in the context of the linear systems. The nonlinear interpolation functions (weighting functions) depend, by assumption, on measured variables (input or output of the system). In the context of fault detection by using banks of observers, it is thus not possible to remove one of the inputs or outputs since they intervene directly in the weighting functions. For that reason it is necessary to work out different models to detect the sensor faults or the actuator faults. An approach to solve this problem is to consider models whose weighting functions depend on the state of the system. Moreover, the Takagi-Sugeno models with unknown premise variables describe a wider class of nonlinear systems compared with the models with measurable premise variables.

In the context of the linear models, fault detection can be carried out by methods using state observers (Maquin and Ragot, 2000) and residual generation. In general, fault isolation methods use banks of observers where each observer is driven by a subset of the inputs u.The preceding technique cannot be immediately extended to the multiple model because of the couplings introduced into the structure. Generally, the design of an observer for a multiple model begins with the design of local observers, then a weighted interpolation is performed to obtain the estimated state. This design allows the extension of the analysis and synthesis tools developed for the linear systems, to the nonlinear systems.

(Tanaka *et al.*, 1998) proposed a study concerning the stability and the synthesis of regulators and observers for multiple models. In (Chadli, 2002), (Tanaka *et al.*, 1998) and (Guerra *et al.*, 2006) tools directly inspired of the study of the linear systems are adapted for the stability study and stabilization of nonlinear systems. (Patton *et al.*, 1998) proposed a multiple observer based on the use of Luenberger observers, which was then used for the diagnosis. In (Akhenak, 2004) observers with sliding mode developed for the linear systems, were transposed to the systems described by multiple model. The principal interest of this type of observers is the robustness with respect to modeling uncertainties . Moreover, the unknown input observers designed for linear systems, are transposed, in the same way, into the case of nonlinear systems and application to fault diagnosis is envisaged in (Marx *et al.*, 2007).

However, in all these works, the authors supposed that the weighting functions depend on measurable premise variables. In the field of diagnosis, this assumption forces to design observers with weighting functions depending on the input $u(t)$, for the detection of the sensors faults, and on the output $y(t)$, for the detection of actuator faults. Indeed, if the decision variables are the inputs, for example in a bank of observers, even if the i^{th} observer is not controlled by the input u_i, this input appears indirectly in the weighting function and it cannot be eliminated. For this reason, it is interesting to consider the case of weighting functions depending on unknown premise variables, like the state of the system. This assumption makes it possible to represent a large class of nonlinear systems. Only few works are based on this approach, nevertheless, one can cite (Bergsten and Palm, 2000), (Palm and Driankov, 1999), (Bergsten *et al.*, 2001) and (Bergsten *et al.*, 2002), in which a Luenberger observer is proposed, by using Lipschitz weighting functions. The stability conditions of the observer are formulated in the form of linear matrix inequalities (LMI) (Boyd *et al.*, 1994). Unfortunately, the Lipschitz constant appears in the LMIs to be solved and reduces the applicability of the method if this constant has an important value. In (Palm and Bergsten, 2000) and (Bergsten and Palm, 2000), the observer with sliding mode compensates the unknown terms of the system.

In this paper, a discrete time nonlinear system described by the Takagi-Sugeno structure is considered. The state estimation error dynamics is written as a perturbed system. So, we propose a first method based on the use of the second method of Lyapunov and some assumptions on the weighting functions. The second proposed method is based on the use of $\mathcal{L}_2$ design (which is an extension of the H_∞ design), the influence of the unknown terms on the state estimation error is minimized. According to this objective, we propose a new observer design for multiple model with unknown premise variables. The observer synthesis is carried out using the second method of Lyapunov with a quadratic function and $\mathcal{L}_2$ optimization. The paper is organized as follows : in section 3, the proposed observer is presented, convergence conditions of the proposed observer are established. In section 4, an other method of state estimation, based on $\mathcal{L}_2$ techniques, is proposed. Simulation results are presented in section 5, where a comparison between the two methods is presented and some conclusions and perspectives are given in section 5.

2. MULTIPLE MODEL APPROACH

The form of Takagi-Sugeno systems studied in this paper is :

$$x(k+1) = \sum_{i=1}^{r} \mu_i(x(k)) \left(A_i x(k) + B_i u(k)\right) \quad (1)$$

$$y(k) = Cx(k) \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input of the system, $y(t) \in \mathbb{R}^p$ is the output of yhe system. $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{p \times n}$ are real known constant matrices, and r is the number of sub-models. The weighting functions μ_i depend on unknown premise variables (state of a system), and verify:

$$\begin{cases} \sum_{i=1}^{r} \mu_i(x(k)) = 1 \\ 0 \leqslant \mu_i(x(k)) \leqslant 1 \;\; \forall i \in \{1, ..., r\} \end{cases} \quad (3)$$

Few works can be found concerning the class of nonlinear system with the assumption of unknown premise variables but only for nonlinear continuous time systems.

3. OBSERVER DESIGN PROCEDURE

3.1 Structure of the multiple observer

The matrices A_i are decomposed into:

$$A_i = A_0 + \overline{A}_i \quad (4)$$

where A_0 is defined by:

$$A_0 = \frac{1}{r}\sum_{i=1}^{r} A_i \quad (5)$$

By substituting (4) in the equation of the multiple model (1) we obtain:

$$x(k+1) = A_0 x(k) + \sum_{i=1}^{r} \mu_i(x(k))(\overline{A}_i x(k) + B_i u(k)) \quad (6)$$

$$y(k) = Cx(k) \quad (7)$$

Based on this model, the following multiple observer is proposed:

$$\begin{aligned}\hat{x}(k+1) = A_0\hat{x}(k) + \sum_{i=1}^{r} \mu_i(\hat{x}(k))(\overline{A}_i\hat{x}(k) + B_i u(k) \\ + G(y(k) - \hat{y}(k))) \quad (8)\\ \hat{y}(k) = C\hat{x}(k) \quad (9)\end{aligned}$$

3.2 Observer Design

In this section, the following assumption is made:
Assumption 1. Suppose that the weighting functions are Lipschitz and verify:

$$\begin{cases} |\mu_i(x)x - \mu_i(\hat{x})\hat{x}| \leqslant \gamma_{1i}\,|x - \hat{x}| \\ |\mu_i(x)u - \mu_i(\hat{x})u| \leqslant \gamma_{2i}\,|x - \hat{x}| \end{cases}$$

with $\gamma_{2i} = M_i\rho$, where M_i is the Lipschitz constant of the weighting function and ρ the upper bound of the input $u(t)$.
The observer error is given by:

$$e(k) = x(k) - \hat{x}(k) \quad (10)$$

and its dynamic is described by:

$$e(k+1) = \Phi e(k) + \sum_{i=1}^{r} \overline{A}_i\delta_i(k) + B_i\Delta_i(k) \quad (11)$$

where:

$$\begin{cases} \delta_i = \mu_i(x)x - \mu_i(\hat{x})\hat{x} \\ \Delta_i = (\mu_i(x) - \mu_i(\hat{x}))u \\ \Phi = A_0 - GC \end{cases} \quad (12)$$

Theorem 1. The state estimation error between the multiple model (1)-(2) and the multiple observer (8)-(9) converges globally asymptotically toward zero, if there exist a matrix $P = P^T > 0$, gain matrix K and positive scalars τ, ε_1,ε_2 and ε_3 such that the following conditions hold for $i = 1, \ldots, r$:

$$\begin{bmatrix} \Theta_i & \Gamma^T & \Gamma^T & \Gamma^T & \overline{A}_i^T P \\ \Gamma & -rP & 0 & 0 & 0 \\ \Gamma & 0 & -r\varepsilon_1 I & 0 & 0 \\ \Gamma & 0 & 0 & -r\varepsilon_2 I & 0 \\ P\overline{A}_i & 0 & 0 & 0 & -\frac{\varepsilon_3}{r\gamma_{2i}^2} I \end{bmatrix} < 0 \quad (13)$$

where:

$$\begin{aligned}\Theta_i &= -r^{-1}P + \tau\gamma_{2i}^2 I + \gamma_{1i}^2(r\varepsilon_1 + 1)\overline{A}_i^T\overline{A}_i \\ &\quad + \gamma_{1i}^2(r-1)\overline{A}_i^T P\overline{A}_i \\ \Gamma &= P\Phi \quad (14)\end{aligned}$$

and:

$$(r\varepsilon_2 + r\varepsilon_3)B_i^T B_i + rB_i^T PB_i - \tau I < 0 \quad (15)$$

The observer gain is given by $G = P^{-1}K$.

Proof. To prove the convergence of the estimation error toward zero, let us consider the following quadratic function of Lyapunov :

$$V(e(k)) = e(k)^T Pe(k), P = P^T > 0 \quad (16)$$

The variation of V along the trajectory of (11) is given by:

$$\Delta V = e(k+1)^T Pe(k+1) - e(k)^T Pe(k) \quad (17)$$

and by using (11): (The time k is omitted for sake of brevity)

$$\begin{aligned}\Delta V = (\Phi e + \sum_{i=1}^{r} \overline{A}_i\delta_i + B_i\Delta_i)^T P(\Phi e + \sum_{i=1}^{r} \overline{A}_i\delta_i \\ + B_i\Delta_i) - e^T Pe \quad (18)\end{aligned}$$

$$\begin{aligned}\Delta V &= e^T\Phi^T P\Phi e + \sum_{i=1}^{r} e^T\Phi^T P\overline{A}_i\delta_i + \sum_{i=1}^{r} e^T\Phi^T PB_i\Delta_i \\ &+ \sum_{i=1}^{r}\delta_i^T\overline{A}_i^T P\sum_{j=1}^{r}\overline{A}_j\delta_j + \sum_{i=1}^{r}\delta_i^T\overline{A}_i^T P\Phi e \\ &+ \sum_{i=1}^{r}\delta_i^T\overline{A}_i^T P\sum_{j=1}^{r} B_j\Delta_j + \sum_{i=1}^{r}\Delta_i^T B_i^T P\Phi e \\ &+ \sum_{i=1}^{r}\Delta_i^T B_i^T P\sum_{j=1}^{r}\overline{A}_j\delta_j + \sum_{i=1}^{r}\Delta_i^T B_i^T P\sum_{j=1}^{r} B_j\Delta_j \\ &- e^T Pe \quad (19)\end{aligned}$$

Lemma 1. For any matrices X and Y with appropriate dimensions, the following property holds for any positive scalar ε :

$$X^T Y + Y^T X < \varepsilon X^T X + \varepsilon^{-1} Y^T Y \quad (20)$$

To reduce a double sum into a simple one, we proceed as follows :

$$\sum_{i=1}^{r} X_i^T \sum_{j=1}^{r} X_j = \sum_{i=1}^{r} X_i^T X_i + \sum_{i=1}^{r} X_i^T \sum_{\substack{j=1 \\ j\neq i}}^{r} X_j \quad (21)$$

Applying lemma 1 and taking $\varepsilon = 1$, thus we obtain :

$$\sum_{i=1}^{r} X_i^T \sum_{\substack{j=1\\ j\neq i}}^{r} X_j \leqslant (r-1)\sum_{i=1}^{r} X_i^T X_i \tag{22}$$

The inequality (21) becomes:

$$\sum_{i=1}^{r} X_i^T \sum_{j=1}^{r} X_j \leqslant r\sum_{i=1}^{r} X_i^T X_i \tag{23}$$

by using (23), ΔV can be reduced as follows:

$$\begin{aligned}\Delta V \leqslant\, & e^T\Phi^T P\Phi e + \varepsilon_1^{-1} e^T\Phi^T PP\Phi e \\ & + r\varepsilon_1 \sum_{i=1}^{r} \delta_i^T \overline{A}_i^T \overline{A}_i \delta_i + \varepsilon_2^{-1} e^T \Phi^T PP\Phi e \\ & + r\varepsilon_2 \sum_{i=1}^{r} \Delta_i^T B_i^T B_i \Delta_i + \sum_{i=1}^{r} \delta_i^T \overline{A}_i^T P \sum_{j=1}^{r} \overline{A}_j \delta_j \\ & + \varepsilon_3^{-1} \sum_{i=1}^{r} \delta_i^T \overline{A}_i^T PP \sum_{j=1}^{r} \overline{A}_j \delta_j \\ & + \varepsilon_3 \sum_{i=1}^{r} \Delta_i^T B_i^T \sum_{j=1}^{r} B_j \Delta_j \\ & + \sum_{i=1}^{r} \Delta_i^T B_i^T P \sum_{j=1}^{r} B_j \Delta_j - e^T P e \end{aligned} \tag{24}$$

ΔV can be re-written as:

$$\Delta V \leqslant e^T \Psi_1 e + \sum_{i=1}^{r} \delta_i^T \Psi_{2i} \delta_i + \sum_{i=1}^{r} \Delta_i^T \Psi_{3i} \Delta_i \tag{25}$$

where:

$$\Psi_1 = \Phi^T P\Phi + \varepsilon_1^{-1}\Phi^T PP\Phi + \varepsilon_2^{-1}\Phi^T PP\Phi - P \tag{26}$$

$$\Psi_{2i} = r\varepsilon_1 \overline{A}_i^T \overline{A}_i + r\overline{A}_i^T P\overline{A}_i + \varepsilon_3^{-1} r\overline{A}_i^T PP\overline{A}_i \tag{27}$$

and:

$$\Psi_{3i} = r(\varepsilon_1 + \varepsilon_2) B_i^T B_i + rB_i^T PB_i \tag{28}$$

Lipschitz property of δ_i in assumption 1 gives:

$$\sum_{i=1}^{r} \delta_i^T \delta_i \leqslant \sum_{i=1}^{r} \gamma_{1i}^2 e^T e \tag{29}$$

Using (23) and (29) we obtain:

$$\begin{aligned}\sum_{i=1}^{r} \delta_i^T \Psi_{2i} \delta_i < & \sum_{i=1}^{r} e^T (\gamma_{1i}^2 r\varepsilon_1 \overline{A}_i^T \overline{A}_i \\ & + \gamma_{1i}^2 r\overline{A}_i^T P\overline{A}_i + \gamma_{1i}^2 \varepsilon_3^{-1} r\overline{A}_i^T PP\overline{A}_i) e \end{aligned} \tag{30}$$

We have also:

$$\begin{aligned}\sum_{i=1}^{r} \Delta_i^T \Psi_{3i} \Delta_i < & (r\varepsilon_2 + r\varepsilon_3) \sum_{i=1}^{r} (\Delta_i^T B_i^T B_i \Delta_i) \\ & + r\sum_{i=1}^{r} (\Delta_i^T B_i^T PB_i \Delta_i) \end{aligned} \tag{31}$$

Lipschitz property of Δ_i in assumption 1 gives:

$$\sum_{i=1}^{r} \Delta_i^T \Delta_i \leqslant \sum_{i=1}^{r} \gamma_{2i}^2 e^T e \tag{32}$$

that can be written in the form:

$$-\sum_{i=1}^{r} \gamma_{2i}^2 e^T e + \sum_{i=1}^{r} \Delta_i^T \Delta_i \leqslant 0 \tag{33}$$

Applying the $\mathcal{S}$-procedure:

$$\Delta V < \Delta V - \tau\Gamma \tag{34}$$

with:

$$\Gamma = -\sum_{i=1}^{r} \gamma_{2i}^2 e^T e + \sum_{i=1}^{r} \Delta_i^T \Delta_i \leqslant 0 \tag{35}$$

we obtain :

$$\Delta V < \sum_{i=1}^{r} e^T \Omega_{1i} e + \sum_{i=1}^{r} \Delta_i^T \Omega_{2i} \Delta_i \tag{36}$$

where:

$$\begin{aligned}\Omega_{1i} = & r^{-1}\Phi^T P\Phi + r^{-1}\varepsilon_1^{-1}\Phi^T PP\Phi \\ & + r^{-1}\varepsilon_2^{-1}\Phi^T PP\Phi - r^{-1}P + \tau\gamma_{2i}^2 I \\ & + \gamma_i^2 r\varepsilon_1 \overline{A}_i^T \overline{A}_i + \gamma_i^2 r\overline{A}_i^T P\overline{A}_i \\ & + \gamma_i^2 \varepsilon_3^{-1} r\overline{A}_i^T PP\overline{A}_i \end{aligned} \tag{37}$$

and :

$$\Omega_{2i} = (r\varepsilon_2 + r\varepsilon_3) B_i^T B_i + rB_i^T PB_i - \tau I \tag{38}$$

The negativity of ΔV is guaranteed if:

$$\Omega_{1i} < 0 \tag{39}$$

$$\Omega_{2i} < 0 \tag{40}$$

$$i \in \{1, \ldots, r\}$$

We use the Schur complement on (39), and we perform a change of variable $K = PG$ to eliminate the nonlinearity between P and G. Then, we obtain the inequalities expressed in theorem 1.

4. $\mathcal{L}_2$ APPROACH

In this section a new method to synthesize an observer is proposed. It is based on the $\mathcal{L}_2$ techniques. The state estimation error (11) can be written as a perturbed system:

$$e(k+1) = \Phi e(k) + H\omega(k) \tag{41}$$

where :

$$\begin{cases} \Phi = A_0 - GC \\ H = [H_1 \ \ \ldots \ \ H_r] \\ \omega = [v_1^T \ \ \ldots \ \ v_r^T]^T \\ H_i = \left[\, \overline{A}_i \ \ B_i \,\right] \\ v_i = \left[\, \delta_i^T \ \ \Delta_i^T \,\right]^T \end{cases}$$

Theorem 2. The state estimation error between the multiple model (1)-(2) and the multiple observer (8)-(9) converges globally asymptotically toward zero, if there exists matrices $P = P^T > 0$ and K such that the following conditions hold :

$$\begin{bmatrix} -P+I & \Psi_1 & \Psi_2 \\ \Psi_1^T & H^TPH-\tilde{\gamma}I & 0 \\ \Psi_2^T & 0 & -P \end{bmatrix} < 0 \quad (42)$$

where:

$$\Psi_1 = (A_0^T P - C^T K^T)H \quad (43)$$

$$\Psi_2 = A_0^T P - C^T K^T \quad (44)$$

The observer gains are given by $G = P^{-1}K$.

Proof. To show the convergence of the estimation error toward zero, let us consider the following quadratic Lyapunov function :

$$V(e(k)) = e(k)^T P e(k), P = P^T > 0 \quad (45)$$

The observer converges and the $\mathcal{L}_2$-gain from $\omega(k)$ to e(k) is bounded by γ if the following holds :

$$\Delta V(e) + e(k)^T e(k) - \gamma^2 \omega(k)^T \omega(k) < 0 \quad (46)$$

Then, by using (41):

$$\Delta V(e) = e^T \Phi^T P \Phi e - e^T P e + e^T \Phi^T P H \omega$$
$$+ \omega^T H^T P \Phi e + \omega^T H^T P H \omega \quad (47)$$

Inequality (46) can then be written in the following way :

$$e^T \Phi^T P \Phi e - e^T P e + e^T \Phi^T P H \omega + \omega^T H^T P \Phi e$$
$$+ \omega^T H^T P H \omega + e^T e - \gamma^2 \omega^T \omega < 0 \quad (48)$$

That can be expressed under the following form :

$$\begin{bmatrix} e \\ \omega \end{bmatrix}^T \mathcal{M} \begin{bmatrix} e \\ \omega \end{bmatrix} < 0 \quad (49)$$

where :

$$\mathcal{M} = \begin{bmatrix} \Phi^T P \Phi - P + I & \Phi^T P H \\ H^T P \Phi & H^T P H - \gamma^2 I \end{bmatrix} \quad (50)$$

Inequalities (50) are not linear because of the product PG ($\Phi = A_0 - GC$) and γ^2. This problem can be solved by using the changes of variables $K = PG$ and $\tilde{\gamma} = \gamma^2$. And after resolution of the LMIs (50), the observer gains are computed by $G = P^{-1}K$ and the $\mathcal{L}_2 - gain$ from $\omega(t)$ to $e(t)$ is computed by $\gamma = \sqrt{\tilde{\gamma}}$.

5. SIMULATION RESULTS

5.1 Comparison between theorems 1 and 2

To illustrate the interest of the $\mathcal{L}_2$ method, a comparison between the two methods is performed on the following example with variable parameters a and b:

$$A_1 = \begin{bmatrix} a & -0.3 \\ 0 & -0.5 \end{bmatrix}, A_2 = \begin{bmatrix} 0.4 & 0.1 \\ -0.2 & b \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The weighting functions are

$$\begin{cases} \mu_1(x) = \dfrac{1-\tanh(x_1)}{2} \\ \mu_2(x) = 1 - \mu_1(x) = \dfrac{1+\tanh(x_1)}{2} \end{cases} \quad (51)$$

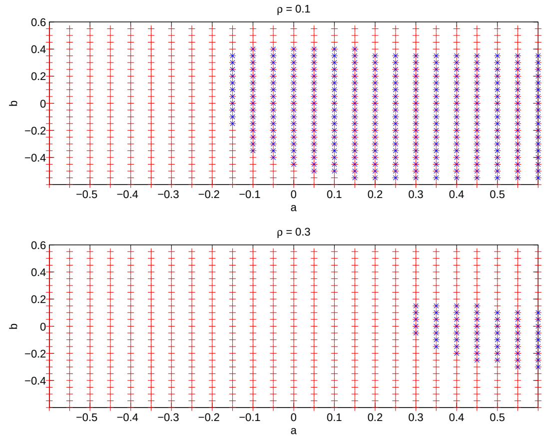

Fig. 1. Comparison between the use of theorem 1 (x) and theorem 2 (+).

Figure 1 shows the set of solutions of LMIs of theorems 1 and 2 according to different values of the parameters a, b and ρ (ρ represents the upper bound of the input of the system). In Figure 1 (top), we set $\rho = 0.1$, and the parameters a and b of the system vary in $[-0.6, 0.6]$. We noticed that the $\mathcal{L}_2$ method provides solutions whatever the values of a and b, whereas the resolution of LMIs in Theorem 1 provides solutions only for some values of a and b. In Figure 1 (bottom), we remark that increasing the value of ρ leads to decreasing the solution set of the LMIs in theorem 1, while the solution set of the LMIs in theorem 2 remains unchanged. It can be concluded, for the proposed example, that the $\mathcal{L}_2$ method is less conservative compared to the method using the theorem 1.

5.2 State estimation with $\mathcal{L}_2$ approach

We consider the previous example with $a = -0.6$ and $b = 0.1$, to show the advantages of the using proposed $\mathcal{L}_2$ observer. A stable observer with $\mathcal{L}_2$ attenuation of the considered perturbation terms for the above system can be designed using Theorem 2. Conditions in Theorem 2 are satisfied with :

$$\mathrm{P} = \begin{bmatrix} 2.55 & -1.76 \\ -1.76 & 2.99 \end{bmatrix}, G = \begin{bmatrix} -0.18 \\ -0.27 \end{bmatrix}$$

Given the initial conditions $x(0) = [0.7 \quad -0.5]^T$, $\hat{x}(0) = [0 \quad 0]^T$, and the input signal in figure 2 (top), the simulation results (The reconstruction state errors) are illustrated in figure 3 and the variation of the weighting functions are illustrated in figure 2 (bottom). The advantages of this method compared to the first one are, on the one hand, the elimination of the assumption of Lipschitz on the weighting functions, which makes it possible to apply it to a more important class of nonlinear systems, and on the other hand, this method does not require the knowledge of the input bound of the system.

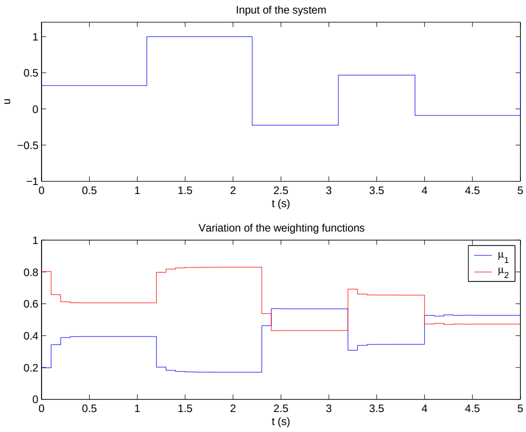

Fig. 2. Input of the system (top) and variation of the weighting functions (bottom)

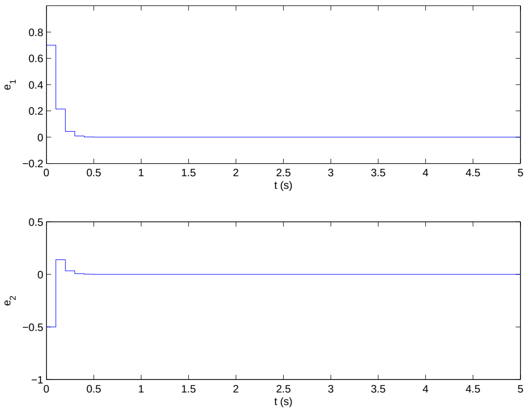

Fig. 3. Estimation error

6. CONCLUSION

In this paper, we have proposed a new method to design an observer for discrete time Takagi-Sugeno systems with unknown premise variables. This representation is very interesting because it can represent a large class of nonlinear systems compared to the representation with measurable premise variables. The estimation error is written like a perturbed system and conditions convergence of the observer are studied by using a quadratic Lyapunov candidate function. An other method using $\mathcal{L}_2$ design to attenuate the effect of the perturbations on the state estimation error is proposed. These conditions are expressed in LMI terms.

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