# ***Finite memory observer for switching systems: application to diagnosis***

Abdelfettah HOCINE, Didier MAQUIN and José RAGOT

Presented by Didier THEILLIOL

Institut National Polytechnique de Lorraine

Centre de Recherche en Automatique de Nancy

UMR 7039 CNRS – Université Henri Poincaré, Nancy 1 – INPL

## Goals

# Introduction (1)

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- To recognize among a set of models, that which describes best the behavior of a system at a given instant
- To use this technic for fault detection purpose

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## Means

- To use a bank of observers, each one of them being tuned on a model and to generate various residuals
- To jointly analyze the residuals with the *a priori* activation probabilities of the models

## Guideline

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- Adaptation of the GBP approach (Generalized Pseudo-Bayesian approach) developed by Bar-Shalom (1989) (recent works : Zhang, Jiang, 2001)

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- Substitution, in the estimator bank, of the Kalman filters by finite memory observers

# Introduction (2)

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- Adaptation of the GBP approach (Generalized Pseudo-Bayesian approach) developed by Bar-Shalom (1989) (recent works : Zhang, Jiang, 2001)
- Substitution, in the estimator bank, of the Kalman filters by finite memory observers
- Taking into account of model with unknown inputs

## Discrete form of a linear dynamical model

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + Gw_k \\ y_k = Cx_k + v_k \end{cases}$$

with centered noises $v_k$ and $w_k$

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## Integration on a finite horizon $[k-m, k]$

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# Finite memory observer (2)

Integration on a finite horizon $[k-m, k]$

$$Y_k = P_m x_{k-m} + B_m U_k + G_m W_k + V_k$$

with

$$X_m = \begin{pmatrix} 0 & 0 & \dots & \dots & 0 \\ CX & 0 & \ddots & \ddots & 0 \\ CAX & CX & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & 0 \\ CA^{m-1}X & CA^{m-2}X & \dots & CX & 0 \end{pmatrix}$$

Integration on a finite horizon $[k-m,k]$

$$Y_k = P_m x_{k-m} + B_m U_k + G_m W_k + V_k$$

# Finite memory observer (3)

Integration on a finite horizon $[k-m, k]$

$$Y_k = P_m x_{k-m} + B_m U_k + G_m W_k + V_k$$

State estimation

$$\hat{x}_{k-m} = \arg \min_{x_{k-m}} \|P_m x_{k-m} + B_m U_k - Y_k\|^2$$

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State estimation    estimation at instant $k-m$

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Solution

$$\hat{x}_{k-m} = (P_m^T P_m)^{-1} P_m^T (Y_k - B_m Uk)$$

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Solution

$$\hat{x}_{k-m} = (P_m^T P_m)^{-1} P_m^T (Y_k - B_m U k)$$

$$\hat{x}_k = A^m \hat{x}_{k-m} + T_m U_k$$

$$T_m = \begin{pmatrix} (A^{m-1}B)^T & (A^{m-2}B)^T & \dots & B^T & 0 \end{pmatrix}^T$$

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$$\hat{x}_{k-m} = (P_m^T P_m)^{-1} P_m^T (Y_k - B_m U k)$$

$$\hat{x}_k = R_m Y_k + S_m U_k$$

$$R_m = A^m (P_m^T P_m)^{-1} P_m^T, \qquad S_m = T_m - A^m (P_m^T P_m)^{-1} P_m^T B_m$$

## Model

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + Ed_k + Gw_k \\ y_k = Cx_k + v_k \end{cases}$$

with $d_{k+1} = d_k + \delta_k$

# Extension : unknown inputs

## Model

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + Ed_k + Gw_k \\ y_k = Cx_k + v_k \end{cases}$$

with $d_{k+1} = d_k + \delta_k$

## Augmented model

$$\begin{cases} x'_{k+1} = A_a x'_k + B_a u_k + G_a w_k \\ y_k = C_a x'_k + v_k \end{cases}$$

$$x'_k = \begin{pmatrix} x_k \\ d_k \end{pmatrix}, w'_k = \begin{pmatrix} w_k \\ \delta_k \end{pmatrix}$$

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with

$$A_a = \begin{pmatrix} A & E \\ 0 & I \end{pmatrix}, G_a = \begin{pmatrix} G & 0 \\ 0 & I \end{pmatrix}, B_a = \begin{pmatrix} B & 0 \\ 0 & I \end{pmatrix}, C_a = \begin{pmatrix} C & 0 \end{pmatrix}$$

## Set of $r$ models

$$M_j : \begin{cases} x_{k+1} = A_j x_k + B_j u_k + G_j w_k \\ y_k = C_j x_k + v_k \end{cases} \qquad j = 1, \ldots, r$$

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Model transitions are governed by a Markovian process

$$\Pi = \begin{pmatrix} p_{11} & \ldots & p_{1r} \\ \vdots & \ddots & \vdots \\ p_{r1} & \ldots & p_{rr} \end{pmatrix}$$

$p_{ij}$ : Transition conditional probability from model $M_i$ to model $M_j$ (a priori information)

## “Local” estimation

$$(Y_k, U_k) \text{ and } M_j \Rightarrow \hat{x}_k^j \qquad j = 1, \ldots, r$$

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“Global” estimation

$$\hat{x}_k = \sum_{j=1}^{r} \hat{x}_k^j \mu_k^j$$

$\mu_k^j$ : a posteriori probability that the system behaves as the $j^{th}$ model

# Estimation of the probabilities $\mu_k^j$ (1)

Definition

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## De Bayes formula

$$\mu_k^j = \frac{p(y_k|M_j(k), \tilde{Y}_{k-1})P\{M_j(k)|\tilde{Y}_{k-1}\}}{\sum_{l=1}^{r} p(y_k|M_l(k), \tilde{Y}_{k-1})P\{M_l(k)|\tilde{Y}_{k-1}\}}$$

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likelihood fct. of mode $j$ at time $k$ (Gaussian assumption)

# Estimation of the probabilities $\mu_k^j$ (2)

## A posteriori probability

$$\mu_k^j = \frac{p(y_k|M_j(k), \tilde{Y}_{k-1}) P\{M_j(k)|\tilde{Y}_{k-1}\}}{\sum_{l=1}^{r} p(y_k|M_l(k), \tilde{Y}_{k-1}) P\{M_l(k)|\tilde{Y}_{k-1}\}}$$

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## Recurrent calculus

$$P\{M_j(k)|\tilde{Y}_{k-1}\} = \sum_{i=1}^{r} P\{M_j(k)|M_i(k-1),\tilde{Y}_{k-1}\}P\{M_i(k-1)|\tilde{Y}_{k-1}\}$$

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## A posteriori probability

$$\mu_k^j = \frac{p(y_k | M_j(k), \tilde{Y}_{k-1}) P\{M_j(k) | \tilde{Y}_{k-1}\}}{\sum_{l=1}^{r} p(y_k | M_l(k), \tilde{Y}_{k-1}) P\{M_l(k) | \tilde{Y}_{k-1}\}}$$

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$$Y_k = \begin{pmatrix} y_{k-m} & \ldots & y_{k-1} & y_k \end{pmatrix}, \quad Y_{k-1} = \begin{pmatrix} y_{k-m-1} & y_{k-m} & \ldots & y_{k-1} \end{pmatrix}$$

# Estimation of the probabilities $\mu_k^j$ (3)

## Recurrent calculus

$$P\{M_j(k)|\tilde{Y}_{k-1}\} = \sum_{i=1}^{r} P\{M_j(k)|M_i(k-1), \tilde{Y}_{k-1}\} P\{M_i(k-1)|\tilde{Y}_{k-1}\}$$

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$$P\{M_j(k)|\tilde{Y}_{k-1}\} = \sum_{i=1}^{r} p_{ij}\mu_{k-1}^{i}$$

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## Recurrent calculus

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$$\mu_k^j = \frac{p(y_k|M_j(k), \tilde{Y}_{k-1}) \sum_{i=1}^{r} p_{ij}\mu_{k-1}^i}{\sum_{l=1}^{r} \left( p(y_k|M_l(k), \tilde{Y}_{k-1}) \sum_{i=1}^{r} p_{il}\mu_{k-1}^i \right)}$$

## Actuator fault

$$x_{k+1} = Ax_k + (B + \Delta B_i)u_k + w_k$$

$\Delta B_i$ : all the columns are null but the $i^{th}$ that characterizes the fault of the $i^{th}$ actuator

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## Sensor fault

$$y_k = (C + \Delta C_i)x_k + v_k$$

$\Delta C_i$ : all the rows are null but the $i^{th}$ that characterizes the fault of the $i^{th}$ sensor

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Assimilation of the faults to changes of models

# Toy example 1 : description

Models

$(A, B_1, C_1)$ : normal functionning [0, 100] $\cup$ [500, 800]
$(A, B_2, C_2)$ : occurrence of actuator fault [100, 500]
$(A, B_3, C_3)$ : occurrence of sensor fault [800, 1000]

## Models

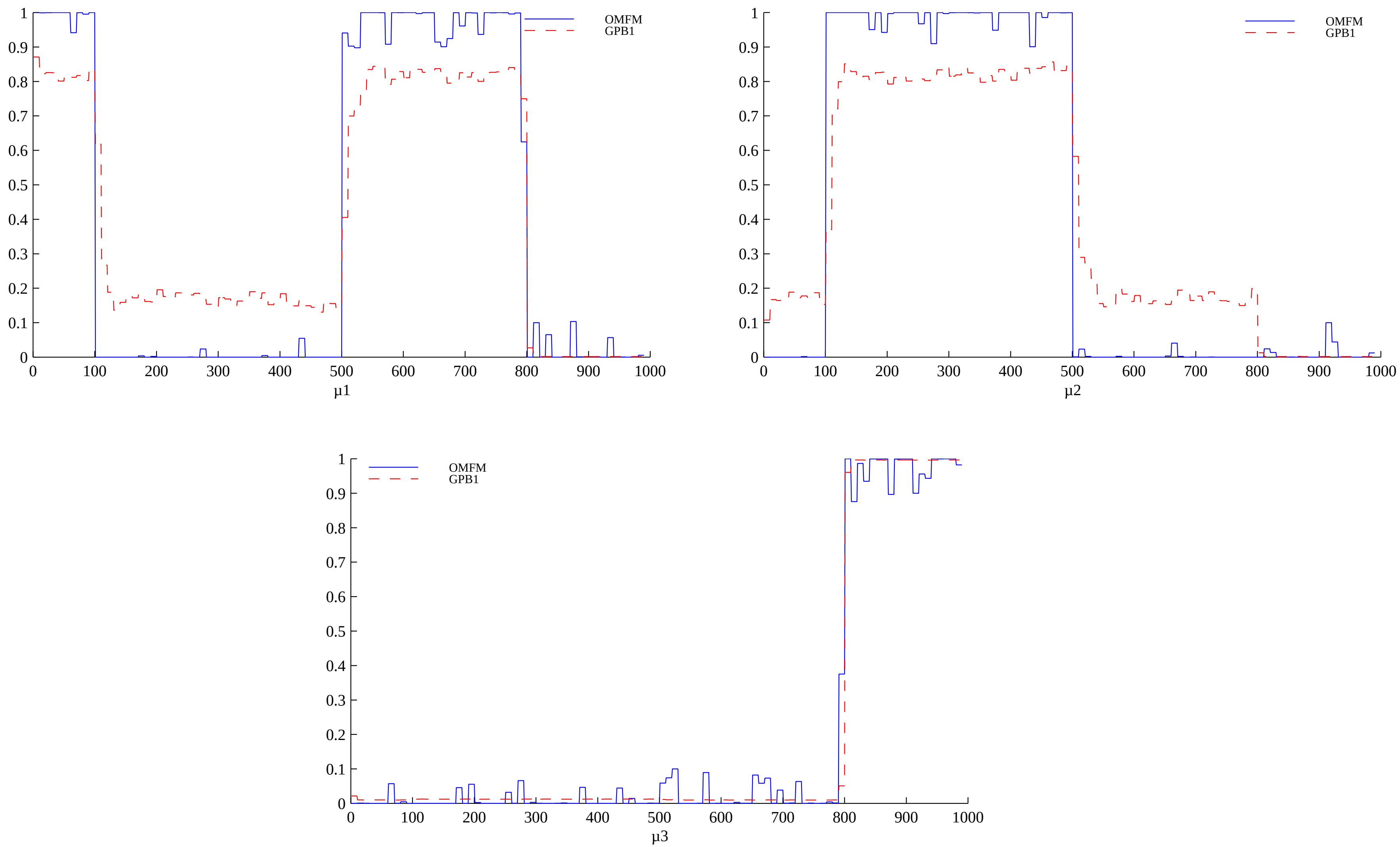
$(A, B_1, C_1)$ : normal functionning $[0, 100] \cup [500, 800]$
$(A, B_2, C_2)$ : occurrence of actuator fault $[100, 500]$
$(A, B_3, C_3)$ : occurrence of sensor fault $[800, 1000]$

## Matrix values

$$A = \begin{pmatrix} 0.45 & 0 \\ 0 & 0.4 \end{pmatrix}, \quad \Pi = \begin{pmatrix} 0.9 & 0.05 & 0.05 \\ 0.05 & 0.9 & 0.05 \\ 0.05 & 0.05 & 0.9 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0.1815 & 1.7902 \end{pmatrix}^T, \quad C_1 = I_{2\times 2}$$
$$B_2 = \begin{pmatrix} 1.1815 & 1.7902 \end{pmatrix}^T, \quad C_2 = I_{2\times 2}$$
$$B_3 = \begin{pmatrix} 0.1815 & 1.7902 \end{pmatrix}^T, \quad C_3 = 1.5 I_{2\times 2}$$

# Toy example 1 : results




Activation probabilities of the different models

# *Toy example 2 : description*

## Models with unknown input

$(A, B_1, C_1)$ : normal functionning [0, 100] $\cup$ [500, 800]
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# Toy example 2 : description

## Models with unknown input

$(A, B_1, C_1)$ : normal functionning [0, 100] ∪ [500, 800]
$(A, B_2, C_2)$ : occurrence of actuator fault [100, 500]
$(A, B_3, C_3)$ : occurrence of sensor fault [800, 1000]

## Matrix values

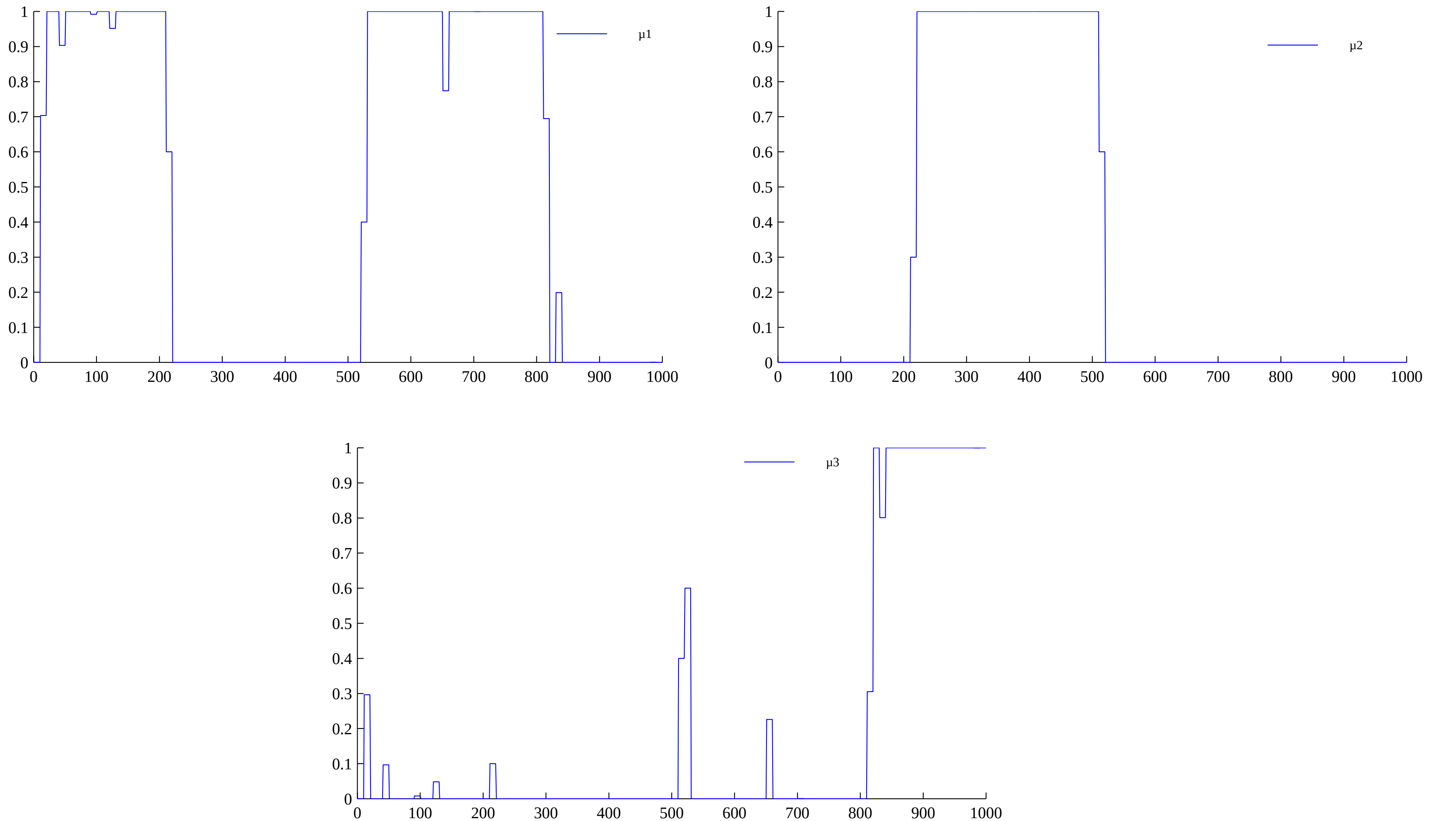
$A, B_i, C_i$ identical as the previous case

$$E = \begin{pmatrix} 0.0129 & -1.2504 \end{pmatrix}^T$$

## Unknown input

Crenel of magnitude 0.5 between instants 100 and 300

# Toy example 2 : results

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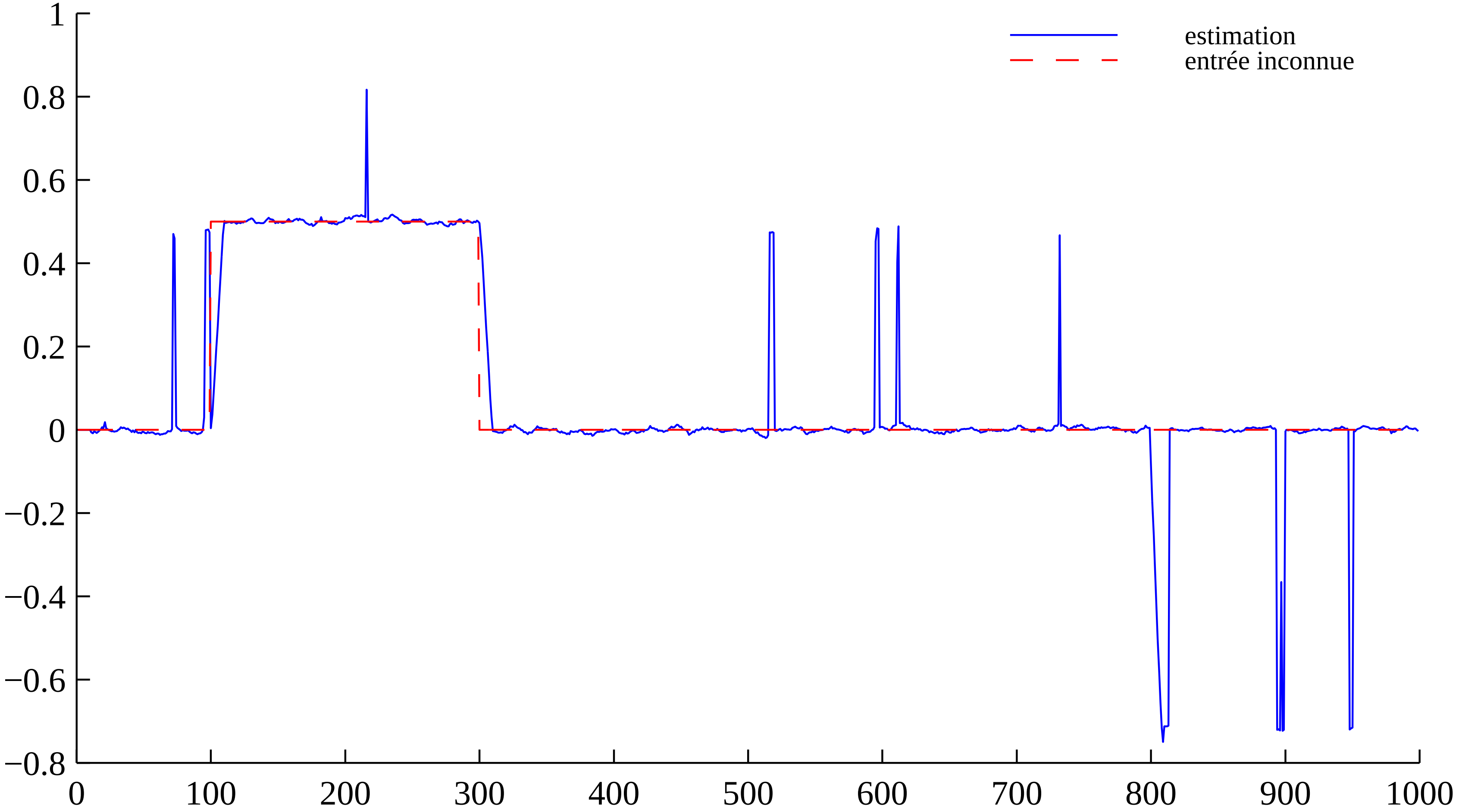


Unknown input estimation

- Comparable performances of the methods based on Kalman filters and finite memory observers

## *Conclusion - outlines*

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- Capacity of taking into account of unknown input when using observers (finite memory observers)

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# Conclusion - outlines

- Comparable performances of the methods based on Kalman filters and finite memory observers
- Capacity of taking into account of unknown input when using observers (finite memory observers)
- Advantages for fault detection strategies (notion of bank of observers)
- Actual research: Estimation of the Markov transition matrix