ELSEVIER Int. J. Miner. Process. 47 (1996) 125–140

INTERNATIONAL JOURNAL OF MINERAL PROCESSING

Observability of systems involving flow circulation

José Ragot *, Marie Luong, Didier Maquin

Centre de Recherche en Automatique de Nancy, Ecole Nationale Supérieure de Géologie de Nancy, 2, Avenue de la Forêt de Haye, 54516 Vandoeuvre lès Nancy, France

Received 24 May 1995; accepted 20 November 1995

Abstract

This paper presents an analysis of processes with regard to their circulating flows and mainly focuses on the characterization of variables depending on their redundancy level. Previous studies in this field were performed on processes described by linear equations (i.e. when considering a model of total conservation of material or energy). Here, the authors propose a novel methodology, permitting extension of observability analysis to processes involving linear and bilinear equations (i.e. partial and total balance equations). The aim here is the classification of process variables according to their observability level and, on a more practical side, the extraction of the subset of unmeasured variables deducible from those measured and from balance equations. The proposed technique is based on the analysis of the circuit matrix associated to the graph of the process under consideration. This analysis is performed depending on the number and position of the measurement devices.

1. Introduction

Values provided from sensors can be validated through comparison with redundant measurement values of the process under consideration. These redundant measurements can be obtained through spatial or temporal redundancy. Static redundancy equations and observability concepts were first utilised for measurement availability, in the mineral processing and chemical industries as well as for electrical distribution networks. The first studies (Ripps, 1962; Vaclavek, 1969) concerned data reconciliation using the now classical technique of production balance equilibration. In the following stages, this data reconciliation principle has been generalized to processes described by algebraic equations which are either linear (Crowe and Garcia Campos, 1983) or non

* Corresponding author.

0301-7516/96/$15.00
SSDI 0301-7516(95)00104-2

linear (Sood et al., 1979; Crowe, 1989). At the same time, rather than just establishing statistically coherent balances, data reconciliation went into use for more general applications. It was then applied to more fundamental problems such as detection and estimation of gross errors (Ragot et al., 1990), diagnosis and observability of systems (Kretsovalis and Mah, 1988a,b; Crowe, 1989; Ragot et al., 1990), optimization of sensors location (Maquin et al., 1989) and study of the reliability of a measurement system (Turbatte et al., 1991). Researchers from the artificial intelligence field also proposed knowledge-based systems approaches that diagnosed sensors along with other system components. In this paper we will present the principles for analysing the observability of variables that generate equations containing only redundant variables. Let us recall that a measured variable may be designed as redundant if it can be calculated uniquely from remaining variables. As previously mentioned, this redundancy generally leads to a discrepancy between the data and the equations which are to be reconciled, hence providing means to ascertain the reliability of a given set of measurements. The methods described in this paper are currently being applied in a project developing a monitoring and diagnosis system for chemical and mineralurgical units.

2. Formulation of the problem and definitions

Cost-wise, not every variable in a process may be measured. However, redundancy, due to an analytical model, permits estimation of the value of the variables from other measurements. It is clear that the ability to perform this estimation strongly depends on the structure of the process flowsheet and on the position of the measurement devices. It is, a priori, desirable to acknowledge if all variables of interest to the control of the process are observable. This is in fact, the aim of our paper and to do so we propose to address ourselves to the classification problems for processes described by linear and bilinear equations.

Consider a process involving flow circulation (material or energy) or a network which may be modelized by a graph. This graph is formed by n nodes (production units) and m arcs (streams ensuring circulation between the production units and also with the environment of the process). In this study, an arc i may be characterized by two variables X_i and Y_i, for example a volumic flowrate and density or otherwise a massic flowrate and mineral content, for example. The process flowsheet may also be modelized by algebraic equations derived from the law of mass and energy conservation. Here, these are restricted to linear and bilinear equations. For one node, these have the following structure:

— a linear constraint (for total flow balance):

$$\sum_{i=1}^{u} \alpha_i X_i = 0 \tag{1}$$

— a bilinear constraint (for partial flow balance):

$$\sum_{i=1}^{v} \beta_i X_i Y_i = 0 \tag{2}$$

where α_i and β_i are coefficients whose values depend on the directions of the flowrates in regard to the considered node (generally, with values 0, -1 and $+1$) and u and v correspond to the number of variables in each equation (u and v may be equal as is the case proposed in this paper).

These equations may be applied to each node of the flowsheet. In the following, let us note S_x and S_{xy} as the systems of equations respectively associated to the linear and bilinear balances. These two subsets form the system S, describing the model of the process, which may be written under a matrix form with:

$$M_x X = 0 \tag{3}$$

$$M_{xy}(X \cdot Y) = 0 \tag{4}$$

where $M_x(n_x, v_x)$ and $M_{xy}(n_y, v_y)$ are the so-called node incidence matrices of the systems S_x and S_{xy}. The vectors $X(v_x,1)$ and $Y(v_y,1)$ are formed from variables X_i and Y_i and the operator $\cdot$ is used for the product of two vectors entry by entry (the ith element of the vector $X \cdot Y$ is the product of the corresponding elements of X and Y).

It is also possible to represent a flowsheet by the cycles of its associated graph. These are gathered in the so-called cycle matrix (a cycle is a closed path connecting several nodes). When the orientation of the circuits is not taken into account, the (i,j)th entry of the cycle matrix is assigned "1" if the jth edge occurs in the ith circuit and "0" otherwise. Let us now define:

C_x as the cycle matrix of the graph associated with system S_x, and

C_{xy} as the cycle matrix of the graph associated with the system S_{xy}.

In certain applications, the matrices M_x and M_{xy} are identical (each node is described simultaneously by a linear and a bilinear equation) and consequently the cycle matrices C_x and C_{xy} are also identical. In other cases, the linear and bilinear parts of the system are not necessarily characterised by the same incidence matrix. This is the case of processes characterised by two different graphs, a first representing the flow conservation, and a second for the energy conservation.

Lastly, the process is characterised by a set of measurements with $\{X_m\}$ and $\{Y_m\}$, lists containing all measured variables.

2.1. Purpose

The analysis of the system previously described deals with the determination of the observable variables $\{X_{ob}\}$ and $\{Y_{ob}\}$, based on the knowledge of the measurements $\{X_m\}$ and $\{Y_m\}$. The observable variables comprise measured variables and variables which are unmeasured but deducible. A secondary aim concerns the search for the unobservable variables i.e. those which are unmeasured and undeducible. The proposed method is based on the novel observability rules presented in section 3 and illustrated in sections 4 and 5.

This analysis has, in particular, lead to the validation of processes, using the knowledge of the measured–deducible variables. This can be achieved by studying the analytical redundancy. This estimation problem is generally known as measurement reconciliation procedure and has received a considerable amount of attention during the last two decades. However, it is clear that estimation may be achieved only if the estimation equations have been generated and the observable variables extracted. This is the aim of our contribution. An important consequence is the use of redundancy equations in measurement fault detection or process malfunction detection. For such a purpose, the residuals generated by the redundancy equations are analysed with respect to their magnitude, in order to detect failures.

3. Observability rules

As previously pointed out, due to technical and economic constraints, it is not possible to obtain measurements of each process variable. The rules given in Table 1, based on the cycle matrix analysis, are well adapted to establish the conditions under which certain unmeasured variables may or may not be deduced. These are merely illustrated (section 4) without a demonstration, except for rule 2, and with examples in order to understand their application.

Before going on to section 4, as an example, let us demonstrate the establishment of the simplest rule, rule 2. Fig. 1 presents a flowsheet with 5 nodes and 7 streams where streams 1 and 2 are measured.

Demonstration, part I. The dashed line, cutting the flowsheet into two parts, S_1 and S_2, which defines the subset C formed by arcs 1, 2 and 3 is called a cutset of the graph. A balance around the S_1 (or S_2) node is the algebraic sum of flows on the arcs of the cutset C such that:

$$a_1 x_1 + a_2 x_2 + a_3 x_3 = 0$$

where $a_i = \pm 1$ depending on directions of the arcs. The missing value of arc 3 can be calculated through the overall balance if and only if Eq. 1 contains exactly one unknown variable. In others words, a necessary condition for x_3 to be deducible through the balance equation is that all the arcs of the cutset, except arc 3, have measured variables.

Demonstration, part II. Considering the subsets S_1 and S_2 and the arcs between these subsets, one end node of an arc between S_1 and S_2 belongs to S_1 and the other to S_2. Indeed, the arc 3 necessarily lies on a cycle C_0 with at least one node belonging to S_1 and another to S_2. As a cycle is a closed path, the considered cycle C_0 has at least another arc joining a node of S_1 to a node of S_2. This arc necessarily has a measured variable.

This analysis is extended to all possible cutsets of the graph and consequently rule 2

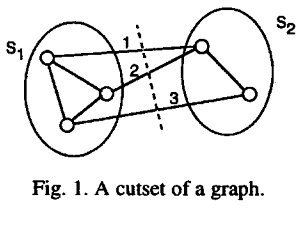

Fig. 1. A cutset of a graph.

Table 1
Rules for observability study

Rule	Statement	Comment
1	An unmeasured variable X_j is not observable if, and only if, it belongs, at least, to a cycle of C_{xy} where no variable X is measured and not more than one variable Y is measured.	This rule is restricted to the case where the matrices C_x and C_{xy} are identical.
2	An unmeasured variable X_j is observable if it belongs to cycles of C_x with a minimum of one measured variable X (observability rule for linear system).	This rule permits determination of the observable variables X_j by taking into account the system S_x only.
3	In a graph where the variables Y are measured, an unmeasured variable X_j is observable if, and only if, it belongs to all cycles of C_{xy}, without a measurement in X, simultaneously with a same variable X_j.	This rule allows the determination of the observable variables X by taking into account the systems of equations S_x and S_{xy}.
4a	An unmeasured variable Y_j is unobservable if, and only if it belongs to a cycle of C_{xy} where no variable Y is measured.	This rule may be applied even if the matrices C_x and C_{xy} are different.
4b	An unmeasured variable Y_j is unobservable if, and only if it belongs to a cycle of C_{xy} where no variable X is measured and not more than one variable Y is measured.	This rule is restricted to the case where the matrices C_x and C_{xy} are identical.
5a	In a graph where the cycles of C_{xy} have at least one measured variable X, one unmeasured variable Y_j is observable if, and only if, it solely belongs to cycles of C_{xy} with a minimum of one measured variable Y.	This rule is restricted to the case where the matrices C_x and C_{xy} are identical.
5b	In a graph where the cycles of C_{xy} have variables X which solely belong in cycles of C_x with a minimum of one measured variable X, one unmeasured variable Y_j is observable if, and only if, it solely belongs to cycles of $C_{xy} C_x$ with a minimum of one measured variable Y.	This rule may be applied even if the matrices C_x and C_{xy} are different.

is established such that an unmeasured variable X_i is observable if and only if it belongs to a cycle of the graph with a minimum of one measured variable X_j.

4. Examples

In order to illustrate the method, let us consider a system extracted from a refinery plant situated at Grandpuits in France. This system, presented in the Fig. 2, is composed of a pair of twinned heat exchangers. In these exchangers, the crude oil is warmed up by

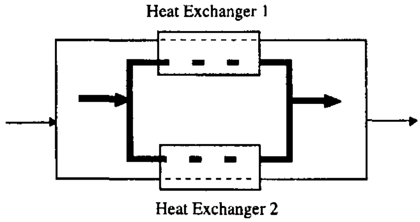

Fig. 2. Flowsheet of a process.

the hot refined products. The system modelling is characterized by the relations of mass, thermal and enthalpy balances. The variables involved are flowrate and temperature. Indeed, the temperature is considered as could be the enthalpy. The latter can be computed from the temperature. Extra data is not given here to lighten the notation.

In the representation using a graph model, this system is characterized by 12 streams and 6 nodes (Fig. 3), corresponding respectively to 24 variables (X and Y) and 12 linear and bilinear equations.

The balance relationships of this process are written as:

$$x_1 - x_2 - x_4 = 0 \quad (1.1)$$

$$x_4 + x_9 - x_5 - x_{11} = 0 \quad (1.2)$$

$$x_2 + x_8 - x_{10} - x_3 = 0 \quad (1.3)$$

$$x_3 + x_5 - x_6 = 0 \quad (1.4)$$

$$x_7 - x_8 - x_9 = 0 \quad (1.5)$$

$$x_{10} + x_{11} - x_{12} = 0 \quad (1.6)$$

$$x_1 \cdot y_1 - x_2 \cdot y_2 - x_4 \cdot y_4 = 0 \quad (1.7)$$

$$x_4 \cdot y_4 + x_9 \cdot y_9 - x_5 \cdot y_5 - x_{11} \cdot y_{11} = 0 \quad (1.8)$$

$$x_2 \cdot y_2 + x_8 \cdot y_8 - x_{10} \cdot y_{10} - x_3 \cdot y_3 = 0 \quad (1.9)$$

$$x_3 \cdot y_3 + x_5 \cdot y_5 - x_6 \cdot y_6 = 0 \quad (1.10)$$

$$x_7 \cdot y_7 - x_8 \cdot y_8 - x_9 \cdot y_9 = 0 \quad (1.11)$$

$$x_{10} \cdot y_{10} + x_{11} \cdot y_{11} - x_{12} \cdot y_{12} = 0 \quad (1.12)$$

Notice that the relations from (1.1) to (1.6) are the mass balances. In particular, the mass balances (1.1) and (1.5) are written at the point of separation of flows, while the

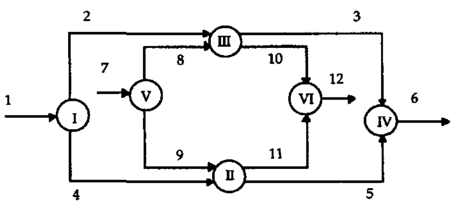

Fig. 3. Network of a process.

mass balances (1.4) and (1.6) involve the mixing of the flows at the outputs of the exchangers. The relations (1.7), (1.10), (1.11) and (1.12) are the thermal balances. Finally, the relations (1.9) and (1.8) are the enthalpy balances around the heat exchangers 1 and 2 respectively. The sensors measuring the variables X and Y are denoted by

Table 2
Matrix C of all the cycles

2	1	3	4	7	10	5	6	8	9	11	12	
1	.	1	1	.	.	1	.	.	.	.	.	1
1	1	1	.	.	.	.	1	.	.	.	.	2
1	1	.	.	1	.	.	.	1	.	.	.	3
.	1	.	1	1	.	.	.	.	1	.	.	4
1	.	.	1	.	1	.	.	.	.	1	.	5
1	1	.	.	.	1	.	.	.	.	.	1	6
.	1	.	1	.	.	1	1	.	.	.	.	7
.	1	1	1	1	.	1	.	1	.	.	.	8
1	1	1	.	1	.	1	.	.	1	.	.	9
.	.	1	.	.	1	1	.	.	.	1	.	10
.	1	1	1	.	1	1	.	.	.	.	1	11
.	.	1	.	1	.	.	1	1	.	.	.	12
1	.	1	1	1	.	.	1	.	1	.	.	13
.	1	1	1	.	1	.	1	.	.	1	.	14
.	.	1	.	.	1	.	1	.	.	.	1	15
1	.	.	1	.	.	.	.	1	1	.	.	16
.	1	.	1	1	1	.	.	1	.	1	.	17
.	.	.	.	1	1	.	.	1	.	.	1	18
1	1	.	.	1	1	.	.	.	1	1	.	19
1	.	.	1	1	1	.	.	.	1	.	1	20
.	1	.	1	.	.	.	.	.	.	1	1	21
1	.	.	1	1	.	1	1	1	.	.	.	22
.	.	.	.	1	.	1	1	.	1	.	.	23
1	1	.	.	.	1	1	1	.	.	1	.	24
1	.	.	1	.	1	1	1	.	.	.	1	25
.	.	1	.	.	.	1	.	1	1	.	.	26
.	.	1	.	1	1	1	.	.	1	.	1	27
1	1	1	.	.	.	1	.	.	.	1	1	28
.	1	1	1	.	.	.	1	1	1	.	.	29
.	.	1	.	1	1	.	1	.	1	1	.	30
1	.	1	1	.	.	.	1	.	.	1	1	31
.	.	.	.	.	1	.	.	1	1	1	.	32
.	1	.	1	.	1	.	.	1	1	.	1	33
1	.	.	1	1	.	.	.	1	.	1	1	34
.	.	.	.	1	.	.	.	.	1	1	1	35
1	1	.	.	.	.	1	1	1	1	.	.	36
.	.	.	.	1	1	1	1	1	.	1	.	37
.	.	.	.	.	.	1	1	.	.	1	1	38
.	.	1	.	1	.	1	.	1	.	1	1	39
1	1	.	.	.	.	.	.	1	1	1	1	40
.	.	.	.	.	1	1	1	1	1	.	1	41
.	.	1	.	.	.	.	1	1	1	1	1	42

filled rectangles and circles respectively. In this example, as the matrices M_x and M_{xy} are identical, the cycle matrices C_x and C_{xy} are also identical and defined by C. This cycle matrix is presented in Table 2. Its first line denotes the variables and its last column the cycle number. This matrix consists of two parts. The first contains the fundamental cycles (for example, the first is composed of streams 2, 3, 4 and 5) and the second all the cycles obtained by aggregation (in deleting common arcs) of the fundamental cycles (aggregation of two cycles, three cycles,...). Hence, as an example, the 7th line results from the aggregation of cycles 1 and 2. The reader is invited to verify that the matrix of fundamental cycles may be obtained by a simple transformation of the incidence matrix of the flowsheet (Mah et al., 1976).

Different situations are now presented and analysed in order to apply and illustrate the observability rules of Table 1. In the following examples, the list of measured variables and the list of unmeasured variables are respectively denoted as L_m and $L_{\bar{m}}$.

Remark: If the flowsheet is modified, a new cycle matrix can easily be obtained from the previous cycle matrix. The fusion of two nodes corresponds to the elimination of their adjacent streams. For the cycle matrix, this would imply the suppression of the columns corresponding to these streams.

4.1. Using rule 1

In this example (Fig. 4), the measured variables X and Y are gathered in the lists below:

$$X_m = [x_2, x_5, x_7] \quad Y_m = [y_1, y_2, y_3, y_5, y_6, y_7, y_{10}]$$

Variables X_i, for which analysis of observability is desired, are defined using the list:

$$X_{\bar{m}} = [x_1, x_3, x_4, x_6, x_8, x_9, x_{10}, x_{11}, x_{12}]$$

In order to apply rule 1, let us first determine the cycles without measurement X. According to Table 1, these are cycles 14, 15, 29, 42, 21, 32 and 33, and are respectively made up of streams (1, 3, 4, 6, 10, 11), (3, 6, 10, 12), (1, 3, 4, 6, 8, 9), (3, 6, 8, 9, 11, 12), (1, 4, 11, 12), (8, 9, 10, 11) and (1, 4, 8, 9, 10, 12). Among these cycles, those containing no more than one Y measurement are (1, 4, 11, 12), (8, 9, 10, 11). Thus, one may conclude from rule 1 that the unmeasured variables $X_{\bar{m}}$ belonging to these cycles are unobservable and:

$$X_{o\bar{b}} = [x_1, x_4, x_8, x_9, x_{10}, x_{11}, x_{12}]$$

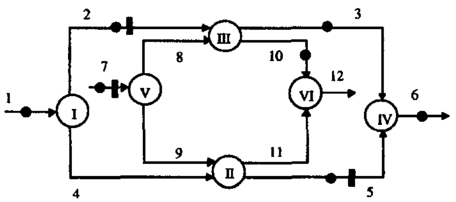

Fig. 4. Flowsheet of a process and associated measurement positions.

4.2. Using rule 2

Two measurements x_4 and x_8 are added (Fig. 5) to the previous list X_m and the measured variables X and Y are stored in the following lists:

$$X_m = [x2, x4, x5, x7, x8] \quad Y_m = [y_1, y_2, y_3, y_5, y_6, y_7, y_{10}]$$

Here we aim to study the observability of the variables:

$$X_{\bar{m}} = [x_1, x_3, x_6, x_9, x_{10}, x_{11}, x_{12}]$$

Consequently, it is necessary to determine the cycles with a minimum of one measured variable X. It is useful to note that the variables X, which do not appear in cycles without a measured variable X, are in fact variables which only appear in cycles with a minimum of one measured variable X. Consequently, for this example, it is more convenient (in order to apply rule 2) to detect cycles without X measurement. Thus only cycle 15 formed of streams (3, 6, 10, 12) is concerned. According to rule 2, the following observable variables in $X_{\bar{m}}$ may be selected and:

$$X_{ob} = [x_1, x_9, x_{11}]$$

4.3. Using rule 3

As for rule 2, the identical measured variables are used such that:

$$X_m = [x_2, x_4, x_5, x_7, x_8] \quad Y_m = [y_1, y_2, y_3, y_5, y_6, y_7, y_{10}]$$

The unmeasured variables X and Y are stored in the following lists:

$$X_{\bar{m}} = [x_1, x_3, x_6, x_9, x_{10}, x_{11}, x_{12}] \quad Y_{\bar{m}} = [y_4, y_8, y_9, y_{11}, y_{12}]$$

Our aim is to determine the observable variables X using rule 3.

First, we look for variables belonging to cycles within which a minimum of two variables Y are measured. When analysing the cycles of Table 2, it is more convenient to list the cycles with no more than one measurement in Y. These are cycles 16, 21, 32 and 35 respectively formed of streams (2, 4, 8, 9), (1, 4, 11, 12), (10, 8, 9, 11) and (7, 9, 11, 12). These cycles do not contain variables 3, 5 and 6 and consequently, this result proves that the variables (3, 5, 6) only belong to cycles with a minimum of two Y measurements.

From a practical point of view, these cycles are obtained from Table 2 in which the

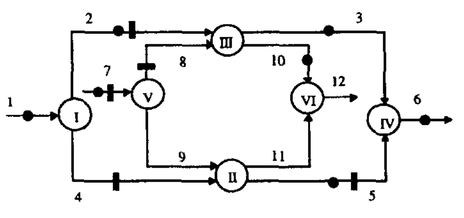

Fig. 5. Flowsheet of a process and its measurement positions.

Table 3
Cycles of the reduced graph

3	5	6	
1	1	.	1
1	.	1	2
.	1	1	7

rows corresponding to streams (1, 2, 4, 7, 8, 9, 10, 11, 12) are suppressed. The resulting submatrix, given in Table 3 only contains cycles 1, 2 and 7 of Table 2 (one may note that this matrix corresponds to the initial flowsheet reduced to node IV):

In the reduced graph of Table 3, applying rule 3 requires the determination of cycles without measurement X (i.e. containing only unmeasured variables). According to the entries of list X_m, merely cycle 2 is made up of streams 3 and 6. Thus from rule 3, one may deduce the list of observable X variables such that:

$$X_{ob} = [x_3, x_6]$$

4.4. Using rule 4

In this example (Fig. 6), the measured variables X and Y are stored in the lists below:

$$X_m = [x_1, x_4, x_{11}, x_{12}] \quad Y_m = [y_1, y_2, y_5, y_6, y_7]$$

and we aim to distinguish the unobservable Y variables. The unmeasured variables, Y, are stored in list: $Y_{\bar{m}} = [y_3, y_4, y_8, y_9, y_{10}, y_{11}, y_{12}]$

According to rule 4a, the unmeasured variables, Y, which belong to cycles without measurement Y are unobservable. Merely cycle (32) is found without Y measurement (made up of streams (8, 9, 10, 11)) and therefore, one may claim that y_8, y_9, y_{10} and y_{11} are unobservable.

Applying rule 4b requires the determination of cycles without measurement X. The analysis of Table 2 shows that cycles 12, 23 and 26 respectively made up of streams (3, 6, 7, 8), (5, 6, 7, 9) and (3, 5, 8, 9) have no measurements in X. Among these cycles, merely cycle 26 contains one or less than one measurement Y. This cycle is made up of streams (3, 5, 8, 9). Therefore, with variable 5 being measured, one may conclude from

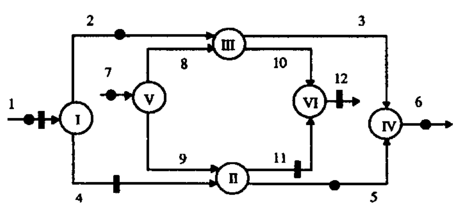

Fig. 6. Flowsheet of a process and its measurement positions.

rule 4b that the unmeasured variables y_3, y_8, y_9 are unobservable. Following rules 4a and 4b, the unmeasured unobservable variables are:

$$Y_{\overline{\text{ob}}} = [y_3, y_8, y_9, y_{10}, y_{11}]$$

4.5. Using rule 5

As in previous examples:

$$X_{\text{m}} = [x_1, x_4, x_{11}, x_{12}] \quad Y_{\text{m}} = [y_1, y_2, y_5, y_6, y_7]$$

The aim is to analyse the observability of the variables Y:

$$Y_{\overline{\text{m}}} = [y_3, y_4, y_8, y_9, y_{10}, y_{11}, y_{12}]$$

The first step deals with the determination of a network comprised of cycles with a minimum of one measured variable X. Firstly, it would be more convenient to determine the cycles without an X measurement. These cycles (lines 12, 23 and 26 of Table 2) are respectively defined by streams (3, 6, 7, 8), (5, 6, 7, 9) and (3, 5, 8, 9). It is now possible to obtain a graph comprised of cycles with a minimum of one measured variable X. For this purpose, one deletes the streams belonging to cycles with no measurement X (3, 5, 6, 7, 8, 9). The nodes connecting to these streams are then aggregated. When considering the cycle matrix, the suppression of the above streams yields a reduced matrix (Table 4). This reduced matrix is obtained from Table 2 by deleting columns 3, 5, 6, 7, 8 and 9. In this matrix, measured Y variables are presented in bold characters.

The observability of the unmeasured variables y_4, y_{10}, y_{11} and y_{12} may now be studied. Analysing the cycles of Table 4, we seek cycles with a minimum of one

Table 4
Cycles resulting from streams deletion

2	**1**	4	10	11	12	
1	.	1	.	.	.	1
1	**1**	.	.	.	.	2
.	**1**	1	.	.	.	4
1	.	1	1	1	.	5
1	**1**	.	1	.	1	6
.	.	.	1	1	.	10
.	**1**	1	1	.	1	11
.	**1**	.	1	1	.	14
.	.	.	1	.	1	15
1	**1**	.	1	1	.	19
1	.	1	1	.	1	20
.	**1**	1	.	1	1	21
1	**1**	.	.	1	1	28
.	**1**	.	.	.	1	33
.	**1**	1	.	1	1	34
.	.	.	.	1	1	42

measured variable Y. As a result, the unmeasured variable y_4 is the sole observable variable.

Remark: Combining Eqs. (1.1),

$$x_1 - x_2 - x_4 = 0$$

and (1.7),

$$x_1 \cdot y_1 - x_2 \cdot y_2 - x_4 \cdot y_4 = 0$$

also yields an identical result. Replacing the unknown variable x_2 by the measurements x_1 and x_4, Eq. (1.7) becomes the deduction equation of the variable y_4.

The proposed examples thus provide a means of using observability rules in order to perform the classification of process variables. It is important to note the simplicity of the numerical computation. Indeed, an analysis of the cycle matrix according to the positions of the measurement devices only involves a search of occurrences of variables. The automation of the treatment is then rather straightforward.

5. Observability algorithm

The previous examples have enabled us to apply the different observability rules. However for physical flowsheets, these rules have to be applied in an iterative way. Indeed, in applying one rule, we can point out a certain number of observable variables, observable, as these may be deduced from other measured variables. These deduced variables play the same role as the measurements and therefore for the total flowsheet the list of measured variables has to be updated. Consequently, the observability analysis must be repeated.

As shown in previous examples, the observability analysis includes two main steps. The first is dedicated to the construction of the cycle matrix of the graph and requires the establishment of the independent cycles, a familiar task. Based on these basic cycles, all the cycles may be obtained by linear combinations. The second step concerns the observability analysis. It is based on the examination of cycles in conjunction with the available measurement and according to the rules summarized in Table 1.

The algorithm comprises two steps. The first points out the unobservable variables (rules 1 and 4). The second permits the iterative determination of observable variables. Let us introduce the definitions of the two sets:

X_{cm} contains unmeasured variables X_j solely belonging to cycles of C_x in which a minimum of one variable X is measured;

X_{cs} contains the remaining unmeasured variables X.

Rule 2 (Table 1) shows that the variables X of X_{cm} may be classed as observable. When analysing the observability of the variables X of X_{cs}, one uses the measured variables Y and also the deducible variables. On another hand, analysing the observability of variables Y not only leads us to consider the bilinear system but also depends on the observability of variables X of X_{cs}. For this, one needs to consider the measurements as well as the observable variables. If the cycle matrices C_x and C_{xy} are different, the observable variables X are updated (rule 2). Consequently, the observability study of step 2 obeys the following structure:

1. the variables X of type X_{cm} are classified as observable (rule 2, matrix C_x);
2. a maximum number of variables Y are classified observable (rule 5, matrix C_{xy});
3. a maximum number of variables X of type X_{cs} are classified as observable (rule 3, matrix C_{xy}).

The above procedure is repeated until no further unmeasured X and Y variables can be classified as observable. During the execution of the procedure, it is understood that whenever an unmeasured variable is classified as observable, it is thereafter treated as measured.

6. Example

Let us consider the process of Fig. 3 described by the following equations:

$$x_1 - x_2 - x_4 = 0$$
$$x_2 - x_3 = 0$$
$$x_4 - x_5 = 0$$
$$x_3 + x_5 - x_6 = 0$$
$$x_7 - x_8 - x_9 = 0$$
$$x_8 - x_{10} = 0$$
$$x_9 - x_{11} = 0$$
$$x_{10} + x_{11} - x_{12} = 0$$
$$x_1 y_1 - x_2 y_2 - x_4 y_4 = 0$$
$$x_4 y_4 - x_5 y_5 + x_9 y_9 = 0$$
$$x_2 y_2 - x_3 y_3 + x_8 y_8 - x_{10} y_{10} = 0$$
$$x_3 y_3 + x_5 y_5 - x_6 y_6 = 0$$
$$x_7 y_7 - x_8 y_8 - x_9 y_9 = 0$$
$$x_{10} y_{10} + x_{11} y_{11} - x_{12} y_{12} = 0$$

As previously explained, these equations may be used to describe mass or energy balance equations of the process and many of the features of this simple network may be

Table 5
Cycle matrix C_x

2	1	3	4	7	8	9	10	5	6	11	12	
1	.	1	1	.	.	.	.	1	.	.	.	1
1	1	1	.	.	.	.	.	.	1	.	.	2
.	.	.	.	.	1	1	1	.	.	1	.	3
.	.	.	.	1	1	.	1	.	.	.	1	4
.	1	.	1	.	.	.	.	1	1	.	.	5
.	.	.	.	1	.	1	.	.	.	1	1	6

analogous to other physical systems dealing with liquids, gases or solids, flowing through pipes, tanks, pumps, etc. One notes that the incidence matrices M_x and M_{xy} are now different. The cycle matrix C_x is presented in Table 5. The cycle matrix C_{xy} has been given in Table 2.

The measured variables are defined in lists:

$$X_{\mathrm{m}} = [x_1, x_2, x_7] \quad Y_{\mathrm{m}} = [y_1, y_2, y_3, y_7, y_8, y_9]$$

and the unmeasured variables in:

$$X_{\bar{\mathrm{m}}} = [x_3, x_4, x_5, x_6, x_8, x_9, x_{10}, x_{11}, x_{12}] \quad Y_{\bar{\mathrm{m}}} = [y_4, y_5, y_6, y_{10}, y_{11}, y_{12}]$$

6.1. *First step: determination of the unobservable variables Y*

As cycle 38 is formed of streams 5, 6, 11 and 13, which are unmeasured in Y, rule 4a yields:

$$Y_{\mathrm{o}\bar{\mathrm{b}}} = [y_5, y_6, y_{11}, y_{12}]$$

6.2. *Second step: determination of the observable variables X_{ob} and Y_{ob}*

6.2.1. *First iteration*

According to rule 2, applied to C_x, the observable variables X are:

$$X_{\mathrm{ob}} = [x_3, x_4, x_5, x_6, x_{12}]$$

Considering the observable variables, the lists of variables X are updated to:

$$X_{\mathrm{m}} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_{12}] \quad X_{\bar{\mathrm{m}}} = [x_8, x_9, x_{10}, x_{11}]$$

According to rule 5, applied to C_{xy}, one searches for cycles with a minimum of one measured variable X. As previously suggested, it is more convenient to delete rows 8, 9, 10 and 11 of Table 2 of C_{xy}. From the remaining cycles, one looks for the variables Y belonging to cycles with a minimum of one measured Y variable. As a result, merely y_4 is observable. Consequently:

$$Y_{\mathrm{ob}} = [y_4]$$

As y_4 is observable, one may modify the definition of the measured and unmeasured Y variables such that:

$$Y_{\mathrm{m}} = [y_1, y_2, y_3, y_4, y_7, y_8, y_9] \quad Y_{\bar{\mathrm{m}}} = [y_5, y_6, y_{10}, y_{11}, y_{12}]$$

According to rule 3, one finds that the observable variables are:

$$X_{\mathrm{ob}} = [x_8, x_9]$$

which allows an update of the lists of variables X yielding:

$$X_{\mathrm{m}} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{12}] \quad X_{\bar{\mathrm{m}}} = [x_{10}, x_{11}]$$

Summarizing the results of iteration 1 leads to the following lists of measured and unmeasured variables:

$$X_{\mathrm{m}} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{12}] \quad X_{\overline{\mathrm{m}}} = [x_{10}, x_{11}]$$

$$Y_{\mathrm{m}} = [y_1, y_2, y_3, y_4, y_7, y_8, y_9] \quad Y_{\overline{\mathrm{m}}} = [y_5, y_6, y_{10}, y_{11}, y_{12}]$$

6.2.2. Second iteration

According to rule 2, the observable X variables are: $X_{\mathrm{ob}} = [x_{10}, x_{11}]$ and the X lists are updated:

$$X_{\mathrm{m}} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}] \quad X_{\overline{\mathrm{m}}} = [\ ]$$

According to rule 5, the observable variables Y are: $Y_{\mathrm{ob}} = [y_{10}]$. Thus, the new lists of measured and unmeasured Y variables are updated:

$$Y_{\mathrm{m}} = [y_1, y_2, y_3, y_4, y_7, y_8, y_9, y_{10}] \quad Y_{\overline{\mathrm{m}}} = [y_5, y_6, y_{11}, y_{12}]$$

One notices that the remaining unmeasured variables Y are unobservable and have been determined during the first step of the algorithm. Moreover, the procedure stops where no X variables are left to classify. As a final classification, one may write:

$$X_{\mathrm{ob}} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}] \quad X_{\overline{\mathrm{ob}}} = [\ ]$$

$$Y_{\mathrm{ob}} = [y_1, y_2, y_3, y_4, y_7, y_8, y_9, y_{10}] \quad Y_{\overline{\mathrm{ob}}} = [y_5, y_6, y_{11}, y_{12}]$$

As stated before, statistical treatment (such as balance equilibration through data reconciliation) may be achieved on these observable variables. The reader should also be aware that the $Y_{\overline{\mathrm{ob}}}$ list, referring to the unobservable Y variables, could involve a reflexion on supplementary sensor definition.

7. Conclusion

The observability analysis of variables of a physical process is proposed. A new classification method based on the cycle matrix of a process has been established and tested on several networks. The aim resulting from such classification is the extraction of observable and unobservable variables. This method is particularly devoted to the analysis of systems modelized by linear and bilinear balance equations and applications may be encountered for processes involving material and energy transportation such as mineral and chemical plants.

The application of the method on a concrete process is composed of two steps. The first concerns the modelisation of the process by its graph from which the cycle matrix is generated. The second deals with the structure analysis of the cycle matrix according to the number and the positions of available sensors. The main result of this second step is the classification of variables in respect to their observability level. It is important to note that a modification of the measurement system (number and position of new sensors, moving existing sensors, deleting sensors) is easily taken into account. Indeed, the cycle matrix is independent of measurements and thus one is merely required to analyse the cycle matrix in accordance with new measurement positions.

For analysis purpose, the proposed method appears to be attractive for characterising the variables and therefore to give the users a list of the observable and unobservable variables. As a consequence of this analysis, one may further investigate the design of the measurement system, i.e. the definition of the number and the position of the sensors in order to satisfy specifications on the observability of certain key variables including constraints on these sensors (cost, precision, reliability).

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