Switching Systems Mode Estimation Using A Model-Based Diagnosis Method

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2007 Diagnostics of Processes and Systems

Introductory Example

Regression model with two modes

$$y(k) = \Theta_{\mu_k} \Psi_k^T + \varepsilon(k)$$

$$\Psi_k = \begin{pmatrix} y(k-1) & y(k-2) & u(k-1) \end{pmatrix}$$

$$\Theta_{\mu_k} = \begin{cases} \begin{pmatrix} a_{11} & a_{12} & b_{11} \end{pmatrix} & \text{if } \mu_k = 1 \\ \begin{pmatrix} a_{21} & a_{22} & b_{21} \end{pmatrix} & \text{if } \mu_k = 2 \end{cases}$$

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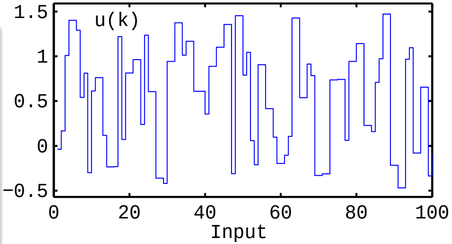

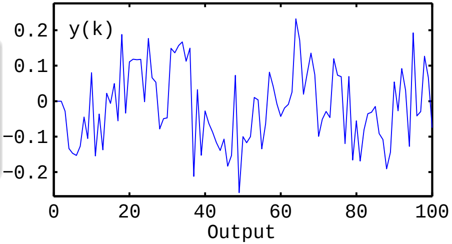

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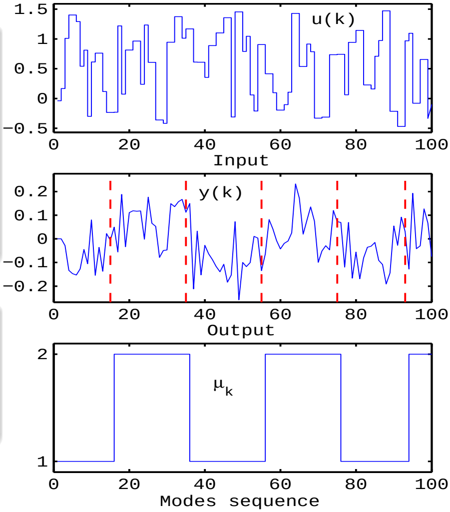

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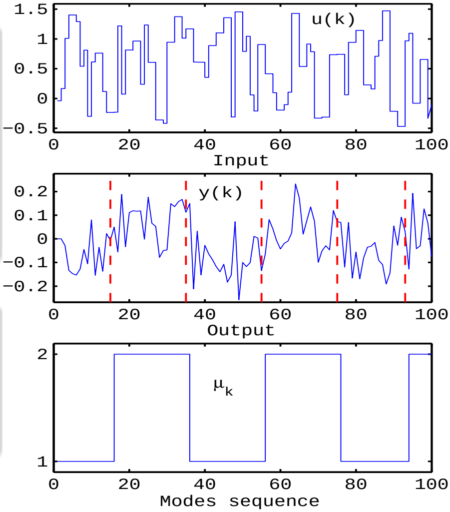

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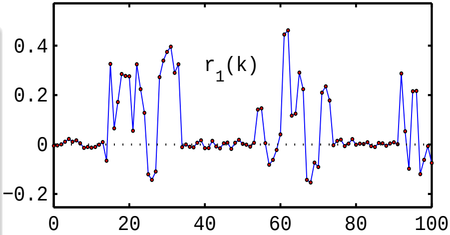

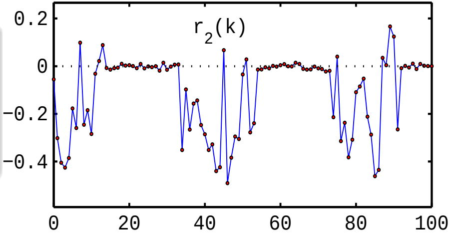

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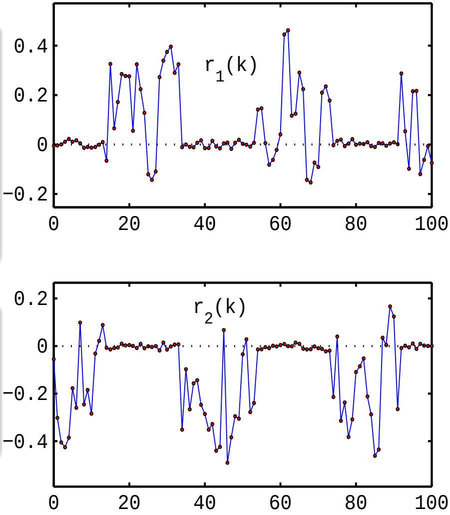

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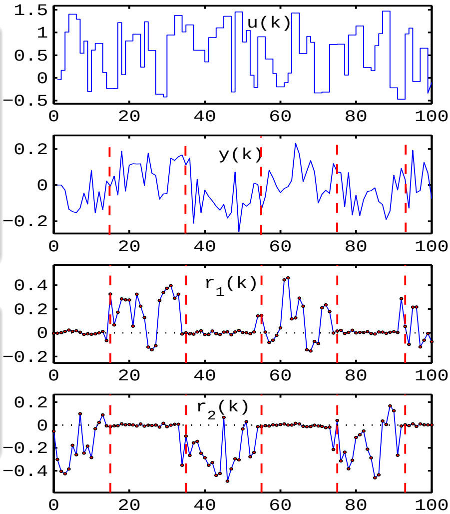

Outline

Problem Statement

Assumptions

- Input
- Output
- Number of modes
- Matrices describing the different modes

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Goals

- Active mode
- Switching times

Adopted Model

Model

$$\begin{cases} x(k+1) = A_{\mu_k} x(k) + Bu(k) \\ y(k) = Cx(k) \end{cases}$$

$$u(\cdot) \in \mathbb{R}^p, y(\cdot) \in \mathbb{R}^q, x(\cdot) \in \mathbb{R}^n$$

$$A_{\mu_k} \in \{A_1, A_2, \ldots, A_s\}, s \in \mathbb{N}^* \setminus \{1\}$$

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Definitions and notations

Definition (Path)

A path μ is a finite sequence of modes : $\mu = (\mu_1 \cdot \mu_2 \cdot \ldots \cdot \mu_N)$, $N \in \mathbb{N}^*$.

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Notations

- $|\mu|$: length of the path μ
  $\mu = (\mu_1 \cdot \mu_2 \cdot \ldots \cdot \mu_N) \Rightarrow |\mu| = N$.
- Θ_N : set of all paths of length N.
- $\mathcal{O}_{\mu,N}$: observability matrix

$$\mathcal{O}_{\mu,N} = \begin{pmatrix} C \\ CA_{\mu_1} \\ \vdots \\ CA_{\mu_{N-1}} \ldots A_{\mu_1} \end{pmatrix}$$

Active Mode Estimation

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$$\Downarrow$$

$$Y_{k-h,k} = \mathcal{O}_{\mu,h} x(k-h) + T_{\mu,h} U_{k-h,k-1}$$

$$T_{\mu,h} = \begin{pmatrix} 0 & 0 & \ldots & 0 \\ CB & 0 & & \\ \vdots & & & \\ CA_{\mu_{k-1}} \ldots A_{\mu_{k-h+1}} B & CA_{\mu_{k-1}} \ldots A_{\mu_{k-h+2}} B & \ldots & CB \end{pmatrix}$$

$$Y_{k-h,k} = \begin{pmatrix} y(k-h) & \ldots & y(k) \end{pmatrix}^T$$
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Active Mode Estimation

$$Y_{k-h,k} = \mathcal{O}_{\mu,h} x\left(k-h\right) + T_{\mu,h} U_{k-h,k-1}$$

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$$r_{\mu,h}\left(k\right) = \Omega_{\mu,h}\left(Y_{k-h,k} - T_{\mu,h} U_{k-h,k-1}\right)$$

with $\Omega_{\mu,h}\mathcal{O}_{\mu,h} = 0$

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$$Y_{k-h,k} = \mathcal{O}_{\mu,h} x(k-h) + T_{\mu,h} U_{k-h,k-1} + E_{k-h,k}$$

Influence on the residuals

$$\mu = \mu^* \Rightarrow r_{\mu^*,h}(k) = \Omega_{\mu,h} E_{k-h,k} \neq 0$$

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Path discernability

Definition (discernible paths : free-noise case)

μ^1 and μ^2 are discernible on a time window $[k-h,\ k]$ if :

$$\mu^1 = \mu^* \Rightarrow \begin{cases} r_{\mu^1,h}(k) = 0 \\ r_{\mu^2,h}(k) \neq 0 \end{cases}$$
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Path discernability

Theorem (Path discernability : free-noise case)

Two paths μ^1 and μ^2 are discernible on an observation window $[k-h,\ k]$ if :

$$\Omega_{\mu^i,h}\mathcal{O}_{\mu^j,h} \neq 0, \quad i,j \in \{1,2\}, i \neq j$$

or

$$\Omega_{\mu^i,h}\left(T_{\mu^j,h} - T_{\mu^i,h}\right) U_{k-h,k} \neq 0 \quad i,j \in \{1,2\}, i \neq j$$

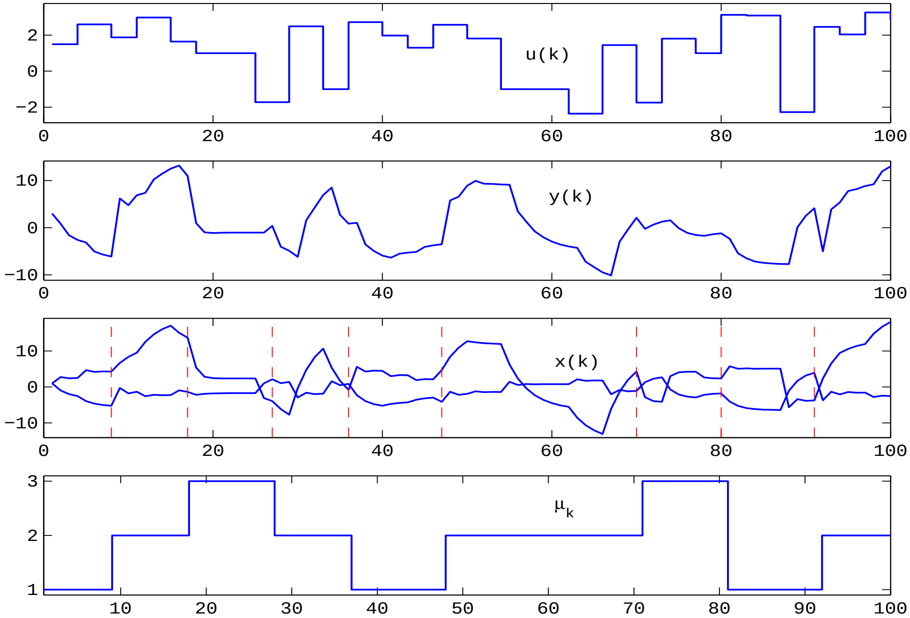
Academic Example

System matrices

$$A_1 = \begin{pmatrix} -0.211 & 0 \\ 0 & 0.521 \end{pmatrix} A_2 = \begin{pmatrix} 0.691 & 0 \\ 0 & -0.310 \end{pmatrix} A_3 = \begin{pmatrix} 0.153 & 0 \\ 0 & 0.410 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & -1 \end{pmatrix}^T C = \begin{pmatrix} 1 & 2 \end{pmatrix}$$

Academic Example



Academic Example

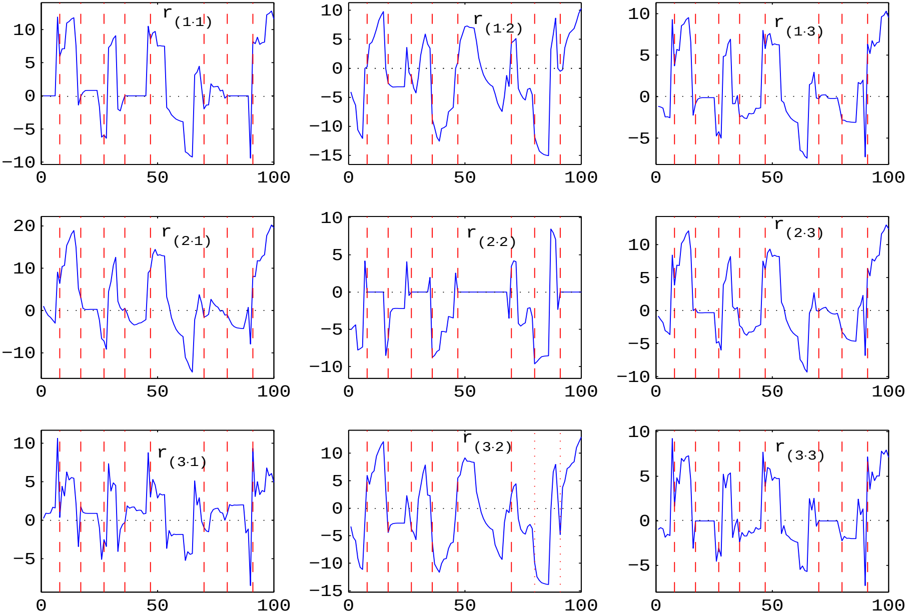
Path of length 2 on $[k-2, k]$

Path number	1	2	3	4	5	6	7	8	9
μ_1	1	1	1	2	2	2	3	3	3
μ_2	1	2	3	1	2	3	1	2	3

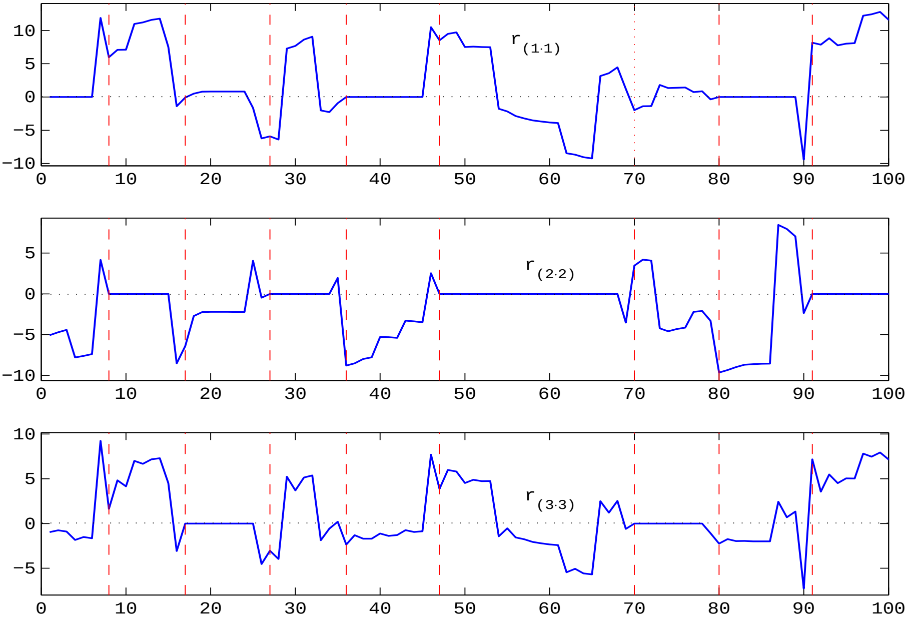
Observability matrices

$$\left\{\begin{array}{lll} \mathcal{O}_{(1\cdot1),h} = \begin{pmatrix} C \\ CA_1 \\ CA_1A_1 \end{pmatrix} & \mathcal{O}_{(1\cdot2),h} = \begin{pmatrix} C \\ CA_2 \\ CA_1A_2 \end{pmatrix} & \mathcal{O}_{(1\cdot3),h} = \begin{pmatrix} C \\ CA_3 \\ CA_1A_3 \end{pmatrix} \\ \mathcal{O}_{(2\cdot1),h} = \begin{pmatrix} C \\ CA_1 \\ CA_2A_1 \end{pmatrix} & \mathcal{O}_{(2\cdot2),h} = \begin{pmatrix} C \\ CA_2 \\ CA_2A_2 \end{pmatrix} & \mathcal{O}_{(2\cdot3),h} = \begin{pmatrix} C \\ CA_3 \\ CA_2A_3 \end{pmatrix} \\ O_{(3\cdot1),h} = \begin{pmatrix} C \\ CA_1 \\ CA_3A_1 \end{pmatrix} & \mathcal{O}_{(3\cdot2),h} = \begin{pmatrix} C \\ CA_2 \\ CA_3A_2 \end{pmatrix} & \mathcal{O}_{(3\cdot3),h} = \begin{pmatrix} C \\ CA_1 \\ CA_3A_3 \end{pmatrix} \end{array}\right.$$

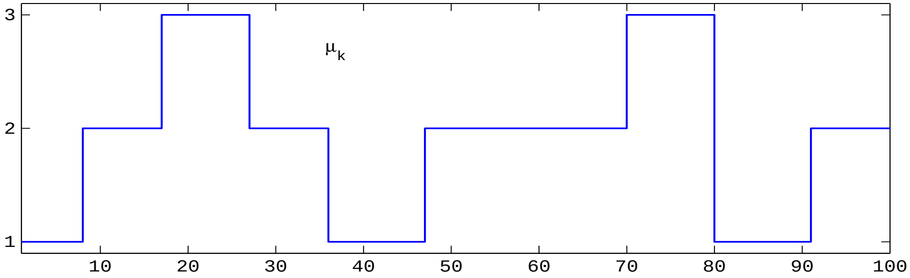
Academic Example

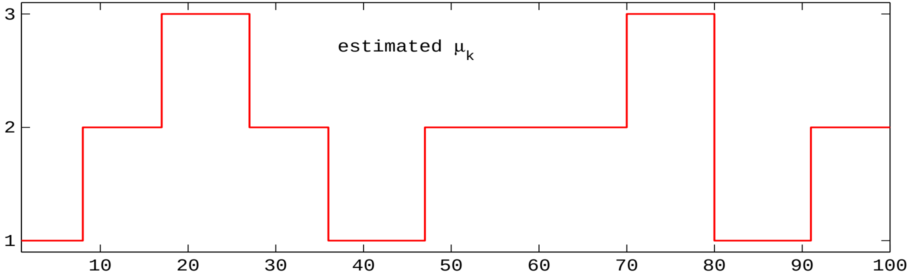


Academic Example

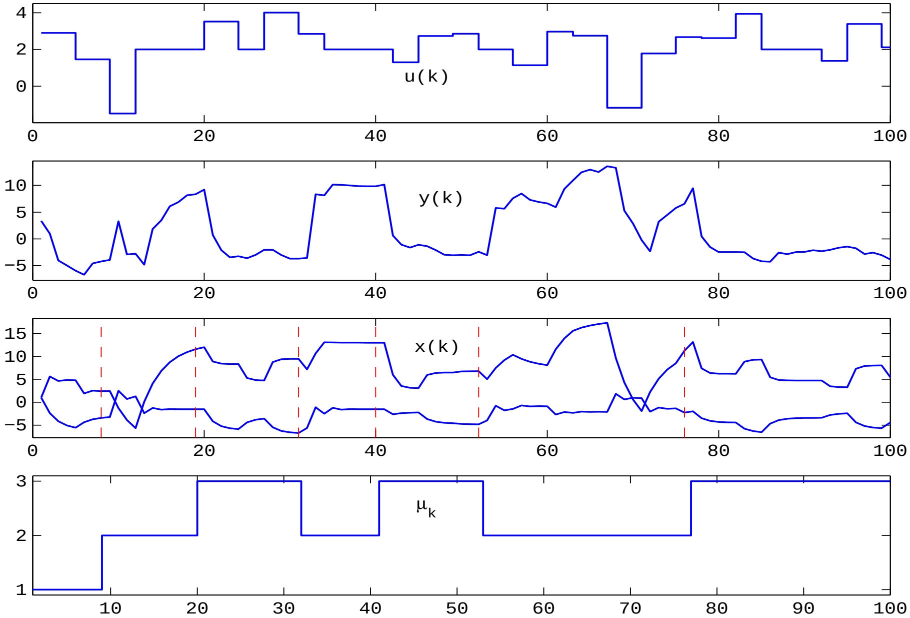


Academic Example

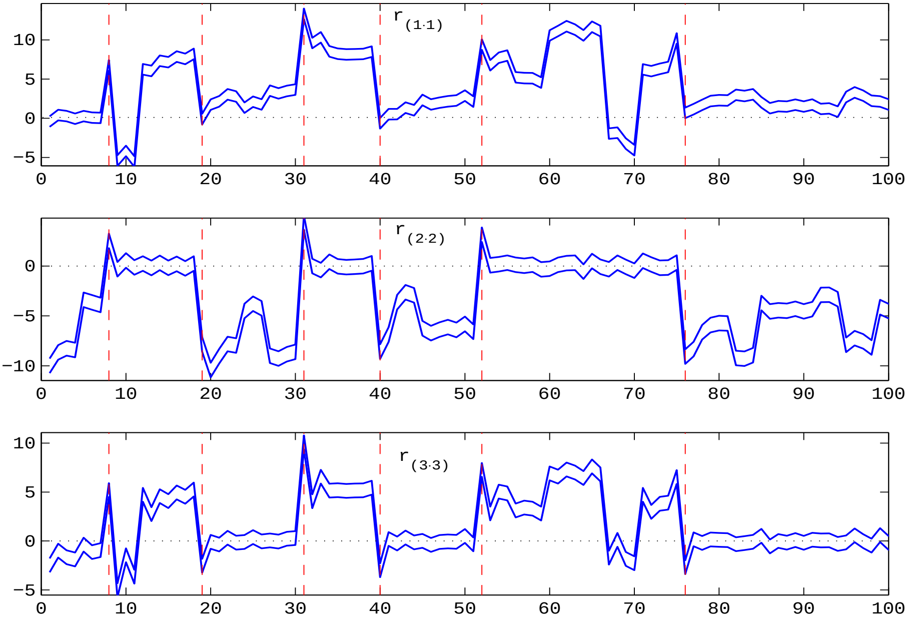




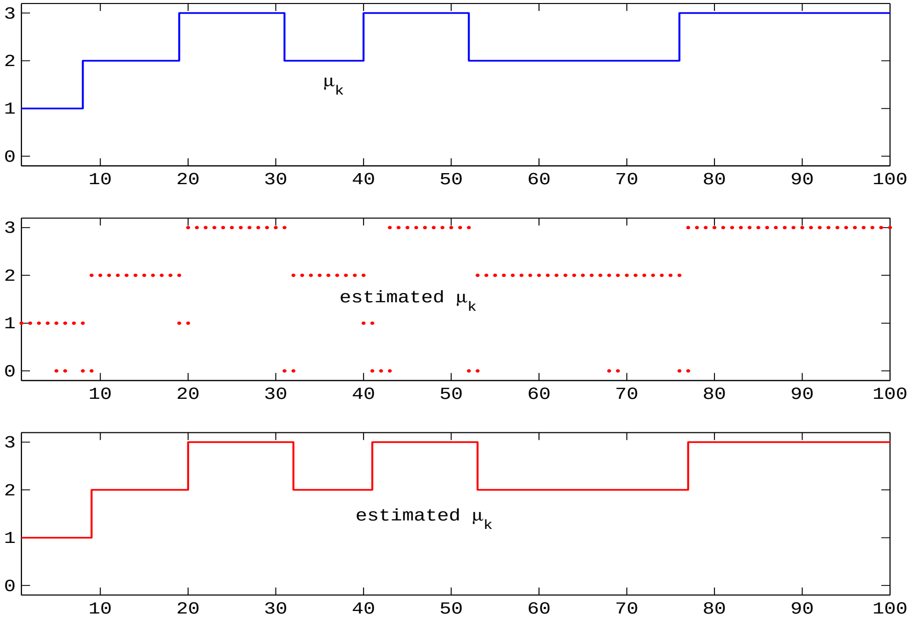
Academic Example



Academic Example



Academic Example



Conclusion

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- Discernability analysis

Future work

- Discernability conditions in the presence of noise
- Continuous state estimation
- Mode estimation with partial knowledge of the modes

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