SOME IDEAS ABOUT THE DESIGN OF MEASUREMENT SYSTEMS

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Abstract

The quality of the state estimation of a system and, consequently, its dependability are strongly conditioned by the number and the location of the measurements. The availability of a system may be increased if this latter is able to function even in the presence of sensor breakdowns. So, the measurement planning (measurement system design) represent a very important stage. This communication presents a method for assessing the availability of the necessary information for the process control and defining the sensor locations such that the variables required for controlling the process remain always observable even if one or more sensors become defective. Underlying, the connections between several concepts such observability, redundancy, sensor location and reliability are emphasised.

1 Introduction

Monitoring a process requires a certain number of measurements which are usually supplied by sensors; unfortunately, some of these sensors may breakdown and the corresponding measurements become unavailable. Moreover if these data are essential for process monitoring, then the breakdown of these sensors may lead to the breakdown of the whole process. However, the lack of measurement due to one or more faulty sensors may be overcome by using the existing relationships between different measured variables.

The problem of measurement planning covers different aspects such as the determination of the number of sensors, their location, their accuracy, their type and so on. Due to increasing accuracy requirements and tighter financial constraints, the problem of measurement system design has received much attention; unfortunately general methods do not exist to solve this problem due to the number of constraints to take into account. For the problem of process diagnosis there are many situations in which modifying the sensor placements will strongly improve the quality of the measurements; indeed modifications in the sensor locations change the observation equations, the variable observability [8] and therefore the performance of the measurement system. At present, only a few papers about designing the measurement system have been published. Some of them use an optimisation of the variance matrix of the estimated variables in respect of the coefficients of the measurement matrix [2]. In [3] the problem of sensor placement is addressed by monitoring the eigenstructure of a multivariable dynamical system. Furthermore, it has been addressed more in view of good parameter and state estimation than for system monitoring and diagnosis. In the field of electric power networks specific developments on this subject have also been published using topological observability algorithms [9], [4].

This study deals with steady state linear systems. These models frequently occurs when considering the mass and energy balance relationships. In this context, some recent studies have been published. Madron *et al.* [11] proposed an approach based on the analysis of the incidence matrix associated to the graph of the process. Immonen [7] formulated the problem as a linear programming one. The resulting optimisation problem is solved using a binary mixed integer programming method in order to take into account the occurrences and locations of sensors. The most developed works are undoubtedly those presented by Ali *et al.* [1] and Luong *et al.* [10]. Based on the analysis of the cycles of the process graph, these studies simultaneously take into account the variable observability and the reliability of the sensors in order to determine an optimal measurement system with regards to certain criteria. The design of a measurement system may be addressed with different goals: minimise the variance of the estimated variables [13], obtain a given redundancy level of the variables or only of a subset of variables, ensure that some variables are observable. This communication deals with a new method for optimising measurement system design based on a reliability and cost analysis in conjunction with observability constraints.

In the second section, we describe the process codification and remember some classical transformations of graph theory. Then, the main results of observability analysis are remembered in the third section. We focus our attention on the method based on the cycle matrix analysis. In the fourth section, we propose to extend the notion of redundancy by defining multiple redundancy and more precisely the degree of redundancy of a variable. This concept is very important in the following when considering the possibility of sensor breakdowns. The fifth section is then dedicated to the presentation of a method for designing a measurement system guaranteeing the degree of redundancy of certain variables whilst minimising the installation cost of the sensors. In the sixth section, after defining the reliability of a process with regard to its measurement system, we point out the relationship between the notions of observability, redundancy and reliability. We analyse the influence of sensor breakdowns on the reliability of the whole measurement system and show that it can be increased taking into account the redundancy equations of the process. Then in section seven, we show how to select the sensor locations in order to have a given value of the reliability function of the whole measurement system; this will be done by taking into account constraints on the observability of given variables. For the simplicity of the presentation, we only use here, at the different steps of the presentation, small dimension processes and present some "by hand" treatments. However all the proposed treatments may be systematised and computerised.

2 Process Codification

Here, we consider only processes which may be described by a carriage network (figure 1), the nodes of which correspond to elementary equations and the arcs to variables. For a physical interpretation, a node may be a processing unit and an arc, a material or energy transfer stream.

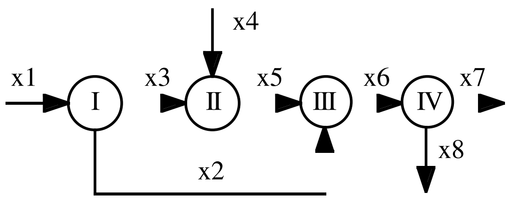

Figure 1: a carriage network

This process may be described by a set of linear equations:

$$A\ X = 0 \tag{1}$$

where A (4,8) is the so-called node incidence matrix and X (8,1) is the variable vector. More generally, the size of A is (n,v) with n giving the number of equations and v the number of variables. It is full row rank. The incidence matrix (table 1) corresponding to the network is written as follows (for convenience, the "." states for the value 0):

	1	2	3	4	5	6	7	8
I	1	-1	-1	.	.	.	.	.
II	.	.	1	1	-1	.	.	.
III	.	1	.	.	1	-1	.	.
IV	.	.	.	.	.	1	-1	-1

Table 1: incidence matrix

In a certain number of applications, it is much easier to use the cycle matrix of the associated graph. In order to transform the network into a graph, all the input/output arcs are linked to the so-called environment node {0}.

The matrix B of the fundamental cycles of the graph may be easily deduced from the incidence matrix [6]. For that purpose, the following transformations may be used. First, the incidence matrix is partitioned (using simple permutation of columns) according to its regular part A_r:

$$A = (A_r \quad A_s) \tag{2}$$

Then, the fundamental cycle matrix may be written:

$$B = (B_s \quad I) \tag{3}$$

where B_s is obtained from the relation:

$$B_s^T = - A_r^{-1} \ A_s \tag{4}$$

and I is the identity matrix with appropriate dimensions.

From this fundamental cycle matrix and using linear combinations of them, it is then possible to generate the matrix of all the cycles of the graph. The matrix of all these cycles associated to the figure 2 is given in table 2. It comprises ten cycles among which only four are independent (fundamental cycles).

	1	2	3	4	5	6	7	8
1	-1	.	-1	1	.	.	.	.
2	.	-1	1	.	1	.	.	.
3	1	1	.	.	.	1	1	.
4	1	1	.	.	.	1	.	1
5	-1	-1	.	1	1	.	.	.
6	.	1	-1	1	.	1	1	.
7	.	1	-1	1	.	1	.	1
8	1	.	1	.	1	1	1	.
9	1	.	1	.	1	1	1	1
10	.	.	.	.	.	.	1	-1

Table 2: matrix of all the cycles of the graph

3 Observability Analysis

Due to technical and economic feasibility, it is not possible to measure each and every process variable. However, it is very important to determine the observability of the whole process and the "state" of each variable. In this section we only report some classical results about the observability of linear systems [12], [14]. From the analysis of the node incidence matrix A, it is possible to classify the variables into different categories.

Consider for example $L_m = \{x1, x2, x4, x6\}$ the list of the measured variables and $L_{\overline{m}} = \{x3, x5, x7, x8\}$ the list of the unmeasured variables. In the equations (5) of the process, the boldfaced characters denote the measured variables:

$$\mathbf{x1} - \mathbf{x2} - x3 = 0 \tag{5a}$$
$$x3 + \mathbf{x4} - x5 = 0 \tag{5b}$$
$$\mathbf{x2} + x5 - \mathbf{x6} = 0 \tag{5c}$$
$$\mathbf{x6} - x7 - x8 = 0 \tag{5d}$$

The variable classification based on the observability analysis consists in pointing out two categories of variables: the observable variables, the value of which may be known (by direct measurement or by deduction from other measurements) and the unobservable variables. Taking into account the small dimensions of the proposed example, this analysis may be done intuitively. The addition of the equations (5a), (5b) and (5c) allow a redundancy equation to be extracted: $\boldsymbol{x1} + \boldsymbol{x4} - \boldsymbol{x6} = 0$. Such equation involves measured variables only. The equations (5a) and (5c) are equations of deduction; they allow unmeasured variables $x3$ and $x5$ to be deduced. Finally, variables $x7$ and $x8$ are unobservable due to a lack of information. This analysis leads to the classification of the variables into the four following distinct categories:

$L_{me} = \{x1, x4, x6\}$ $\qquad L_{m\bar{e}} = \{x2\}$

$L_{\bar{m}e} = \{x3, x5\}$ $\qquad L_{\bar{m}\bar{e}} = \{x7, x8\}$

The first subscript (m or $\bar{m}$) indicates whether a variable is measured or unmeasured and the second one (e or $\bar{e}$) whether it is estimable or non estimable.

The above classification has been easily achieved from a simple exam of the network owing to its small dimensions. In a more general case, the extraction of observable variables must be systematised using an algebraic formulation. This systematisation may be done either from the analysis of the incidence matrix [5] or from that of the cycle matrix. In this latter case, the analysis is based on the four fundamental following rules:

Rule I: A measured variable is estimable (redundant variable) if and only if it only belongs to cycles where at least two variables are measured.

Rule II: A measured variable is not estimable if and only if it belongs at least to a cycle where it is the only measured variable.

Rule III: An unmeasured variable is deducible (estimable) if and only if it only belongs to cycles with at least one measured variable.

Rule IV: An unmeasured variable is not deducible (non estimable) if and only if it belongs at least to a cycle where not any variable is measured.

So, the classification algorithm of variables includes the four following steps:

1) Determine the matrix of all the cycles of the graph, let C_t this matrix.

2) Extract from C_t the sub-matrix which comprises at most two measured variables (the knowledge of cycles which comprise more than two measurements is not necessary at this step)

3) Detect the cycles which do not involve measured variables; the unmeasured variables which belong to these cycles are the unmeasured but estimable variables (*rule IV*). The other unmeasured variables of the process are the unmeasured but deducible variables.

4) Detect the cycles involving only one measured variable; the measured variables that belong to these cycles are the measured but non estimable variables (*rule II*). The other measured variables of the process are the measured and estimable variables).

Let us consider the process described by figure 1 and the lists L_m and $L_{\bar{m}}$ previously defined. The sub-matrix comprises at most two measured variables is given in table 3:

	1	**2**	3	**4**	5	**6**	7	8
1	**-1**	**.**	-1	**1**	.	**.**	.	.
2	**.**	**-1**	1	**.**	1	**.**	.	.
8	**1**	**.**	1	**.**	1	**1**	1	.
9	**1**	**.**	1	**.**	1	**1**	1	1
10	**.**	**.**	.	**.**	.	**.**	1	-1

Table 3: matrix of the cycles involving at most two measured variables

Following the different described steps, the reader will verify that one finds again the previous classification.

4 Degree of Redundancy Concept

From the preceding notion of observability, one may define the concept of degree of redundancy. It will be very useful when taking into account sensor breakdowns. Let us begin by the notion of minimal observability. A variable is redundant of degree 0 (minimal observability) if there exists, at least, a configuration such that the breakdown of only one sensor make this variable inaccessible. For example, it is the case of the measured but estimable variables. Some unmeasured variables may also own this property. Let us consider the network of figure 1 with $L_m = \{x1, x4\}$. Then $x6$ may be deduced from the equation issued from the "aggregation" of nodes I, II and III: $\boldsymbol{x1} + \boldsymbol{x4} - x6 = 0$. In a normal situation, when measurements of $x1$ and $x4$ are available, the variable $x6$ is deducible. But, when one of the sensors measuring $x1$ or $x4$ is faulty, the variable $x6$ become inaccessible. The variable $x6$ is said to be a "redundant variable of degree 0" or a "variable of minimal observability".

This previous notion may be extended. A redundant variable of degree k is an observable variable which value remains deducible even when k whatever sensors simultaneously breakdown. Let us illustrate a redundancy of degree 2 considering again the process of figure 1 and the list $L_m = \{x1, x2, x4, x5, x6\}$. There exist ten distinct combinations of two sensors simultaneously faulty among five. Let us consider the variable $x6$. The combinations involving the breakdowns of two sensors which do not measure the variable $x6$ are not injurious to the observability of this variable because its value remain known by its direct measurement. There exist six combinations of this type, let us analyse the four remaining ones. Let us begin by remembering the equations describing the system:

$$\boldsymbol{x1} - \boldsymbol{x2} - x3 = 0 \qquad (6a)$$
$$x3 + \boldsymbol{x4} - \boldsymbol{x5} = 0 \qquad (6b)$$
$$\boldsymbol{x2} + \boldsymbol{x5} - \boldsymbol{x6} = 0 \qquad (6c)$$
$$\boldsymbol{x6} - x7 - x8 = 0 \qquad (6d)$$

If the sensors measuring the variables $x1$ and $x6$ are faulty, then equation (6c) allows the variable $x6$ to be deduced because it is the only unknown quantity of this equation. The situation is identical if the sensors measuring the variables $x4$ and $x6$ are simultaneously faulty. If one of the following couple of sensor ($x2$, $x6$) or ($x5$, $x6$) is faulty, it is the equation issued from the "aggregation" of nodes I, II and III ($\boldsymbol{x1} + \boldsymbol{x4} - x6 = 0$) that allows the variable $x6$ to be deduced. In conclusion, the variable $x6$, which remains observable even when two whatever sensors are simultaneously faulty, is said to be redundant of degree 2.
The determination of the degree of redundancy of a variable is obtained by applying the following rule which is, in fact, an extension of the *rule I* involving the redundant variables (redundancy degree equal to 1):

Rule V: A variable is redundant of degree k if, and only if, it belongs to cycles comprising at least k+1 measured variables.

The degree of redundancy of a variable is easily determined by counting the minimum number of measured variables in the cycles where it intervenes. One may immediately deduced, for the considered example, that the variables $x1$, $x2$, $x3$, $x4$ and $x5$ are redundant of degree 1; they intervene in cycles comprising at least two measurement. The variable $x6$ is a redundant variable of degree 2 and the variables $x7$ and $x8$ are unobservable.

5 Sensor Placement subject to Redundancy Constraints

Now, we propose a method for designing a measurement system satisfying some redundancy requirements. In order to take into account industrial constraints, we first specify the list of variables which are absolutely necessary for controlling the process (list L_1); these variables ought to be of minimal observability. Then, we specify the lists the variables of which we ought to ensure a given degree of redundancy (list L_{dk} for the variables which must be redundant of degree k). Moreover, a weight, proportional to the installation cost of the corresponding sensor, is associated to each variable. The main goal of the design then consists in determining the variables which must be measured in order to satisfy the constraints on the degrees of redundancy whilst minimising the global cost of installation. Let n the maximum degree of redundancy required, the proposed algorithm then comprises the n+2 following steps:

Step n° 1: *Cycle matrix*
We first generate the matrix of all the cycles of the graph. This matrix is issued from the matrix of the fundamental cycles.
Step n° 2: *Minimal observability of required variables*
According to *rule V*, it is sufficient to measure, at least, one variable per cycle comprising a variable belonging to L_1. This analysis is a combinatorial one; after having considered all the possibility of measurements for each cycle, we generate all the possible combinations of variables to be measured. We then select that corresponding to the minimal cost.

Step n° 2+k: *Degree of redundancy of the variables belonging to* L_{dk} **(for $k = 1, ..., n$)**
According to *rule V* again, it is sufficient to measure k+1 variables per cycle comprising a variable belonging to L_{dk}. We proceed similarly to the previous step, taking into account the measurements already chosen during the previous steps. After having generated all the possible combinations, we keep that of minimal cost.

Consider the example described by the graph of figure 2.

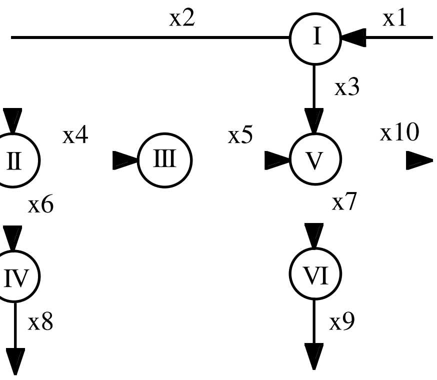

Figure 2: an elementary process

Let us assume that the list of the variables required for the control is the following: $L_1 = \{x1, x4, x6, x9, x10\}$. In order to present a comprehensive example, let us specify a list of variables which must be redundant of degree 1 only: $L_{d1} = \{x1, x9\}$. The costs associated to each sensor are given in table 4:

Var.	Cost	Var.	Cost
$x1$	3	$x6$	4
$x2$	3	$x7$	4
$x3$	4	$x8$	7
$x4$	1	$x9$	2
$x5$	9	$x10$	9

Table 4: costs associated to sensors

The matrix of all the cycles is generated during step n°1; it is given in Table 5.

1	2	3	4	5	6	7	8	9	10
1	1	.	.	.	1	.	1	.	.
.	.	.	.	.	.	1	.	1	1
1	.	1	.	.	.	1	.	1	.
1	1	.	1	1	.	1	.	1	.
1	1	.	.	.	1	1	1	1	1
.	1	1	.	.	1	1	1	1	.
.	.	.	1	1	1	1	1	1	.
1	.	1	.	.	.	.	.	.	1
1	1	.	1	1	.	.	.	.	1
.	1	1	1	1	.	.	.	.	.
.	1	1	.	.	1	.	1	.	1
.	.	.	1	1	1	.	1	.	1
1	.	1	1	1	1	.	1	.	.
.	1	1	1	1	.	1	.	1	1
1	.	1	1	1	1	1	1	1	1

Table 5: matrix C_t of all the cycles of the graph

The number of all the combinations of variables which must be measured for ensuring the minimal observability of the variables belonging to L_1 is too important for being presented here. The reader may verify, generating these combinations, that the minimal cost solution consists in measuring the variables $x1$, $x2$, $x4$ and $x9$. During step n°3, one searches to ensure to variables $x1$ and $x9$, a degree of redundancy equal to one. It is then necessary to consider cycles comprising less than two already measured variables. The sub-matrix of these cycles, where the measured variables are boldfaced, is given table 6.
Only the first two cycles comprise the variables $x1$ and $x9$ and they are already measured. For the first cycle, the possible combinations of variables to be measured are $\{x7\}$ or $\{x10\}$ and for the second one $\{x10\}$ or $\{x3\}$.

1	**2**	3	**4**	5	6	7	8	**9**	10
.	.	.	**.**	.	.	1	.	**1**	1
1	**.**	1	**.**	.	.	.	.	**1**	1
.	**1**	1	**.**	.	1	.	1	**.**	1
.	**.**	.	**1**	1	1	.	1	**.**	1

Table 6: sub-matrix of the useful cycles

The constraints on the degree of redundancy will be satisfied if we measure one of the following sets of variables: $\{x7, x10\}$, $\{x7, x3\}$, $\{x3, x10\}$ or $\{x10\}$. The minimal cost solution then consists in measuring the variables $x3$ and $x7$. It is very important to note that the minimisation of the cost implies the installation of the two sensors measuring $x3$ and $x7$ (cost equal to 8) whilst the sole respect of constraints on the redundancy degrees will lead to the installation of only one sensor measuring the variable $x10$ (cost equal to 9). This remark points out the importance of the choice of the optimised criterion which could, for example, take also into account the number of sensors to be installed. In conclusion, the set of variables which must be measured for satisfying the different given constraints whilst minimising the cost is the following: $\{x1, x2, x3, x4, x7, x9\}$. The corresponding cost is equal to 17 (3+3+4+1+4+2).

6 Reliability Calculation

For a given configuration of the measurement system (i.e. number, location and reliability of the sensors) it is important to analyse the influence of a sensor breakdown; for that purpose, we now compute the whole measurement system reliability. This will takes into account the constraint that the data required for the control be always available (either by direct measurement or by deduction if the observability conditions are satisfied). By definition, sensor reliability is the probability $r(t)$ that at the time t, there is no failure knowing that no failure had occurred at time t=0. By using the law of Poisson, we have:

$$r(t) = \exp(-\lambda t) \qquad (7)$$

where λ is a constant which represents the rate of sensor failure. The reliability of a set of n sensors is then expressed in terms of reliability of each sensor:

$$R(t) = f(r_1(t), ..., r_n(t)) \qquad (8)$$

In the following discussion, the sensor failure rates $r_i(t)$ are equal. However, the proposed strategy may be extended to the general case where specific reliabilities are considered. In practice, the evaluation of the system reliability uses the mean time to failure function (*MTTF*) defined by:

$$MTTF = \int_0^\infty R(t)\ \mathrm{d}t \qquad (9)$$

In order to define the link between reliability and observability, let us return to the example of figure 1 with measured variables $L_m = \{x1, x2, x4, x5\}$.

When considering the case where all the measured variables are necessary for process monitoring, $L_1 = \{x1, x2, x4, x5\}$, the probability that all the sensors are well functioning is given by:

$$R_0(t) = r(t)^4 = \exp(-4\lambda t) \qquad (10)$$

$$MTTF_0 = 0.25/\lambda \qquad (11)$$

Let us now examine how the redundancy due to the process model can be used in order to increase this *MTTF*. With the given measurements there exists only one redundancy equation: $x1 - x2 + x4 - x5$ (it is obtained the aggregation of the two first nodes of the graph). As all the variables in that equation are known, we can remove one of the sensors without modifying variable observability. Indeed, a defective sensor can be tolerated provided the value it measures may be deduced from data obtained from sound sensors. Thus, if one sensor (among the sensors measuring $x1$, $x2$, $x4$ or $x5$) is defective, the corresponding variable may be easily deduced from the remaining measurements. The probability that one sensor is defective and that the three others well function is:

$$R_1(t) = (1 - r(t))\ r(t)^3 \qquad (12)$$

There are four possibilities of one sensor breakdown each ensuring the observability of the corresponding variable. Considering now the whole probability that the system remains controllable, in the sense that the variables of L_1 are observable, yields the result:

$$R(t) = R_0(t) + 4\ R_1(t) \qquad (13)$$

from which we deduce:

$$MTTF = 0.58/\lambda \qquad (14)$$

This value must be compared with that obtained in (11) when the redundancy due to the process model is not taken into account. The reader should now ask if there exist admissible configurations with two sensor breakdowns. The analysis of the cycle matrix (table 2) shows that if two sensors are simultaneously faulty, then the variables of L_1 are not all observable (second rule of observability); therefore equation (14) gives the best reliability. However, more generally the reliability of a measurement system with c sensors is given by:

$$R(\mathrm{t}) = \sum_{i=0}^{c} a_i\ (1 - r(t))^i\ r(t)^{c-i} \qquad (15)$$

where the coefficients a_i give the number of configurations admitting i sensor breakdowns and ensuring the system control. To calculate $R(t)$ one must determine the coefficients a_i; a systematic procedure is developed in [15].

The *MTTF* gives a global characterisation of the reliability of a system. It is obvious that if we want to improve this *MTTF*, one have to reduce the number of "non reliable" components that is to say the cycles having the less number of measured variables. So, this remark emphasised the link between the reliability of a system and the degree of redundancy of the different variables. However, we clearly justify the use of the degree of redundancy previously introduced in section 4 rather than that of the ordinary concept of redundancy (section 3), by presenting the following example. We consider the process of figure 1 with two configurations of measurements. The first one is such that $L_m = \{x1, x2, x4, x5\}$ and the second one such that $L_m = \{x1, x3, x5, x6\}$. The list of variables required for controlling the process is $L_1 = \{x1, x3, x6\}$. Applying the proposed method for assessing the reliability of the whole system, one may ascertain that the *MTTF* corresponding to these two configurations of measurements is identical and is equal to $0.58/\lambda$. However, the classical analysis of observability shows that, for the first configuration, $x1$ is the only redundant variable, whilst for the second one all of the required variables are redundant. Using the proposed classification and analysis based on multiple redundancy, one may establish that, in fact, for both configurations, all the variables are redundant of degree 1 although some of them are not measured at all. This last remark emphasised the importance to use the proposed characterisation of the variables using their degree of redundancy rather than the classical one.

7 Conclusion

A new classification method based on the degree of redundancy of the variables was developed. Based on this analysis, a method for designing a measurement system guaranteeing the degree of redundancy of certain variables whilst minimising the installation cost of the sensors was proposed. After characterising a process by its *MTTF*, we have pointed out the relationships between the concepts of degree of redundancy and that of reliability highlighting the superiority of the proposed classification with respect to the classical one. The formulation of a multicriteria problem including observability, reliability, degree of redundancy, accuracy of deduction, costs of sensors is undoubtedly a fruitful future research direction.

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