Proceedings of the 2005 IEEE
International Symposium on Intelligent Control
Limassol, Cyprus, June 27-29, 2005

Quadratic stability and stabilisation of interval Takagi-Sugeno model : $\mathcal{LMI}$ approach

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***Abstract*— This paper deals with the stability analysis of interval Takagi-Sugeno. Based on a quadratic Lyapunov function, new asymptotic stability conditions for continuous case are presented without any assumption on the norm of matrices uncertainties. This result is obtained directly according to the bounds (minimal and maximum) of the intervals of each element of matrices representing the system. These stability conditions, extended to the design of controller, are formulated in terms of linear matrix inequalities ($\mathcal{LMI}$). Example is given to illustrate the proposed method.**

I. INTRODUCTION

In the last decade, engineers have successfully utilized Takagi-Sugeno (T-S) approach in modelling and control [7]. T-S allow to represent nonlinear dynamic systems and their basic structure includes a number of approaches: multiple model [4], PLDI [8].
Analysis and synthesis studies of T-S based on quadratic Lyapunov functions lead to result which are often conservative [18], [6], [2]. To overcome these conservatism non quadratic Lyapunov functions may be used. Among these functions, we can quote the piecewise quadratic function [17]. The stability analysis using this type of function was studied these last years by using the uncertain system techniques [10]. In [11] another class of non quadratic Lyapunov functions of the form was also considered. Some works also propose another type of non quadratic Lyapunov function - polyquadratic and piecewise Lyapunov functions- [9], [19], [12], [16], [13]. The obtained results make it possible to also reduce the conservatism of the quadratic approach. However, in engineering problems, systems are often complex, uncertain and ill-defined. Recently, some works have proposed methods of studying stability analysis and designing controllers for nonlinear systems with uncertainties [6], [15]. The use of intervals allows one to take account of uncertainties, the parameters of a system being considered as variable but belonging to a bounded domain [1], [3], [5]. Another works has been also published on the use of interval arithmetic [1], [3], [5], mainly in modelling, identification [20] and control [14].

In this paper, the interval T-S ($\mathcal{ITS}$) representation, allowing to take account of bounded uncertainties affecting the parameters of the process, is used. The idea consists on representing the uncertainty by means of interval parameters. The paper, based on the work of [3] in the case of linear system, is organized as follows. In the next section, $\mathcal{ITS}$ is defined. In section 3, using the quadratic Lyapunov approach, stability analysis of such system is performed leading to conditions expressed in $\mathcal{LMI}$ formulation [8]. The section 4 is devoted to the design of robust controller of $\mathcal{ITS}$. Two illustrative examples are then developed in the last section.

Notations - Throughout the paper, the following useful notation is used: X^T denotes the transpose of the matrix X, $X > 0$ $(X \geq 0)$ means that X is a symmetric positive definite (semidefinite) matrix, I_n denotes the $n.n$ identity matrix, $I_s = \{1, 2, .., s\}$, $diag(X_1, ..., X_n)$ is a diagonal matrix which the diagonal elements are X_i, $i \in I_n$ and $[X]$, with $X = (x_{ij}) \in R^{u.v}$, denotes an interval matrix such that $[X] \in [\underline{X},\ \overline{X}]$ and $[\underline{X},\ \overline{X}] = \{[x_{ij}] : \underline{x}_{ij} \leq [x_{ij}] \leq \overline{x}_{ij},\ (i,j) \in I_u \times I_v\}$.

II. INTERVAL TAKAGI-SUGENO REPRESENTATION

The $\mathcal{ITS}$ is represented as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{p=1}^{s} \mu_p(z(t))([A_p]x(t) + [B_p]u(t)) \\ y(t) &= \sum_{p=1}^{s} \mu_p(z(t))[C_p]x(t) \end{aligned} \tag{1}$$

with s is the number of local models, $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input vector, $y(t) \in R^l$ is the output vector, $z(t) \in R^w$ is the vector of the so-called decision variables, $[A_p] \in R^{n.n}$, $[B_p] \in R^{n.m}$ and $[C_p] \in R^{l.n}$ are interval matrices characterizing the p^{th} local model. Uncertainties affecting the parameters of the T-S are taken into account when considering the lower and upper bounds of matrices $[A_p]$ and $[B_p]$ respectively defined by:

$$\begin{aligned} \underline{A}_p &= \begin{pmatrix} \underline{a}_{11p} & & \underline{a}_{1np} \\ \vdots & \ddots & \\ \underline{a}_{n1p} & & \underline{a}_{nnp} \end{pmatrix}, \quad \overline{A}_p = \begin{pmatrix} \overline{a}_{11p} & & \overline{a}_{1np} \\ \vdots & \ddots & \\ \overline{a}_{n1p} & & \overline{a}_{nnp} \end{pmatrix} \\ \underline{B}_p &= \begin{pmatrix} \underline{b}_{11p} & & \underline{b}_{1mp} \\ \vdots & \ddots & \\ \underline{b}_{n1p} & & \underline{b}_{nmp} \end{pmatrix}, \quad \overline{B}_p = \begin{pmatrix} \overline{b}_{11p} & & \overline{b}_{1mp} \\ \vdots & \ddots & \\ \overline{b}_{n1p} & & \overline{b}_{nmp} \end{pmatrix} \end{aligned} \tag{2}$$

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The activation functions $\mu_p(.)$ are such that:

$$\left\{ \begin{array}{l} \sum\limits_{p=1}^{s} \mu_p(z(t)) = 1 \\ \mu_p(z(t)) \geq 0, \quad \forall\, p \,\in\, I_s \end{array} \right. \tag{3}$$

The choice of the variable $z(t)$ leads to different classes of models. It can depend on the measurable state variables, be a function of the measurable outputs of the system and possibly on the input. In this case, the $\mathcal{ITS}$ (1) describes an uncertain nonlinear system. It can also be an unknown constant value, $\mathcal{ITS}$ (1) then represents a PLDI [8].

In the following, the bounds are assumed to be known. Equivalent forms of the matrices $[A_p]$ and $[B_p]$ can be written such that:

$$\begin{array}{c} [A_p] = A_{p0} + \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{n} e_i [f_{ijp}] e_j^T \\ |[f_{ijp}]| \leq \delta_{ijp}^a, \quad \delta_{ijp}^a = \frac{1}{2}\left(\overline{a}_{ijp} - \underline{a}_{ijp}\right) \end{array} \tag{4}$$

$$\begin{array}{c} [B_p] = B_{p0} + \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{m} e_i [g_{ijp}] h_j^T \\ |[g_{ijp}]| \leq \delta_{ijp}^b, \quad \delta_{ijp}^b = \frac{1}{2}\left(\overline{b}_{ijp} - \underline{b}_{ijp}\right) \end{array} \tag{5}$$

with the definitions:

$$\begin{array}{l} A_{p0} = \frac{1}{2}\left(\underline{A}_p + \overline{A}_p\right), \ \ \Delta A_p = \frac{1}{2}\left(\overline{A}_p - \underline{A}_p\right) \\ B_{p0} = \frac{1}{2}\left(\underline{B}_p + \overline{B}_p\right), \ \ \Delta B_p = \frac{1}{2}\left(\overline{B}_p - \underline{B}_p\right) \\ e_i = \begin{pmatrix} 0 & \cdots & \underset{i^{th}\ element}{1} & \cdots & 0 \end{pmatrix}^T, \quad e_i \in R^n \\ h_i = \begin{pmatrix} 0 & \cdots & \underset{i^{th}\ element}{1} & \cdots & 0 \end{pmatrix}^T, \quad h_i \in R^m \end{array} \tag{6}$$

In the following, we are interested by the stability of the unforced $\mathcal{ITS}$ (1) writen in the equivalent form:

$$\dot{x}(t) = \sum_{p=1}^{s} \mu_p(z(t)) \left(A_{p0} + \sum_{i=1}^{n} \sum_{j=1}^{n} e_i [f_{ijp}] e_j^T \right) x(t) \tag{7}$$

III. Stability analysis of $\mathcal{ITS}$

In this section, we investigate the stability of the autonomous $\mathcal{ITS}$ (7) using the quadratic Lyapunov function $V(x(t)) = x(t)^T P^{-1} x(t), P > 0$. The unforced continuous $\mathcal{ITS}$ (7) is globally asymptotically stable if there exists a symmetric matrix $X > 0$ such that:

$$P[A_p]^T + [A_p]P < 0 \qquad \forall p \in I_s \tag{8}$$

The following lemma is useful for the proof of theorem 1 that establishes the asymptotic stability of the $\mathcal{ITS}$ (7).

Lemma 1: Given real matrices X, Y, F of appropriate dimensions with $F^T F \leq I$ then for any positive scalar $\lambda > 0$, the following inequality holds:

$$XFY + Y^T F^T X^T \leq \lambda X X^T + \frac{1}{\lambda} Y^T Y \tag{9}$$

Theorem 1: The $\mathcal{ITS}$ (7) is quadratically asymptotically stable if there exists matrices $P > 0$ and $V = diag\begin{pmatrix} \lambda_{11}...\lambda_{1n} & ... & \lambda_{n1}...\lambda_{nn} \end{pmatrix} > 0$ satisfying the following $\mathcal{LMI}$ $\forall p \in I_s$:

$$\begin{pmatrix} PA_{p0}^T + A_{p0}P + E\Delta_p^a FVF\Delta_p^a E^T & PE \\ E^T P & -V \end{pmatrix} < 0 \tag{10}$$

with

$$\begin{array}{c} \Delta_p^a = diag\begin{pmatrix} \delta_{11p}^a...\delta_{n1p}^a & \delta_{12p}^a...\delta_{n2p}^a & ... & \delta_{1np}^a...\delta_{nnp}^a \end{pmatrix} \\ E = (\underbrace{I_n \quad ... \quad I_n}_{n \quad times}) \end{array} \tag{11}$$

and F is permutation matrix such that

$$FVF = diag\begin{pmatrix} \lambda_{11}...\lambda_{n1} & \lambda_{12}...\lambda_{n2} & ... & \lambda_{1n}...\lambda_{nn} \end{pmatrix} \tag{12}$$

Proof:Taking into account the values of the bounds on $[f_{ijp}]$, the lemma 1 allows to write the successive expressions:

$$\begin{array}{c} P[A_p]^T + [A_p]P = PA_{p0}^T + A_{p0}P + \\ \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{n} \left(P\left(e_i[f_{ijp}]e_j^T\right)^T + \left(e_i[f_{ijp}]e_j^T\right)P \right) \end{array} \tag{13}$$

$$\begin{array}{c} P[A_p]^T + [A_p]P \leq PA_{p0}^T + A_{p0}P + \\ \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{n} \left(\lambda_{ij}(\delta_{ijp}^a)^2 e_i e_i^T + \frac{1}{\lambda_{ij}}\left(Pe_j e_j^T P\right) \right) \end{array} \tag{14}$$

Let us remark that

$$\begin{array}{c} \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{n} \frac{1}{\lambda_{ij}}\left(Pe_j e_j^T P\right) = (P \dots P)\, V^{-1} (P \dots P)^T \\ (P \dots P) = PE \\ \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{n} \lambda_{ij}(\delta_{ijp}^a)^2 e_i e_i^T = E\Delta_p^a FVF\Delta_p^a E^T \end{array} \tag{15}$$

The stability condition (10), taking into account (15), is only the Schur complement of (14), that gives:

$$\begin{array}{c} P[A_p]^T + [A_p]P \leq \\ \begin{pmatrix} PA_{p0}^T + A_{p0}P + E\Delta_p^a FVF\Delta_p^a E^T & PE \\ E^T P & -V \end{pmatrix} < 0 \end{array} \tag{16}$$

with Δ_p^a, E and F defined in (11) and (12). An example of construction of the permutation matrix F is given in the section 5.

IV. CONTROLLER DESIGN OF $\mathcal{ITS}$

The dynamic of the closed loop $\mathcal{ITS}$ with the control law:

$$u(t) = -\sum_{p=1}^{s} \mu_p(z(t)) K_p x(t) \tag{17}$$

is described by:

$$\dot{x}(t) = \sum_{p=1}^{s}\sum_{q=1}^{s} \mu_p\left(z\left(t\right)\right)\mu_q(z(t))\left([A_p] - [B_p]K_q\right)x(t) \tag{18}$$

The closed loop $\mathcal{ITS}$ (18) is globally quadratically stabilisable if there exists matrices $P > 0$ and K_p satisfying the following matrix inequalities $\forall (p,q) \in I_s^2$:

$$P\left([A_p] - [B_p]K_q\right)^T + \left([A_p] - [B_p]K_q\right)P < 0 \tag{19}$$

Theorem 2 : The closed loop $\mathcal{ITS}$ (18) is globally quadratically stabilisable if there exist matrices $P > 0$, $V_1 = diag\begin{pmatrix} \lambda_{11}...\lambda_{1n} & ... & \lambda_{n1}...\lambda_{nn}\end{pmatrix} > 0$, $V_2 = diag\begin{pmatrix} \nu_{11}...\nu_{1m} & ... & \nu_{n1}...\nu_{nm}\end{pmatrix} > 0$ and Y_p satisfying the following $\mathcal{LMI}$ $\forall (p,q) \in I_s^2$:

$$\begin{pmatrix} \begin{pmatrix} PA_{p0}^T + A_{p0}P + B_{p0}Y_q + (B_{p0}Y_q)^T + \\ E_1\Delta_p^a F_1 V_1 F_1 \Delta_p^a E_1^T + \\ E_2 \Delta_p^b F_2 V_2 F_2 \Delta_p^b E_2^T \end{pmatrix} & PE_1 & Y_q^T E_2 \\ E_1^T P & -V_1 & 0 \\ E_2^T Y_q & 0 & -V_2 \end{pmatrix} < 0 \tag{20}$$

with

$$\begin{array}{c} \Delta_p^a = diag\begin{pmatrix} \delta_{11p}^a...\delta_{n1p}^a & \delta_{12p}^a...\delta_{n2p}^a & ... & \delta_{1np}^a...\delta_{nnp}^a \end{pmatrix} \\ \Delta_p^b = diag\begin{pmatrix} \delta_{11p}^b...\delta_{n1p}^b & \delta_{12p}^b...\delta_{n2p}^b & ... & \delta_{1mp}^b...\delta_{nmp}^b \end{pmatrix} \\ E_1 = (\underbrace{I_n \quad ... \quad I_n}_{n \quad times}) \\ E_2 = (\underbrace{I_m \quad ... \quad I_m}_{n \quad times}) \end{array} \tag{21}$$

and F_1, F_2 are permutation matrices such that

$$\begin{array}{l} F_1V_1F_1 = diag\begin{pmatrix} \lambda_{11}...\lambda_{n1} & \lambda_{12}...\lambda_{n2} & ... & \lambda_{1n}...\lambda_{nn}\end{pmatrix} \\ F_2V_2F_2 = diag\begin{pmatrix} \nu_{11}...\nu_{n1} & \nu_{12}...\nu_{n2} & ... & \nu_{1m}...\nu_{nm}\end{pmatrix} \end{array} \tag{22}$$

The controller gain is defined by

$$K_q = -Y_q P^{-1} \tag{23}$$

Proof : Taking into account the definition (4) and (5), with $Y_q = -K_q P$, we obtain from (19):

$$\begin{array}{l} P\left([A_p] - [B_p]K_q\right)^T + \left([A_p] - [B_p]K_q\right)P = \\ \quad (A_{p0}P + B_{p0}Y_q)^T + (A_{p0}P + B_{p0}Y_q) + \\ \quad \left(\sum_{i=1}^{n}\sum_{j=1}^{n} e_i[f_{ijp}]e_j^T P + \sum_{i=1}^{n}\sum_{j=1}^{m} e_i[g_{ijp}]h_j^T Y_q\right)^T + \\ \quad \left(\sum_{i=1}^{n}\sum_{j=1}^{n} e_i[f_{ijp}]e_j^T P + \sum_{i=1}^{n}\sum_{j=1}^{m} e_i[g_{ijp}]h_j^T Y_q\right) \end{array} \tag{24}$$

The lemma 1 allows to write

$$\begin{array}{l} P\left([A_p] - [B_p]K_q\right)^T + \left([A_p] - [B_p]K_q\right)P \leq \\ \quad (A_{p0}P + B_{p0}Y_q)^T + (A_{p0}P + B_{p0}Y_q) + \\ \quad \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_{ij}(\delta_{ijp}^a)^2 e_i e_i^T + \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{\lambda_{ij}} P e_j e_j^T P + \\ \quad \sum_{i=1}^{n}\sum_{j=1}^{m} \nu_{ij}(\delta_{ijp}^b)^2 e_i e_i^T + \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{1}{\nu_{ij}} Y_q^T h_j h_j^T Y_q \end{array} \tag{25}$$

The $\mathcal{LMI}$ (20), with definitions (21) and (22), is only the Schur complement of the RHS of this last expression.

Remark: In the case of certain commun input matrix $B_p = B_{p0}$, the closed loop $\mathcal{LMI}$ (18) is globally quadratically stabilisable if there exists matrix $P > 0$, diagonal matrix $V_1 > 0$ and Y_q satisfying the following $\mathcal{LMI}$:

$$\begin{pmatrix} \begin{pmatrix} PA_{p0}^T + A_{p0}P + \\ B_{p0}Y_q + (B_{p0}Y_q)^T + \\ E_1\Delta_p^a F_1 V_1 F_1 \Delta_p^a E_1^T \end{pmatrix} & E_1 P \\ E_1^T P & -V_1 \end{pmatrix} < 0, (p,q) \in I_s^2 \tag{26}$$

The observer gain is defined by

$$K_q = -Y_q P^{-1} \tag{27}$$

with Δ_p^a, E_1 and F_1 defined in (21) and (22).

V. NUMERICAL EXAMPLES: CONTROLLER SYNTHESIS

Consider the following example which correspond to the example given in [6] where a is a parameter:

$$\begin{array}{ll} A_{10} = \begin{pmatrix} -1 & -1.155 \\ 1 & 0 \end{pmatrix}, & \Delta A_1 = \begin{pmatrix} 0.0 & 0.655\text{a} \\ 0.0 & 0.0 \end{pmatrix} \\ A_{20} = \begin{pmatrix} -1 & -1.155 \\ 1 & 0 \end{pmatrix}, & \Delta A_2 = \begin{pmatrix} 0.0 & 0.655\text{a} \\ 0.0 & 0.0 \end{pmatrix} \end{array} \tag{28}$$

$$\begin{array}{ll} B_{10} = \begin{pmatrix} 1.4687 \\ 0 \end{pmatrix}, & \Delta B_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ B_{20} = \begin{pmatrix} 0.5613 \\ 0 \end{pmatrix}, & \Delta B_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \end{array} \tag{29}$$

The stability conditions derived in theorem 2 in the particular case $B_p = B_{p0}$ with $a = 10$ and:

$$n = 2, E_1 = \begin{pmatrix} I_2 & I_2 \end{pmatrix}, V_1 = diag\left(\lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}\right) \quad (30)$$

lead to the resolution of four $\mathcal{LMI}$ which are feasible in $V_1 > 0$ and $P > 0$ and give:

$$P = \begin{pmatrix} 6.6110 & -2.4923 \\ -2.4923 & 1.6386 \end{pmatrix} \quad (31)$$

$$V_1 = (8.7579, \quad 8.7579, \quad 8.7579, \quad 8.7579) \quad (32)$$

and the controller gain

$$\begin{array}{l} K_1 = \begin{pmatrix} 127.9163 & 197.0453 \end{pmatrix} \\ K_2 = \begin{pmatrix} 127.9163 & 197.0453 \end{pmatrix} \end{array} \quad (33)$$

Which shows that our $\mathcal{LMI}$ conditions, compared with result given in [6], allow to compute a robust controller for a large parameter $a = 10(>> 1)$.

VI. Conclusion

In this paper, the stability analysis and the synthesis of robust controller for an uncertain nonlinear model described by an $\mathcal{ITS}$ are considered. Such models consist of a weighted sum of linear systems involving uncertain bounded parameters and allow to describe a large class of nonlinear systems. Using the quadratic Lyapunov function candidate, sufficient conditions for global asymptotic stability are established. The result is extended to design controller in $\mathcal{LMI}$ formulation. Examples are given to illustrate the proposed results.

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