Finite memory observer for switching systems: application to diagnosis

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Goals
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• To recognize among a set of models, that which describes best the behavior of a system at a given instant

• To use this technic for fault detection purpose
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Means
Goals

• To recognize among a set of models, that which describes best the behavior of a system at a given instant
• To use this technic for fault detection purpose

Means

• To use a bank of observers, each one of them being tuned on a model and to generate various residuals
• To jointly analyze the residuals with the \textit{a priori} activation probabilities of the models
Guideline
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• Adaptation of the GBP approach (Generalized Pseudo-Bayesian approach) developed by Bar-Shalom (1989) (recent works : Zhang, Jiang, 2001)
Guideline and contribution

- Adaptation of the GBP approach (Generalized Pseudo-Bayesian approach) developed by Bar-Shalom (1989) (recent works : Zhang, Jiang, 2001)

- Substitution, in the estimator bank, of the Kalman filters by finite memory observers
Guideline and contribution

- Adaptation of the GBP approach (Generalized Pseudo-Bayesian approach) developed by Bar-Shalom (1989) (recent works: Zhang, Jiang, 2001)
- Substitution, in the estimator bank, of the Kalman filters by finite memory observers
- Taking into account of model with unknown inputs
Finite memory observer (1)

Discrete form of a linear dynamical model

\[
\begin{align*}
  x_{k+1} &= Ax_k + Bu_k + Gw_k \\
  y_k &= Cx_k + v_k
\end{align*}
\]

with centered noises $v_k$ and $w_k$
Discrete form of a linear dynamical model

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    x_{k+1} &= Ax_k + Bu_k + Gw_k \\
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\end{align*}
\]

with centered noises \(v_k\) and \(w_k\)

Integration on a finite horizon \([k - m, k]\)

\[
Y_k = P_m x_{k-m} + B_m U_k + G_m W_k + V_k
\]
Finite memory observer (1)

Discrete form of a linear dynamical model

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Integration on a finite horizon \([k - m, k]\)

\[
Y_k = P_m x_{k-m} + B_m U_k + G_m W_k + V_k
\]

with

\[
P_m = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^m \end{pmatrix} \quad \quad Z_k = \begin{pmatrix} z_{k-m} \\ z_{k-m+1} \\ \vdots \\ z_k \end{pmatrix}
\]
Finite memory observer (1)

Discrete form of a linear dynamical model

\[
\begin{align*}
    x_{k+1} &= Ax_k + Bu_k + Gw_k \\
y_k &= Cx_k + v_k
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with centered noises \(v_k\) and \(w_k\)

Integration on a finite horizon \([k - m, k]\)

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with

\[
\begin{align*}
P_m &= \begin{pmatrix}
    C \\
    CA \\
    \vdots \\
    CA^m
\end{pmatrix} \\
Z_k &= \begin{pmatrix}
z_{k-m} \\
z_{k-m+1} \\
\vdots \\
z_k
\end{pmatrix}
\end{align*}
\]
Integration on a finite horizon $[k - m, k]$

$$Y_k = P_m x_{k-m} + B_m U_k + G_m W_k + V_k$$
Integration on a finite horizon $[k - m, k]$

$$Y_k = P_m x_{k-m} + B_m U_k + G_m W_k + V_k$$

with

$$X_m = \begin{pmatrix}
0 & 0 & \ldots & \ldots & 0 \\
CX & 0 & \ddots & \ddots & 0 \\
CAX & CX & \ddots & \ddots & 0 \\
\vdots & \vdots & \ddots & \ddots & 0 \\
CA^{m-1}X & CA^{m-2}X & \ldots & CX & 0 \\
\end{pmatrix}$$
Integration on a finite horizon $[k - m, k]$

$$Y_k = P_m x_{k-m} + B_m U_k + G_m W_k + V_k$$
Integration on a finite horizon \([k - m, k]\)

\[
Y_k = P_m x_{k-m} + B_m U_k + G_m W_k + V_k
\]

State estimation

\[
\hat{x}_{k-m} = \arg \min_{x_{k-m}} \| P_m x_{k-m} + B_m U_k - Y_k \|^2
\]
Finite memory observer (3)

Integration on a finite horizon $[k - m, k]$

$$Y_k = P_m x_{k-m} + B_m U_k + G_m W_k + V_k$$

State estimation estimation at instant $k - m$

$$\hat{x}_{k-m} = \arg \min_{x_{k-m}} \|P_m x_{k-m} + B_m U_k - Y_k\|^2$$
Integration on a finite horizon \([k - m, k]\)

\[ Y_k = P_m x_{k-m} + B_m U_k + G_m W_k + V_k \]

State estimation

\[ \hat{x}_{k-m} = \arg \min_{x_{k-m}} \| P_m x_{k-m} + B_m U_k - Y_k \|^2 \]

Solution

\[ \hat{x}_{k-m} = (P_m^T P_m)^{-1} P_m^T (Y_k - B_m U_k) \]
Integration on a finite horizon \([k - m, k]\)

\[ Y_k = P_m x_{k-m} + B_m U_k + G_m W_k + V_k \]

State estimation

\[
\hat{x}_{k-m} = \arg \min_{x_{k-m}} \| P_m x_{k-m} + B_m U_k - Y_k \|^2
\]

Solution

\[
\hat{x}_{k-m} = (P_m^T P_m)^{-1} P_m^T (Y_k - B_m U_k)
\]

\[
\hat{x}_k = A^m \hat{x}_{k-m} + T_m U_k
\]

\[
T_m = \begin{pmatrix}
(A^{m-1} B)^T & (A^{m-2} B)^T & \cdots & B^T & 0
\end{pmatrix}^T
\]
Integration on a finite horizon $[k - m, k]$

\[ Y_k = P_m x_{k-m} + B_m U_k + G_m W_k + V_k \]

State estimation

\[ \hat{x}_{k-m} = \arg \min_{x_{k-m}} \| P_m x_{k-m} + B_m U_k - Y_k \|^2 \]

Solution

\[ \hat{x}_{k-m} = (P^T_m P_m)^{-1} P^T_m (Y_k - B_m U_k) \]

\[ \hat{x}_k = R_m Y_k + S_m U_k \]

\[ R_m = A^m (P^T_m P_m)^{-1} P^T_m, \quad S_m = T_m - A^m (P^T_m P_m)^{-1} P^T_m B_m \]
Model

\[
\begin{cases}
    x_{k+1} = Ax_k + Bu_k + Ed_k + Gw_k \\
    y_k = Cx_k + v_k
\end{cases}
\]

with \( d_{k+1} = d_k + \delta_k \)
Model

\[
\begin{align*}
    x_{k+1} &= A x_k + B u_k + E d_k + G w_k \\
    y_k &= C x_k + v_k
\end{align*}
\]

with \( d_{k+1} = d_k + \delta_k \)

Augmented model

\[
\begin{align*}
    x'_{k+1} &= A_a x'_k + B_a u_k + G_a w_k \\
    y_k &= C_a x'_k + v_k \\
    x'_k &= \begin{pmatrix} x_k \\ d_k \end{pmatrix}, w'_k = \begin{pmatrix} w_k \\ \delta_k \end{pmatrix}
\end{align*}
\]
Extension: unknown inputs

Model

\[
\begin{cases}
  x_{k+1} = Ax_k + Bu_k + Ed_k + Gw_k \\
y_k = Cx_k + v_k
\end{cases}
\]

with \( d_{k+1} = d_k + \delta_k \)

Augmented model

\[
\begin{cases}
  x'_{k+1} = A_ax'_k + B_au_k + G_aw_k \\
y_k = C_ax'_k + v_k
\end{cases}
\]

\[
x'_{k} = \begin{pmatrix} x_k \\ d_k \end{pmatrix}, \quad w'_{k} = \begin{pmatrix} w_k \\ \delta_k \end{pmatrix}
\]

\[
A_a = \begin{pmatrix} A & E \\ 0 & I \end{pmatrix}, \quad G_a = \begin{pmatrix} G & 0 \\ 0 & I \end{pmatrix}, \quad B_a = \begin{pmatrix} B & 0 \\ 0 & I \end{pmatrix}, \quad C_a = \begin{pmatrix} C & 0 \end{pmatrix}
\]
Set of $r$ models

\[ M_j : \begin{cases} 
  x_{k+1} = A_j x_k + B_j u_k + G_j w_k \\
  y_k = C_j x_k + v_k 
\end{cases} \quad j = 1, \ldots, r \]
Switching systems

Set of $r$ models

$$
M_j : \begin{cases} 
    x_{k+1} = A_j x_k + B_j u_k + G_j w_k \\
    y_k = C_j x_k + v_k 
\end{cases} \quad j = 1, \ldots, r
$$

Model transitions are governed by a Markovian process

$$
\Pi = \begin{pmatrix} 
    p_{11} & \cdots & p_{1r} \\
    \vdots & \ddots & \vdots \\
    p_{r1} & \cdots & p_{rr} 
\end{pmatrix}
$$

$p_{ij}$: Transition conditional probability from model $M_i$ to model $M_j$ (a priori information)
“Global” state estimation

“Local” estimation

\[(Y_k, U_k) \text{ and } M_j \Rightarrow \hat{x}_k^j, \quad j = 1, \ldots, r\]
“Global” state estimation

“Local” estimation

\[(Y_k, U_k) \text{ and } M_j \Rightarrow \hat{x}_k^j \quad j = 1, \ldots, r\]

“Global” estimation

\[\hat{x}_k = \sum_{j=1}^{r} \hat{x}_k^j \mu_k^j\]

\[\mu_k^j : \text{a posteriori probability that the system behaves as the } j^{th} \text{ model}\]
Estimation of the probabilities $\mu_{jk}^j$ (1)

Definition

$$\mu_{jk}^j = P\{M_j(k) | Y_k\}$$
Estimation of the probabilities $\mu_k^j$ (1)

Definition

$$\mu_k^j = P\{M_j(k)|Y_k\}$$

Measurement vector partition

$$Y_k = \begin{pmatrix} \tilde{Y}_{k-1} \\ y_k \end{pmatrix}$$
Estimation of the probabilities $\mu_j^k$ (1)

Definition

$$\mu_j^k = P\{M_j(k) | Y_k\}$$

Measurement vector partition

$$Y_k = \begin{pmatrix} \tilde{Y}_{k-1} \\ y_k \end{pmatrix} \Rightarrow \mu_j^k = P\{M_j(k) | \tilde{Y}_{k-1}, y_k\}$$
Estimation of the probabilities $\mu_{jk}^j$ (1)

Definition

$$\mu_{jk}^j = P\{M_j(k)|Y_k\}$$

Measurement vector partition

$$Y_k = \left(\begin{array}{c} \tilde{Y}_{k-1} \\ y_k \end{array}\right) \quad \Rightarrow \quad \mu_{jk}^j = P\{M_j(k)|\tilde{Y}_{k-1}, y_k\}$$

De Bayes formula

$$\mu_{jk}^j = \frac{p(y_k|M_j(k), \tilde{Y}_{k-1})P\{M_j(k)|\tilde{Y}_{k-1}\}}{\sum_{l=1}^r p(y_k|M_l(k), \tilde{Y}_{k-1})P\{M_l(k)|\tilde{Y}_{k-1}\}}$$
Estimation of the probabilities $\mu_k^j$ \hspace{1cm} (1)

Definition

$$\mu_k^j = P\{M_j(k)|Y_k\}$$

Measurement vector partition

$$Y_k = \begin{pmatrix} \tilde{Y}_{k-1} \\ y_k \end{pmatrix} \Rightarrow \mu_k^j = P\{M_j(k)|\tilde{Y}_{k-1}, y_k\}$$

De Bayes formula

$$\mu_k^j = \frac{p(y_k|M_j(k), \tilde{Y}_{k-1})P\{M_j(k)|\tilde{Y}_{k-1}\}}{\sum_{l=1}^r p(y_k|M_l(k), \tilde{Y}_{k-1})P\{M_l(k)|\tilde{Y}_{k-1}\}}$$

likelihood fct. of mode $j$ at time $k$ (Gaussian assumption)
A posteriori probability

\[
\mu_{jk}^i = \frac{p(y_k | M_j(k), \tilde{Y}_{k-1}) P\{M_j(k) | \tilde{Y}_{k-1}\}}{\sum_{l=1}^{r} p(y_k | M_l(k), \tilde{Y}_{k-1}) P\{M_l(k) | \tilde{Y}_{k-1}\}}
\]
A posteriori probability

\[ \mu^j_k = \frac{p(y_k|M_j(k), \tilde{Y}_{k-1}) P\{M_j(k)|\tilde{Y}_{k-1}\}}{\sum_{l=1}^{r} p(y_k|M_l(k), \tilde{Y}_{k-1}) P\{M_l(k)|\tilde{Y}_{k-1}\}} \]

Recurrent calculus

\[ P\{M_j(k)|\tilde{Y}_{k-1}\} = \sum_{i=1}^{r} P\{M_j(k)|M_i(k-1), \tilde{Y}_{k-1}\} P\{M_i(k-1)|\tilde{Y}_{k-1}\} \]
Estimation of the probabilities $\mu^j_k$ (2)

A posteriori probability

$$\mu^j_k = \frac{p(y_k|M_j(k), \tilde{Y}_{k-1}) P\{M_j(k)|\tilde{Y}_{k-1}\}}{\sum_{l=1}^{r} p(y_k|M_l(k), \tilde{Y}_{k-1}) P\{M_l(k)|\tilde{Y}_{k-1}\}}$$

Recurrent calculus

$$P\{M_j(k)|\tilde{Y}_{k-1}\} = \sum_{i=1}^{r} P\{M_j(k)|M_i(k-1), \tilde{Y}_{k-1}\} P\{M_i(k-1)|\tilde{Y}_{k-1}\}$$

$$P\{M_i(k-1)|\tilde{Y}_{k-1}\} \approx P\{M_i(k-1)|Y_{k-1}\} = \mu^i_{k-1}$$
Estimation of the probabilities $\mu_{jk}^j$ (2)

A posteriori probability

$$\mu_{jk}^j = \frac{p(y_k|M_j(k), \tilde{Y}_{k-1}) P\{M_j(k)|\tilde{Y}_{k-1}\}}{\sum_{l=1}^r p(y_k|M_l(k), \tilde{Y}_{k-1}) P\{M_l(k)|\tilde{Y}_{k-1}\}}$$

Recurrent calculus

$$P\{M_j(k)|\tilde{Y}_{k-1}\} = \sum_{i=1}^r P\{M_j(k)|M_i(k-1), \tilde{Y}_{k-1}\} P\{M_i(k-1)|\tilde{Y}_{k-1}\}$$

$$P\{M_i(k-1)|\tilde{Y}_{k-1}\} \approx P\{M_i(k-1)|Y_{k-1}\} = \mu_{k-1}^i$$

$$Y_k = \begin{pmatrix} y_{k-m} & \cdots & y_{k-1} & y_k \end{pmatrix}, \quad Y_{k-1} = \begin{pmatrix} y_{k-m-1} & y_{k-m} & \cdots & y_{k-1} \end{pmatrix}$$
Recurrence calculus

\[ P\{M_j(k)|\tilde{Y}_{k-1}\} = \sum_{i=1}^{r} P\{M_j(k)|M_i(k-1), \tilde{Y}_{k-1}\} P\{M_i(k-1)|\tilde{Y}_{k-1}\} \]
Recall calculus

\[ P\{M_j(k) | \tilde{Y}_{k-1}\} = \sum_{i=1}^{r} P\{M_j(k) | M_i(k-1), \tilde{Y}_{k-1}\} P\{M_i(k-1) | \tilde{Y}_{k-1}\} \]

\[ P\{M_j(k) | \tilde{Y}_{k-1}\} = \sum_{i=1}^{r} p_{ij} \mu^i_{k-1} \]
Recurrent calculus

\[
P\{M_j(k)|\tilde{Y}_{k-1}\} = \sum_{i=1}^{r} P\{M_j(k)|M_i(k-1), \tilde{Y}_{k-1}\} P\{M_i(k-1)|\tilde{Y}_{k-1}\}
\]

\[
P\{M_j(k)|\tilde{Y}_{k-1}\} = \sum_{i=1}^{r} p_{ij} \mu_{k-1}^i
\]

\[
\mu_{k}^j = \frac{p(y_k|M_j(k), \tilde{Y}_{k-1}) P\{M_j(k)|\tilde{Y}_{k-1}\}}{\sum_{l=1}^{r} p(y_k|M_l(k), \tilde{Y}_{k-1}) P\{M_l(k)|\tilde{Y}_{k-1}\}}
\]

\[
\mu_{k}^j = \frac{p(y_k|M_j(k), \tilde{Y}_{k-1}) \sum_{i=1}^{r} p_{ij} \mu_{k-1}^i}{\sum_{l=1}^{r} \left(p(y_k|M_l(k), \tilde{Y}_{k-1}) \sum_{i=1}^{r} p_{il} \mu_{k-1}^i\right)}
\]
Actuator fault

\[ x_{k+1} = Ax_k + (B + \Delta B_i)u_k + w_k \]

\( \Delta B_i \): all the columns are null but the \( i^{th} \) that characterizes the fault of the \( i^{th} \) actuator
Actuator fault

\[ x_{k+1} = Ax_k + (B + \Delta B_i)u_k + w_k \]

\( \Delta B_i \) : all the columns are null but the \( i^{th} \) that characterizes the fault of the \( i^{th} \) actuator

Sensor fault

\[ y_k = (C + \Delta C_i)x_k + v_k \]

\( \Delta C_i \) : all the rows are null but the \( i^{th} \) that characterizes the fault of the \( i^{th} \) sensor
Actuator fault

\[ x_{k+1} = Ax_k + (B + \Delta B_i)u_k + w_k \]

\( \Delta B_i \): all the columns are null but the \( i^{th} \) that characterizes the fault of the \( i^{th} \) actuator

Sensor fault

\[ y_k = (C + \Delta C_i)x_k + v_k \]

\( \Delta C_i \): all the rows are null but the \( i^{th} \) that characterizes the fault of the \( i^{th} \) sensor

Assimilation of the faults to changes of models
Toy example 1 : description

Models

\((A, B_1, C_1)\) : normal functioning \([0, 100] \cup [500, 800]\)
\((A, B_2, C_2)\) : occurrence of actuator fault \([100, 500]\)
\((A, B_3, C_3)\) : occurrence of sensor fault \([800, 1000]\)
Toy example 1: description

Models

\((A, B_1, C_1)\): normal functioning \([0, 100] \cup [500, 800]\)

\((A, B_2, C_2)\): occurrence of actuator fault \([100, 500]\)

\((A, B_3, C_3)\): occurrence of sensor fault \([800, 1000]\)

Matrix values

\[
A = \begin{pmatrix} 0.45 & 0 \\ 0 & 0.4 \end{pmatrix}, \quad \Pi = \begin{pmatrix} 0.9 & 0.05 & 0.05 \\ 0.05 & 0.9 & 0.05 \\ 0.05 & 0.05 & 0.9 \end{pmatrix}
\]

\[
B_1 = \begin{pmatrix} 0.1815 & 1.7902 \end{pmatrix}^T, \quad C_1 = I_{2 \times 2}
\]

\[
B_2 = \begin{pmatrix} 1.1815 & 1.7902 \end{pmatrix}^T, \quad C_2 = I_{2 \times 2}
\]

\[
B_3 = \begin{pmatrix} 0.1815 & 1.7902 \end{pmatrix}^T, \quad C_3 = 1.5I_{2 \times 2}
\]
Activation probabilities of the different models
Models with unknown input

$(A, B_1, C_1)$: normal functioning $[0, 100] \cup [500, 800]$

$(A, B_2, C_2)$: occurrence of actuator fault $[100, 500]$

$(A, B_3, C_3)$: occurrence of sensor fault $[800, 1000]$
Toy example 2: description

Models with unknown input

\((A, B_1, C_1)\) : normal functioning \([0, 100] \cup [500, 800]\)

\((A, B_2, C_2)\) : occurrence of actuator fault \([100, 500]\)

\((A, B_3, C_3)\) : occurrence of sensor fault \([800, 1000]\)

Matrix values

\(A, B_i, C_i\) identical as the previous case

\[E = \begin{pmatrix} 0.0129 & -1.2504 \end{pmatrix}^T\]

Unknown input

Crenel of magnitude 0.5 between instants 100 and 300
Toy example 2: results

Activation probabilities of the different models
Toy example 2: results

Unknown input estimation

estimation
entrée inconnue
• Comparable performances of the methods based on Kalman filters and finite memory observers
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• Capacity of taking into account of unknown input when using observers (finite memory observers)
Conclusion - outlines

• Comparable performances of the methods based on Kalman filters and finite memory observers

• Capacity of taking into account of unknown input when using observers (finite memory observers)

• Advantages for fault detection strategies (notion of bank of observers)
Conclusion - outlines

- Comparable performances of the methods based on Kalman filters and finite memory observers
- Capacity of taking into account of unknown input when using observers (finite memory observers)
- Advantages for fault detection strategies (notion of bank of observers)
- Actual research: Estimation of the Markov transition matrix