# Observer-based controller for Takagi-Sugeno models


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Abstract-**This paper studies the stabilisation of T-S (Takagi-Sugeno) model using an output feedback controller. Both measurable and estimated decision variables cases are considered. In the case of measurable decision variables a T-S observer and T-S controller are designed separately to stabilise globally exponentially the closed loop T-S model. When the decision variables depend on the state variables estimated by the T-S observer, a procedure to design a stabilising output feedback controller is proposed. An example is given to illustrate the result.**



Key words-**T-S model, regulators, observers, Lyapunov method, LMI technique, separation principle.**


## I. Introduction

The issue of stability, the design of state feedback T-S controller as well as the design of state T-S observer for nonlinear systems described by T-S models [6] have been considered actively during the last decade [2][4][9]. Having the property of universal approximation [7][12], this approach includes the multiple model approach [6] and can be seen also as Polytopic Linear Differential Inclusions (PLDI) [13]. The T-S model consists to construct nonlinear dynamic system by means of interpolating the behaviour of several LTI submodels. Each submodel contributes to the global model in a particular subset of the operating space throughout *activation functions*.

Many works have been carried out to investigate the stability analysis of T-S systems using a quadratic Lyapunov function and sufficient conditions for the stability and stabilisability have been established [1][2][18]. The stability depends on the existence of a common positive definite matrix guarantying the stability of all local subsystems. These stability conditions may be expressed in linear matrix inequalities (LMIs) form [13]. The obtaining of a solution is then facilitated by using numerical toolboxes for solving such problems. For less of conservatism of the quadratic method, activation functions have been took into account [19][24]. To obtain relaxed stability conditions, piecewise quadratic Lyapunov function [3][23] and nonquadratic Lyapunov function [20]-[22] formulated as a set of LMIs are used. A certain form of T-S observers has been proposed and sufficient conditions for the asymptotic convergence are obtained which are dual to those for the stability of T-S controllers. LMIs constraints have been also used for pole assignment in LMI regions to achieve desired performances of T-S controllers and T-S observers [8][11]. Once a T-S observer is obtained, one might be tempted to think that this T-S observer can be used together with a state feedback T-S controller as in case of linear systems. It's well proved, in case of linear systems, that if only the constructed state is available one can combine state feedback controller and observer to obtain a stabilising output feedback controller [14]. For the T-S model some results on the separation property have been studied using the PDC (Parallel Distributed Compensation) controller in [16][15] and the CDF (Compensation and Division for Fuzzy models) control law in [17][21]. However, In [16][15][17][21][10] the decision variables are restricted to be measurable. In this paper, both measurable and estimated decision variables cases are considered to design an output feedback stabilising controller.

This paper is organised as follows. Section 2 recalls the structure of continuous T-s models. In section 3, under the assumption that the T-S model is locally stabilisable and locally detectable, sufficient conditions for the global exponential stability are derived in LMIs form for T-S observer (which are dual with those of the state feedback T-S controller). In the case of measurable decision variables, it is shown in the section 4.1 that a convergent T-S observer and stabilising T-S controller can be designed separately. In the section 4.2 a procedure to design a stabilising output feedback controller when the decision variables depend on the state variables estimated by the T-S observer is proposed. A numerical example is given to illustrate the result.

***Notation***: In this paper, we denote the minimum and maximum eigenvalues of the matrix $X$ by $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ respectively, the symmetric positive definite matrix $X$ by $X>0$, the transpose of $X$ by $X^T$, $\sum_{i<j}^{n} x_i x_j = \sum_{i=1}^{n} \sum_{j=1, j>i}^{n} x_i x_j$ .

## II. Continuous T-S model

A continuous T-S [5] model is based on the interpolation between several LTI local models as follows:

$$\dot{x}(t) = \sum_{i=1}^{n} \mu_i(z(t))(A_i x(t) + B_i u(t)) \qquad (1)$$

where $n$ is the number of submodels, $x(t) \in \mathbb{R}^p$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $A_i \in \mathbb{R}^{p.p}$, $B_i \in \mathbb{R}^{p.m}$ and $z(t) \in \mathbb{R}^q$ is the decision variable vector.
The choice of the decision variables $z(t)$ leads to different class of models. It can depend on the measurable (or estimated) state variables, be a function of the measurable outputs of the system and possibly on the input. In this case, the system (1) describes a nonlinear system. It can also be an unknown constant value, system (1) then represents a PLDI.

The normalized activation function $\mu_i(z(t))$ in relation with the $i^{th}$ submodel is such that:

$$\begin{cases} \sum_{i=1}^{n} \mu_i(z(t)) = 1 \\ \mu_i(z(t)) \geq 0 \quad \forall\, i \in \{1,..,n\} \end{cases} \tag{2}$$

The global output of T-S model is interpolated as follows:

$$y(t) = \sum_{i=1}^{n} \mu_i(z(t)) C_i x(t) \tag{3}$$

where $y(t) \in \mathbb{R}^l$ is the output vector and $C_i \in \mathbb{R}^{l.p}$. More detail about this type of representation can be found in [1][2]. In the sequel, we denote by $r$ the number of submodels simultaneously activated.

## III. Stability analysis

The open loop T-S model of (2) is defined as:

$$\dot{x}(t) = \sum_{i=1}^{n} \mu_i(z(t)) A_i x(t) \tag{4}$$

Basic stability conditions based on the quadratic Lyapunov functions are given by the following result

The continuous T-S model described by (4) is globally asymptotically stable if there exists a common matrix $P = P^T > 0$ such that [2]:

$$A_i^T P + P A_i < 0 \quad \forall\, i \in \{1,..,n\} \tag{5}$$

### A. T-S controller design

In order to stabilise the T-S model (2) a T-S controller can be designed using the PDC technique [1]. In this case, the global control law is obtained by interpolation of local linear feedback laws related with each submodel.
For the T-S controller design, it is supposed that the system (1) is locally stabilisable, i.e. the pairs $(A_i, B_i)$, $\forall\, i \in \{1,..,n\}$ are stabilisable.

The resulting global controller when all decision variables are measurable is:

$$u(t) = -\sum_{i=1}^{n} \mu_i(z(t)) K_i x(t) \tag{6}$$

where $\mu_i(z(t))$ has to respect constraint (3). Substituting (6) in (1), we obtain the closed loop continuous T-S model:

$$\dot{x}(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_i(z(t)) \mu_j(z(t)) R_{ij} x(t) \tag{7}$$

where

$$R_{ij} = A_i - B_i K_j \tag{8}$$

In order to simplify the notation of the forthcoming equations, lets us denote:

$$L(X_{ij}, P) = \left( \frac{X_{ij} + X_{ji}}{2} \right)^T P + P \left( \frac{X_{ij} + X_{ji}}{2} \right) \tag{9}$$

Results in [2] gives sufficient stability conditions for (7). In theorem 1 we extend these conditions to exponential stability

***Theorem 1***: Suppose that there exists symmetric positive definite matrices $P_1$ and $Q_1$ such that

$$L(R_{ii}, P_1) + (r - 1/2) Q_1 < 0 \tag{10a}$$

$$L(R_{ij}, P_1) - Q_1 / 2 \leq 0 \tag{10b}$$

$\forall i < j \in \{1,..,n\}$ and $\mu_i(z(t)) \mu_j(z(t)) \neq 0$. Then the closed loop continuous T-S model described by (7) is globally exponentially stable.

*Proof*: The proof is obtained by considering the derivative of the quadratic Lyapunov function candidate $V(x(t)) = x(t)^T P_1 x(t)$, $P_1 > 0$, along the trajectory of the T-S model (7), and corollary 4 of [2]. We obtain after some elementary operations:

$$\dot{V}(x(t)) < -\frac{\lambda_{\min}(Q_1)}{2\lambda_{\max}(P_1)} V(x(t)). \qquad \blacksquare$$

The control design problem is to find the feedback gains $K_i$ such that the closed loop system (7) is stable. The conditions (10) are not convex in $P_1$ and $K_i$. In order to convert them into an LMI problem, these inequalities are multiplied in the left and the right by $P_1^{-1}$. Then, taking into account the definition (8), the constraints (10) become

$$A_i X_1 + X_1 A_i^T - B_i Y_i - Y_i^T B_i^T + \left( r - \frac{1}{2} \right) S_1 < 0 \tag{11a}$$

$$\begin{aligned} &(A_i + A_j) X_1 + X_1 (A_i + A_j)^T - B_i Y_j - B_j Y_i - \\ &Y_i^T B_j^T - Y_j^T B_i^T - S_1 < 0 \end{aligned} \tag{11b}$$

$\forall\, i < j \in \{1,..,n\}$ and $\mu_i(z(t)) \mu_j(z(t)) \neq 0$.

which are LMIs in $X_1, Y_i$ and $S$ with $X_1 = P_1^{-1}, Y_i = K_i X_1$ and $S = X_1 Q X_1$.

### B. T-S observer design

The T-S controller proposed in previous section is based on a state feedback. However, in practice, all the states of a system are not fully measurable. Thus, the problem addressed in this section is the construction of a T-S observer to estimate the states of the T-S model (1).

It is supposed that the decision variables $z(t)$ are measurable and the T-S model (1) is locally detectable, i.e. the pairs $(A_i, C_i), \forall\, i \in \{1,..,n\}$ are detectable. Using the same structure as the one for T-S controller design, the T-S observer for the T-S model (1) is written as follows

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{n} \mu_i(z(t))(A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))) \\ \hat{y}(t) = \sum_{i=1}^{n} \mu_i(z(t)) C_i \hat{x}(t) \end{cases} \tag{12}$$

where $\hat{x}(t)$ and $\hat{y}(t)$ denote the estimated state vector and output vector respectively. The activation functions $\mu_i(z(t))$ are the same than those used in the T-S model (1).

Denoting the state estimation error by

$$\tilde{x}(t) = x(t) - \hat{x}(t) \tag{13}$$

it follows from (1) and (12) that the observer error dynamic is given by the differential equation

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i(z(t))\mu_j(z(t))\Theta_{ij}\tilde{x}(t) \tag{14}$$

where

$$\Theta_{ij} = A_i - L_i C_j \tag{15}$$

The design of the observer consists to determine the local gains $L_i$ to ensure the convergence to zero of the estimation error. To prove the global exponential stability conditions of the T-S observer (14), it suffices to find symmetric matrices $P_2 > 0$ and $Q_2 > 0$ such that

$$L(\Theta_{ii}, P_2) + (r - 1/2)Q_2 < 0 \tag{16a}$$

$$L(\Theta_{ij}, P_2) - Q_2 / 2 \le 0 \tag{16b}$$

$\forall i < j \in \{1,..,n\}$ and $\mu_i(z(t))\mu_j(z(t)) \ne 0$.

## IV. Output feedback control design

### A. Case of measurable decision variables

In this section, we show that the separation property studied in [15][16] holds also for the global exponential stability stated above. We prove using the LMI formulation that the combination of the global exponential stability of T-S observer and the global exponential stability of T-S controller guarantees the global exponential stability of the closed loop system. A systematic method to compute a quadratic Lyapunov function showing that the separation principle holds for the suggested quadratic stability conditions is given.

If, instead of the actual state, the estimated state $\hat{x}(t)$ is available, the control law with the PDC technique (7) becomes

$$u(t) = -\sum_{i=1}^{n} \mu_i(z(t)) K_i \hat{x}(t) \tag{17}$$

Taking into account (12) and (17), we have

$$\dot{\hat{x}}(t) = \sum_{j=1}^{n}\sum_{i=1}^{n} \mu_i(z(t))\mu_j(z(t))\left(R_{ij}\hat{x}(t) + L_i C_j \tilde{x}(t)\right) \tag{18}$$

where $R_{ij}$ and $\tilde{x}(t)$ are defined in (9) and (13) respectively. Combining (18) and (14) we obtain the following augmented system

$$\dot{\bar{x}} = \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i(z(t))\mu_j(z(t))\bar{A}_{ij}\bar{x} \tag{19a}$$

where:

$$\bar{A}_{ij} = \begin{pmatrix} R_{ij} & L_i C_j \\ 0 & \Theta_{ij} \end{pmatrix}, \; \bar{x}(t) = \left(\hat{x}(t)^T, \quad \tilde{x}(t)^T\right)^T \tag{19b}$$

and $\Theta_{ij}$ is defined in (15).

Thus it is possible to apply theorem 1 in order to check the global exponential stability of the closed loop T-S system (19). It suffices to find symmetric matrices $P > 0$ and $Q > 0$ such that

$$L(\bar{A}_{ii}, P) + (r - 1/2)Q < 0 \tag{20a}$$

$$L(\bar{A}_{ij}, P) - Q/2 < 0 \tag{20b}$$

$\forall i < j \in \{1,..,n\}$ and $\mu_i(z(t))\mu_j(z(t)) \ne 0$.

Thus to prove the global exponential stability, we need to compute the controller gains $K_i$, the observer gains $L_i$ and the symmetric positive definite matrices $P$ and $Q$ respecting the constraints (20). These latter, which are non convex in the variables $K_i$, $L_i$ and $P$, are difficult to convert into an LMI problem using the linearisation method described at the end of paragraph 3.1. In order to overcome this difficulty, the following theorem shows that it suffices to prove the stability of both the T-S controller and the T-S observer independently for proving the global exponential stability of the augmented systems (19). By the same way, we will show that this property guarantees the existence of a

Lyapunov function parameterised by a positive scalar $\sigma$ of the form:

$$V(x(t)) = x(t)^T P(\sigma)x(t),\ P = \begin{pmatrix} P_1 & 0 \\ 0 & \sigma P_2 \end{pmatrix} \quad (21)$$

allowing to prove the stability of the augmented system (19).

***Theorem 2***: Suppose that there exists symmetric matrices $P_1 > 0$, $P_2 > 0$, $Q_1 > 0$ and $Q_2 > 0$ such that

$$L(R_{ii}, P_1) + (r - 1/2)Q_1 < 0 \quad (22a)$$

$$L(R_{ij}, P_1) - Q_1 / 2 < 0 \quad (22b)$$

$$L(\Theta_{ii}, P_2) + (r - 1/2)Q_2 < 0 \quad (23a)$$

$$L(\Theta_{ij}, P_2) - Q_2 / 2 < 0 \quad (23b)$$

$\forall i{:}1,..,n$, $\forall i < j \in \{1,..,n\}$ and $\mu_i(z(t))\mu_j(z(t)) \neq 0$. Then the function (21), with $\sigma \geq Max(\sigma_1, \ \sigma_2) > 0$ respecting the conditions (24)-(25), is a Lyapunov function of the augmented system (19).

$$\sigma_1 = \frac{\lambda_{\min}\left( P_1 L_i C_i \left( L(\Theta_{ii}, P_2) + (r-1/2)Q_2 \right)^{-1} (P_1 L_i C_i)^T \right)}{\lambda_{\max}\left( L(R_{ii}, P_1) + (r-1/2)Q_1 \right)} \quad (24)$$

$$\sigma_2 = \frac{\lambda_{\min}\left( P_1 (L_i C_j + L_j C_i)\left(2L(\Theta_{ij}, P_2) - Q_2\right)^{-1} \left(P_1 (L_i C_j + L_j C_i)\right)^T \right)}{\lambda_{\max}\left( 2L(R_{ij}, P_1) - Q_1 \right)} \quad (25)$$

$\forall\ i < j \in \{1,..,n\}$ and $\mu_i(z(t))\mu_j(z(t)) \neq 0$.

*Proof*: With the following structure of $P$ and $Q$

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & \sigma P_2 \end{pmatrix},\ Q = \begin{pmatrix} Q_1 & 0 \\ 0 & \sigma Q_2 \end{pmatrix} > 0,\ \sigma \in \mathbb{R}^{+*} \quad (26)$$

the inequalities (20) with the definition (19b) allow writing

$$\begin{pmatrix} L(R_{ii}, P_1) & P_1 L_i C_i \\ (P_1 L_i C_i)^T & \sigma L(\Theta_{ii}, P_2) \end{pmatrix} + (r - 1/2)\begin{pmatrix} Q_1 & 0 \\ 0 & \sigma Q_2 \end{pmatrix} < 0 \quad (27a)$$

$$\begin{pmatrix} 2L(R_{ij}, P_1) & P_1(L_i C_j + L_j C_i) \\ \left(P_1(L_i C_j + L_j C_i)\right)^T & 2\sigma L(\Theta_{ij}, P_2) \end{pmatrix} - \begin{pmatrix} Q_1 & 0 \\ 0 & \sigma Q_2 \end{pmatrix} < 0 \quad (27b)$$

$\forall\ i < j \in \{1,..,n\}$ and $\mu_i(z(t))\mu_j(z(t)) \neq 0$.

Applying the Schur complement [13] to the constraints (27), we prove that it suffices to respect the following sufficient conditions to guarantee the separation property:

$$\sigma \geq Max(\sigma_1, \ \sigma_2) > 0 \quad (28)$$

where $\sigma_1$ and $\sigma_2$ are defined in (24) and (25). The condition (28) shows that $\sigma$ always exists if the inequalities (22)-(23) are satisfied. ■

To summarise, it's ensured, in the case where all the decision variables of the T-S model (1) are measurable, that if the exponential stability of T-S controller (inequalities (22)) and those of the T-S observer (inequalities (23)) are satisfied independently then the augmented T-S model (19) is always exponentially stable and accept the function (21) as Lyapunov function.

### B. *Case of estimated decision variables*

All the decision variables of the T-S model (1) are assumed to be measurable in the above parts as in [15][16][17][21][10]. However, in general, this assumption is not verified. In the following part, we assume that the decision variables depend on states variables estimated by a T-S observer. Therefore, the activation functions of the controller are different from the activation functions of the T-S model (1) as they depend on estimated state variables. In the sequel the estimated decision variable vector is denoted by $\hat{z}(t)$.

The T-S observer (12) becomes

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{n} \mu_i(\hat{z}(t))(A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))) \\ \hat{y}(t) = \sum_{i=1}^{n} \mu_i(\hat{z}(t)) C_i \hat{x}(t) \end{cases} \quad (29)$$

with

$$u(t) = -\sum_{i=1}^{n} \mu_i(\hat{z}(t)) K_i \hat{x}(t) \quad (30)$$

where $\hat{z}(t)$ is the vector of estimated decision variables depending on the estimated state variables $\hat{x}(t)$ and possibly on the input $u(t)$. The augmented system (19) becomes in this case

$$\dot{\bar{x}} = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{h=1}^{n} \mu_i(z(t))\mu_j(\hat{z}(t))\mu_h(\hat{z}(t))\bar{A}_{ijh}\bar{x}(t) \quad (31)$$

where:

$$\bar{A}_{ijh} = \begin{pmatrix} R_{ih} & B_i K_h \\ S_{ijh} & \Theta_{jh} + \Delta B_{ij} K_h \end{pmatrix} \quad (32)$$

$$S_{ijh} = \Delta A_{ij} - \Delta B_{ij} K_h + L_j \Delta C_{hi} \quad (33a)$$

$$\Delta A_{ij} = A_i - A_j, \Delta B_{ij} = B_i - B_j, \Delta C_{hi} = C_h - C_i \quad (33b)$$

and $\bar{x}(t), R_{ih}$ and $\Theta_{jh}$ are defined in (19b), (8) and (15) respectively. The asymptotic stability of the augmented system (31) can be derived easily as follows [2].

***Theorem 3***: Suppose that there exists symmetric matrix $P > 0$ such that

$$\bar{A}_{ijj}^T P + P\bar{A}_{ijj} < 0 \quad (34a)$$

$$\left(\frac{\bar{A}_{ijh} + \bar{A}_{ihj}}{2}\right)^T P + P\left(\frac{\bar{A}_{ijh} + \bar{A}_{ihj}}{2}\right) < 0 \quad (34b)$$

$\forall i, j < h \in \{1,..,n\}$ and $\mu_i(z(t))\mu_j(\hat{z}(t))\mu_h(\hat{z}(t)) \neq 0$. Then the closed loop continuous T-S model described by (31) is globally asymptotically stable. ■

It should be noted that it is not possible to relax conditions (34) of theorem 3 as those of theorem 1 due to $\mu_i(z(t)) \neq \mu_i(\hat{z}(t))$.
Since conditions (34) are non convex, it is difficult to transform them into LMIs in $P, K_i$ and $L_i$. To solve those constraints we propose the following technique. To design the T-S controller and the T-S observer separately, we chose

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \quad (35)$$

Substituting (35) into (34), we obtain

$$\begin{pmatrix} R_{ij}^T P_1 + P_1 R_{ij} & P_1 B_i K_j + S_{ijj}^T P_2 \\ P_2 S_{ijj} + K_j^T B_i^T P_1 & \Xi_{ijj} \end{pmatrix} < 0 \quad (36a)$$

$$\begin{pmatrix} \left(R_{ih} + R_{ij}\right)^T P_1 + P_1\left(R_{ih} + R_{ij}\right) & (\bullet)^T \\ P_2\left(S_{ijh} + S_{ihj}\right) + \left(K_h + K_j\right)^T B_i^T P_1 & \Xi_{ijh} + \Xi_{ihj} \end{pmatrix} < 0 \quad (36b)$$

where $(\bullet)^T$ represent $\left(P_2\left(S_{ijh} + S_{ihj}\right) + \left(K_h + K_j\right)^T B_i^T P_1\right)^T$ with definition (33) and

$$\Xi_{ijh} = \left(\Theta_{jh} + \Delta B_{ij} K_h\right)^T P_2 + P_2\left(\Theta_{jh} + \Delta B_{ij} K_h\right) \quad (37)$$

The obtained matrices inequalities (36) are still BMIs (Bilinear Matrix Inequalities) in $P_1, P_2, K_i$ and $L_i$ which are difficult to solve simultaneously. Indeed the BMIs (36) imply that

$$R_{ij}^T P_1 + P_1 R_{ij} < 0 \quad (38)$$

$$\left(R_{ih} + R_{ij}\right)^T P_1 + P_1\left(R_{ih} + R_{ij}\right) < 0 \quad (39)$$

which are easy to transform into LMIs form with the same procedure as stated at the end of section 3.A. Once $P_1$ and $K_i, \forall\, i \in \{1,..,n\}$ are obtained, we substitute them into (36). The obtained conditions are LMIs in $P_2$ and $L_i, \forall\, i \in \{1,..,n\}$ and can be solved easily by a convex optimisation technique such as the interior point method.

## V. Numerical example

The following example illustrates the case of measurable decision variables. Let us consider the T-S model (1)-(3) where $r = n = 2$ and

$$A_1 = \begin{pmatrix} 2 & -10 \\ 1 & 0 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad C_1 = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad (40a)$$

$$A_2 = \begin{pmatrix} 49 & -10 \\ 1 & 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 10 \\ 0.5 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad (40b)$$

$$\mu_1(x(t)) = \begin{cases} 1 - \dfrac{|x_1(t)|}{3} & \forall\, x_1(t) \in [-3, \ 3] \\ 0 & \text{otherwise} \end{cases} \quad (40c)$$

$$\mu_2(x(t)) = \begin{cases} \dfrac{|x_1(t)|}{3} & \forall\, x_1(t) \in [-3, \ 3] \\ 1 & \text{otherwise} \end{cases} \quad (40d)$$

From conditions (22) given in theorem 2 and with definition (8) we obtain the following feedback gains and the positive definite matrices (after linearisation as it is described at the end of paragraph 3.A):

$$K_1 = (4.9225 \quad -0.5300),\ K_2 = (5.1846 \quad -0.0877) \quad (41)$$

$$P_1 = \begin{pmatrix} 0.0277 & 0.0343 \\ 0.0343 & 0.1475 \end{pmatrix},\ Q_1 = \begin{pmatrix} 0.0320 & 0.0645 \\ 0.0645 & 0.1710 \end{pmatrix} \quad (42)$$

And from conditions (23) and with definition (15) we obtain the following observer gains which ensure the exponential convergence of state and the definite positive matrices:

$$L_1 = \begin{pmatrix} 3.0398 \\ -9.0539 \end{pmatrix},\ L_2 = \begin{pmatrix} 50.0398 \\ -9.0539 \end{pmatrix} \quad (43)$$

$$P_2 = \begin{pmatrix} 29.9427 & 1.5509 \\ 1.5509 & 29.9427 \end{pmatrix},\ Q_2 = \begin{pmatrix} 8.4514 & 0.0000 \\ 0.0000 & 8.4275 \end{pmatrix} \quad (44)$$

The conditions (24) and (25) allow to compute respectively $\sigma_1 = 9.6869$ and $\sigma_2 = 1.9098$. Then, as it is shown in (28), the choice of the following symmetric positive definite matrices with for instance $\sigma = 10$:

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & \sigma P_2 \end{pmatrix},\ Q = \begin{pmatrix} Q_1 & 0 \\ 0 & \sigma Q_2 \end{pmatrix},\ \sigma \geq Max(\sigma_1, \sigma_2) \quad (45)$$

guarantees the global exponential stability of the augmented system of (40) and prove that the design of T-S controllers and T-S observers can be done separately. The simulation result of the closed loop T-S model (40) with the control law (17) is given in figure 1.

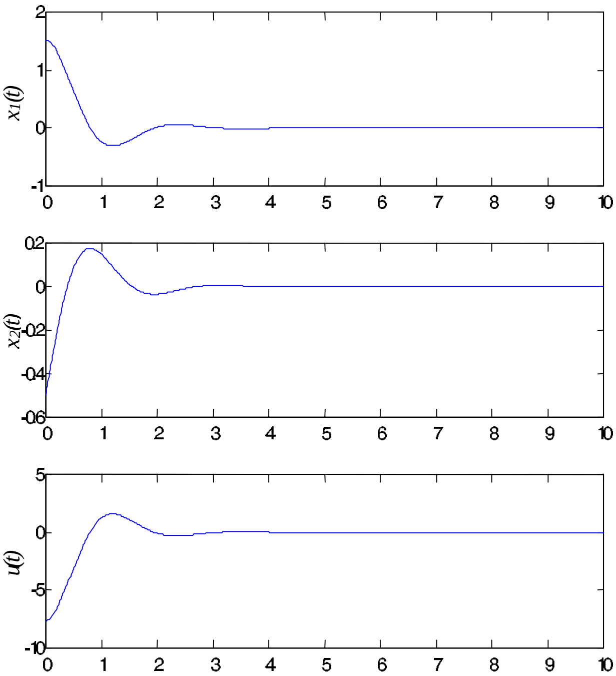


Figure 1. Closed loop system of (40) with $x(0)=(1.5 \quad -0.5)$

## VI. Conclusion

In this paper, the stabilisation of T-S model using an output feedback controller for both measurable and estimated decision variables cases are considered. In the case of measurable decision variables a T-S observer and T-S controller are designed separately to stabilise globally exponentially the closed loop T-S model. When the decision variables depend on the state variables estimated by the T-S observers, a procedure to design (separately but not simultaneously) a stabilising output feedback controller is proposed.

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