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# OUTPUT STABILISATION IN MULTIPLE MODEL APPROACH TO MODELLING

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**Abstract:** In this paper, the separation principle for discrete nonlinear systems in multiple model representation is investigated. The separation principle deals with a property which allows the multiple observer and the multiple controller to be designed separately. Using the quadratic Lyapunov technique and LMIs (Linear Matrix Inequalities) formulation, sufficient conditions for the global exponential stability of discrete multiple controllers are derived which are dual to those for the global exponential convergence of discrete multiple observers. A numerical example is given to illustrate the method.

**Key words:** Discrete nonlinear systems, multiple model approach, Takagi-Sugeno system, stability analysis, regulators, observers, Lyapunov method, LMI technique, separation principle.

## 1. INTRODUCTION

There have been several studies concerning the issue of stability, the design of state feedback multiple controller as well as the design of state multiple observer [2][4][11] for nonlinear systems in multiple model representation [8]. The multiple model approach uses the Takagi-Sugeno (T-S) modelling [1][7] shown to be a universal approximator [9][16]. The multiple model representation consists to construct nonlinear dynamic system by means of interpolating the behaviour of several LTI submodels. Each submodel contributes to the global model in a particular subset of the operating space.

Many works have been carried out to investigate the stability analysis of such multiple models. Sufficient conditions for the stability and stabilisability have been established using a global quadratic Lyapunov function [2][4][5][12][15][11]. The stability depends on the existence of a common positive definite matrix guarantying the stability of all local subsystems. These stability conditions may be expressed in Linear Matrix Inequalities (LMIs) form. The obtaining of a solution is then facilitated by using numerical toolboxes for solving such problems. Moreover, a certain form of multiple observers has been proposed and sufficient conditions for the asymptotic convergence are obtained which are dual to those for the stability of multiple controllers. LMIs constraints have been also used for pole assignment in LMI regions to achieve desired performances of multiple controllers [10] and multiple observers [17]. Once a multiple observer for nonlinear system in multiple model representation is obtained, one might be tempted to think that this one can be used together with a state feedback multiple controller as in case of linear systems. It's well proved, in case of linear systems, that if only the constructed state is available one can combine state feedback controller and observer to obtain a stabilising output feedback controller. Moreover the spectrum of the closed loop linear system consists of the spectrum of the observer and the spectrum of the feedback system [13]. This fact known as the separation principle has been studied for nonlinear systems (see for example [6]; and herein references). The separation principle of multiple model has been studied in [3][12][15]. However, it is not known if the combination of convergent multiple observer with stabilising multiple controller guarantees the stability of the closed loop nonlinear system in any form of multiple model stability [18].

In this paper, using the quadratic Lyapunov technique, sufficient conditions for the global exponential convergence of discrete multiple observer are derived in LMIs form which are dual to those for the stability of the state feedback multiple controller. Under the assumption that discrete nonlinear system in multiple model representation is locally stabilisable and locally detectable, the separation principle is studied and a parametric quadratic Lyapunov function is computed for the augmented multiple model. An example is given to illustrate the theory.

***Notation***: In this paper, we denote the minimum and the maximum eigenvalues of the matrix $X$ by $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ respectively, the definite positive matrix $X$ by $X > 0$, the transpose of $X$ by $X^T$, $\sum_{i<j}^{n} x_i x_j = \sum_{i=1}^{n} \sum_{j=1, j>i}^{n} x_i x_j$ and $\begin{pmatrix} X & (*)^T \\ Z & Y \end{pmatrix} = \begin{pmatrix} X & Z^T \\ Z & Y \end{pmatrix}$

## 2. MULTIPLE MODEL REPRESENTATION

Consider the following nonlinear dynamic system in the multiple model representation:

$$x(k+1) = \sum_{i=1}^{n} \mu_i(z(k))(A_i x(k) + B_i u(k)) \qquad (1)$$

where $x(k) \in \mathbb{R}^p$ is the state vector, $u(k) \in \mathbb{R}^m$ is the input vector, $n$ is the number of submodels, $y(k) \in \mathbb{R}^l$ is the

output vector, $A_i \in \mathbb{R}^{p.p}$, $B_i \in \mathbb{R}^{p.m}$ and $z(k) \in \mathbb{R}^q$ is the *decision variable* vector. The choice of the variable $z(k)$ leads to different class of systems. It can depend on the measurable state variables and possibly on the input; in this case, the system (1) describes a nonlinear system. It can also be an unknown constant value, system (1) then represents a polytopic linear differential inclusion (PLDI) [14].
The activation function $\mu_i(z(k))$ in relation with the $i^{th}$ submodel is such that

$$\begin{cases} \sum_{i=1}^{n} \mu_i(z(k)) = 1 \\ \mu_i(z(k)) \geq 0 \quad \forall\, i \in \{1,..,n\} \end{cases} \tag{2}$$

The final output of discrete multiple model is also interpolated as follows:

$$y(k) = \sum_{i=1}^{n} \mu_i(z(k)) C_i x(k) \tag{3}$$

Where $C_i \in \mathbb{R}^{l.p}$ are the output matrices. More detail about this type of representation can be found in [1].
It should be point out that at a specific time, only a number $r$ of local models are activated, depending on the structure of the activation functions $\mu_i(.)$.

## 3. STABILITY ANALYSIS

The unforced multiple model of (1) is defined as:

$$x(k+1) = \sum_{i=1}^{n} \mu_i(z(k)) A_i x(k) \tag{4}$$

The discrete system described by (4) is globally asymptotically stable if there exists a common matrix $P = P^T > 0$ such that [4]

$$A_i^T P A_i - P < 0 \quad \forall\, i \in \{1,..,n\} \tag{5}$$

The existence of such a common positive definite matrix described by LMIs (5) is a key to check the global stability of the discrete multiple model (4).

### *3.1 Multiple model controller*

In order to stabilise the multiple model (1) a multiple controller is designed using the PDC technique [2]. In the PDC technique, the global control law is obtained by interpolation of local linear feedback laws as the multiple model representation of the nonlinear system. For the multiple controller design, it is supposed that the system (1) is locally stabilisable, i.e. the pairs $(A_i, B_i)$, $\forall\, i \in \{1,..,n\}$ are stabilisable. The resulting global controller, which is nonlinear in general, is:

$$u(k) = -\sum_{i=1}^{n} \mu_i(z(k)) K_i x(k) \tag{6}$$

where $\mu_i(z(k))$ has to respect constraint (2).

Substituting (6) in (1), we obtain the closed-loop discrete multiple model:

$$x(k+1) = \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_i(z(k)) \mu_j(z(k)) R_{ij} x(k) \tag{7}$$

where

$$R_{ij} = A_i - B_i K_j \tag{8}$$

Next, we will extend stability conditions derived in [2] to global exponential stability. For demonstration the following results are needed.

***Lemma*** : Let $X$ be a positive definite matrix and $A, B$ square matrices, then

$$A^T X B + B^T X A \leq A^T X A + B^T X B \tag{9}$$

*Proof*: It follows directly from the following quadratic property: $(A-B)^T X (A-B) \geq 0$, $\forall\, X > 0$ ■

***Corollary*** [2]: Let $r$ be the number of submodels simultaneously activated such that $2 \leq r \leq n$, then

$$\sum_{i=1}^{n} \mu_i(z(k))^2 \geq \frac{1}{r-1} \sum_{i<j}^{n} 2\mu_i(z(k)) \mu_j(z(k)) \tag{10}$$

***Theorem 1***: The closed-loop discrete multiple model described by (7) is globally exponentially stable if there exist matrices $K_i$ and symmetric positive definite matrices $P_1$ and $Q_1$ such that

$$R_{ii}^T P_1 R_{ii} - P_1 + \left(r - \frac{1}{2}\right) Q_1 < 0 \quad \forall\, i \in \{1,..,n\} \tag{11a}$$

$$\left(\frac{R_{ij} + R_{ji}}{2}\right)^T P_1 \left(\frac{R_{ij} + R_{ji}}{2}\right) - P_1 < \frac{Q_1}{2} \tag{11b}$$

with $R_{ij} = A_i - B_i K_j$ $\forall\, i < j \in \{1,..,n\}$ and $\mu_i(z(k)) \mu_j(z(k)) \neq 0$.

*Proof*: let us consider the Lyapunov candidate function

$$V(x(k)) = x(k)^T P_1 x(k), \; P_1 > 0 \tag{12}$$

This is a radialy unbounded Lyapunov function since that $\forall\, x(k) \in \mathbb{R}^p$

$$\lambda_{\min}(P_1) \|x(k)\|^2 \leq V(x(k)) \leq \lambda_{\max}(P_1) \|x(k)\|^2 \tag{13}$$

Then tacking into account the trajectory of the multiple model (7):

$$\Delta V(x(k)) = V(x(k+1)) - V(x(k))$$
$$= x(k)^T \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} \mu_i\mu_j\mu_k\mu_l \left(R_{ij}{}^T P_1 R_{kl} - P_1\right) x(k)$$

The property (9) allows to bound the above equality by the following inequality

$$\Delta V(x(k)) \le \frac{1}{4} x(k)^T \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i(z(k))\mu_j(z(k))\left(G_{ij}\right)x(k) \quad (14)$$

where $G_{ij} = \left(R_{ij} + R_{ji}\right)^T P_1 \left(R_{ij} + R_{ji}\right) - 4P_1$. The right member of (14) can be developed leading to:

$$\Delta V(x(k)) \le x(k)^T \sum_{i=1}^{n} \mu_i{}^2(z(k))\left(R_{ii}{}^T P_1 R_{ii} - P_1\right)x(k) +$$
$$2x(k)^T \sum_{i<j}^{n} \mu_i(z(k))\mu_j(z(k)) \left( \frac{\left(R_{ij}+R_{ji}\right)^T}{2} P_1 \frac{\left(R_{ij}+R_{ji}\right)}{2} - P_1 \right) x(k)$$

Taking into account the conditions (10) and (11b), we deduce

$$\Delta V(x(k)) < x(k)^T \sum_{i=1}^{n} \mu_i{}^2(z(k))\left(R_{ii}{}^T P_1 R_{ii} - P_1 + (r-1)Q_1\right)x(k)$$
$$- x(k)^T 2 \sum_{i<j}^{n} \mu_i(z(k))\mu_j(z(k)) \frac{Q_1}{2} x(k)$$

From (11a) we obtain

$$\Delta V(x(k)) < -x(k)^T \frac{Q_1}{2} x(k) \quad (15)$$

And finally from (13) and (15) we deduce:

$$\Delta V(x(k)) < -\frac{\lambda_{\min}(Q_1)}{2\lambda_{\max}(P_1)} V(x(k))$$

From the condition (11a) (which is equivalent to 16b), the following property holds : $0 < \dfrac{\lambda_{\min}(Q_1)}{2\lambda_{\max}(P_1)} < \dfrac{1}{2r-1}$.

Knowing that $2 \le r \le n$, the exponential stability is ensured. ■

The control design problem is to find the feedback gains $K_i$ such that the closed loop system (7) is stable. The conditions (12) are not convex in $P_1$ and $K_i$. In order to convert them into an LMI problem, these inequalities are multiplied in the left and the right by $P_1^{-1}$ and after the Schur complement [14] is used. Then, taking into account the definition (8), the constraint (12) become

$$X_1 > 0,\ S_1 > 0 \quad (16a)$$

$$\begin{pmatrix} X_1 - \left(r - \frac{1}{2}\right)S_1 & (*)^T \\ A_i X_1 - B_i Y_i & X_1 \end{pmatrix} > 0 \quad \forall i \in \{1,\dots,n\} \quad (16b)$$

$$\begin{pmatrix} X_1 + \frac{1}{2}S_1 & (*)^T \\ \frac{1}{2}\left(\left(A_i + A_j\right)X_1 - B_i Y_j - B_j Y_i\right) & X_1 \end{pmatrix} > 0 \quad (16c)$$

$\forall\ i < j \in \{1,\dots,n\}$ such that $\mu_i(z(k))\mu_j(z(k)) \ne 0$, which are LMIs in $X_1$, $S_1$ and $Y_i$ $\forall i \in \{1,\dots,n\}$ with $P_1 = X_1^{-1}$, $Q_1 = X_1^{-1}S_1X_1^{-1}$, $K_i = Y_i X_1^{-1}$

### *3.2. Multiple observer design*

The multiple controller proposed in previous section is based on a state feedback. However, in practice, all the states of a system are not fully measurable. Thus, the problem addressed in this section is the construction of a multiple observer to estimate states of the multiple model (1). It is supposed that the decision variables $z(k)$ are measurable and the multiple model (1) is locally detectable, i.e. the pairs $(A_i, C_i), \forall\, i \in \{1,..,n\}$ are detectable.

Using the same structure as the one for multiple controller design, the multiple observer for the multiple model (1) is written as follows

$$\begin{cases} \hat{x}(k+1) = \sum_{i=1}^{n} \mu_i(z(k))(A_i\hat{x}(k) + B_i u(k) + L_i(y(k) - \hat{y}(k))) \\ \hat{y}(k) = \sum_{i=1}^{n} \mu_i(z(k))C_i\hat{x}(k) \end{cases} \quad (17)$$

where $\hat{x}(k)$ and $\hat{y}(k)$ denote the estimated state vector and output vector respectively. The activation functions $\mu_i(z(k))$ are the same that those used in the multiple model (1). Denoting the state estimation error by

$$\tilde{x}(k) = x(k) - \hat{x}(k) \quad (18)$$

it follows from (1) and (17) that the observer error dynamic is given by the following equation:

$$\tilde{x}(k+1) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i(z(k))\mu_j(z(k))\Theta_{ij}\tilde{x}(k) \quad (19)$$

where

$$\Theta_{ij} = A_i - L_i C_j \quad (20)$$

The design of the observer consists to determine the local gains $L_i$ to ensure the convergence to zero of the estimation error (18). To prove the global exponential stability conditions of the estimation error (18), the following result which is derived from theorem 1 is proposed.

***Theorem 2***: Suppose that there exist matrices $L_i$ and symmetric positive definite matrices $P_2$ and $Q_2$ such that

$$\Theta_{ii}^T P_2 \Theta_{ii} - P_2 + \left(r - \frac{1}{2}\right) Q_2 < 0 \quad \forall\, i \in \{1,..,n\} \tag{21a}$$

$$\frac{\left(\Theta_{ij} + \Theta_{ji}\right)^T}{2} P_2 \frac{\left(\Theta_{ij} + \Theta_{ji}\right)}{2} - P_2 < \frac{Q_2}{2} \tag{21b}$$

with $\Theta_{ij} = A_i - L_i C_j$ $\forall\, i < j \in \{1,..,n\}$ and $\mu_i(z(k))\mu_j(z(k)) \neq 0$. Then there exists an observer such that the error estimation (18) is globally exponentially stable. ■

With the definition in (20), the constraints (21) are bilinear in $L_i$ and $P_2$. The linearisation of (22) gives:

$$P_2 > 0, Q_2 > 0 \tag{22a}$$

$$\begin{pmatrix} P_2 - \left(r - \frac{1}{2}\right) Q_2 & \left(P_2 A_i - Y_i C_i\right)^T \\ P_2 A_i - Y_i C_i & P_2 \end{pmatrix} > 0, \forall i \in \{1,\dots,n\} \tag{22b}$$

$$\begin{pmatrix} P_2 + \frac{1}{2} Q_2 & (*)^T \\ \frac{1}{2}\left(\left(A_i + A_j\right) P_2 - Y_j C_i - Y_i C_j\right) & P_2 \end{pmatrix} > 0$$

$$\forall\, i < j \in \{1,\dots,n\} \tag{22c}$$

which are LMIs in $P_2, Q_2$ and $Y_i$ $\forall i \in \{1,\dots,n\}$ with

$$L_i = P_2^{-1} Y_i. \tag{23}$$

## 4. SEPARATION PRINCIPLE

In the case of linear system, it's proved that if the constructed state is available one can combine state feedback controller and observer to obtain a stabilising output feedback controller. Moreover the spectrum of the closed loop system consists of the spectrum of the observer and the spectrum of the feedback system. This fact is known as the separation principle. However in the multiple model representation which is nonlinear, the property of separation principle depends on the used method for proving the stability [3][15]. In this section, we show that for the form of multiple model stability stated above, the combination of the global exponential convergent multiple observer and the global exponential stabilising multiple controller guarantees the global exponential stability of the closed loop system.
If instead of the actual state the constructed state $\hat{x}(k)$ is available, the control law with the PDC technique (6) becomes

$$u(k) = -\sum_{i=1}^{n} \mu_i(z(k)) K_i \hat{x}(k) \tag{24}$$

Taking into account (17) and (24), we have

$$\hat{x}(k+1) = \sum_{j=1}^{n} \sum_{i=1}^{n} \mu_i(z(k))\mu_j(z(k))\left(R_{ij}\hat{x}(k) + L_i C_j \tilde{x}(k)\right) \tag{25}$$

where $R_{ij}$ and $\tilde{x}(k)$ are defined in (8) and (18) respectively. Combining (25) and (19) we obtain the following augmented system

$$\bar{x}(k+1) = \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_i(z(k))\mu_j(z(k)) \bar{A}_{ij} \bar{x}(k) \tag{26}$$

where:

$$\bar{x}(k) = \left(\hat{x}(k)^T \quad \tilde{x}(k)^T\right)^T \text{ and } \bar{A}_{ij} = \begin{pmatrix} R_{ij} & L_i C_j \\ 0 & \Theta_{ij} \end{pmatrix} \tag{27}$$

To prove the global exponential stability of the augmented system (26), it suffices to find matrices $K_i$, $L_i$ and symmetric positive definite matrices $P$ and $Q$ such that

$$\bar{A}_{ii}^T P \bar{A}_{ii} - P + \left(r - \frac{1}{2}\right) Q < 0 \quad \forall\, i \in \{1,..,n\} \tag{28a}$$

$$\frac{\left(\bar{A}_{ij} + \bar{A}_{ji}\right)^T}{2} P \frac{\left(\bar{A}_{ij} + \bar{A}_{ji}\right)}{2} - P < \frac{Q}{2} \tag{28b}$$

$\forall\, i < j \in \{1,..,n\}$ and $\mu_i(z(k))\mu_j(z(k)) \neq 0$.

Thus to prove the global exponential stability, we need to compute the controller gains $K_i$, the observer gains $L_i$ and the symmetric positive definite matrices $P$ and $Q$ respecting the constraints (28). These latter, which are non convex in the variables $K_i$, $L_i$ and $P$, is difficult to convert into an LMI problem using the linearisation method described at the end of paragraph 3.1. In order to overcome this difficulty, the following theorem shows that it suffices to prove the stability of both the multiple controller and the multiple observer independently for proving the stability of the augmented systems (26). Similar approach can be found in [3][15]. The new of our method is to propose a systematic way to derive a parametric quadratic Lyapunov function proving the stability of the augmented multiple model (26). An example is given in section 5.

***Theorem 3***: If there exist symmetric positive definite matrices $P_1$, $P_2$, $Q_1$ and $Q_2$ such that (12) and (22) are satisfied, then we can always construct a quadratic Lyapunov function which proves the global exponential stability of the augmented system (26).

*Proof*: With the following structure of $P$ and $Q$:

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & \sigma P_2 \end{pmatrix} > 0,\ Q = \begin{pmatrix} Q_1 & 0 \\ 0 & \sigma Q_2 \end{pmatrix} > 0,\ \sigma \in \mathbb{R}^{+*}$$

Taking into account (8), (20) and (27), the inequalities (28) allow writing :

$$\begin{pmatrix} R_{ii}^T P_1 R_{ii} - P_1 & R_{ii}^T P_1 L_i C_i \\ (P_1 L_i C_i)^T R_{ii} & (L_i C_i)^T P_1 L_i C_i + \sigma\left(\Theta_{ii}^T P_2 \Theta_{ii} - P_2\right) \end{pmatrix} +$$

$$\left(r - \frac{1}{2}\right)\begin{pmatrix} Q_1 & 0 \\ 0 & \sigma Q_2 \end{pmatrix} < 0 \qquad \forall\, i \in \{1,..,n\} \tag{29a}$$

$$\begin{pmatrix} \left(\frac{R_{ij}+R_{ji}}{2}\right)^T P_1 \frac{R_{ij}+R_{ji}}{2} - P & (*)^T \\ \left(\frac{L_iC_j + L_jC_i}{2}\right)^T P_1 \frac{R_{ij}+R_{ji}}{2} & \Xi_{ij} \end{pmatrix} - \frac{1}{2}\begin{pmatrix} Q_1 & 0 \\ 0 & \sigma Q_2 \end{pmatrix} < 0$$

$\forall\, i < j \in \{1,..,n\}$ and $\mu_i(z(k))\mu_j(z(k)) \neq 0$. (29b)

where

$$\Xi_{ij} = \sigma\left(\left(\frac{\Theta_{ij}+\Theta_{ji}}{2}\right)^T P_2 \frac{\Theta_{ij}+\Theta_{ji}}{2} - P_2\right) + \left(\frac{L_iC_j+L_jC_i}{2}\right)^T P_1 \frac{L_iC_j+L_jC_i}{2}$$

the Schur complement [14] applied to the inequalities (29a) and (29b) allows to prove that $\sigma$ will always exist if the inequalities (11) and (21) are satisfied. Thus it suffices to choose this one sufficiently large as follows:

$$\sigma \geq Max(\sigma_1,\ \sigma_2) \tag{30}$$

where $\sigma_1$ and $\sigma_2$ are defined in (31a) and (31b) respectively. ■

5. NUMERICAL EXAMPLE

The following example demonstrates the utility of the proposed method. Let us consider the multiple model (1) where $r = n = 2$, $z(k) = x_1(k)$ and

$$A_1 = \begin{pmatrix} 1.16 & -1.09 \\ 0.11 & 0.94 \end{pmatrix}, B_1 = \begin{pmatrix} 0.1 \\ 0 \end{pmatrix}, C_1 = \begin{pmatrix} 1 & 0 \end{pmatrix} \tag{32a}$$

$$A_2 = \begin{pmatrix} 132.11 & -26.87 \\ 2.67 & 0.47 \end{pmatrix}, B_2 = \begin{pmatrix} 26.87 \\ 0.53 \end{pmatrix}, C_2 = \begin{pmatrix} 1 & 0 \end{pmatrix} \tag{32b}$$

$$\mu_1(x_1(k)) = \frac{(1 - \tanh(x_1(k)))}{2}, \mu_2(x_1(k)) = 1 - \mu_1(x_1(k)) \tag{32c}$$

From conditions (11) given in theorem 2 and with definition (8) we obtain the following feedback gains (after linearisation as it is described in (16)):

$$K_1 = \begin{pmatrix} 4.9518 & -0.9669 \end{pmatrix},\ K_2 = \begin{pmatrix} 4.9189 & -0.9645 \end{pmatrix}$$

and the definite positive matrices:

$$P_1 = \begin{pmatrix} 0.0692 & 0.0608 \\ 0.0608 & 0.3845 \end{pmatrix},\ Q_1 = \begin{pmatrix} 0.0075 & 0.0080 \\ 0.0080 & 0.0176 \end{pmatrix} \tag{33}$$

And from conditions (21) given in theorem 4 and with definition (20) we obtain, (after linearisation as it is described in (22)), the following observer gains which ensure the exponential convergence of state:

$$L_2 = \begin{pmatrix} 1.1849 & 0.0909 \end{pmatrix}^{\mathrm{T}},\ L_2 = \begin{pmatrix} 132.6410 & 2.6608 \end{pmatrix}^{\mathrm{T}}$$

and the definite positive matrices:

$$P_2 = \begin{pmatrix} 0.0306 & 0.3905 \\ 0.3905 & 19.2415 \end{pmatrix},\ Q_2 = \begin{pmatrix} 0.0064 & 0.0153 \\ 0.0153 & 0.7610 \end{pmatrix} \tag{34}$$

The choice of the symmetric positive definite matrices: $P = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix}$ and $Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}$

where $P_1$, $P_2$, $Q_1$ and $Q_2$ are defined in (33) and (34) respectively, fails to prove the global exponential stability (formulated in theorem 5) of the closed loop system of the numerical example (32). But the choice of the following symmetric positive definite matrix

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & \sigma P_2 \end{pmatrix} \text{ and } Q = \begin{pmatrix} Q_1 & 0 \\ 0 & \sigma Q_2 \end{pmatrix}$$

$$\sigma_1 = \underset{\forall\, i \in \{1,..,n\}}{Max}\left(\frac{\lambda_{\min}\left(\left(R_{ii}^T P_1 L_i C_i\right)^T \left(R_{ii}^T P_1 R_{ii} - P_1 + (r - 1/2)Q_1\right)^{-1} R_{ii}^T P_1 L_i C_i - (L_i C_i)^T P_1 L_i C_i\right)}{\lambda_{\max}\left(\Theta_{ii}^T P_2 \Theta_{ii} - P_2 + (r - 1/2)Q_2\right)}\right) \tag{31a}$$

$$\sigma_2 = \underset{\forall i<j \in \{1,..,n\}}{Max}\left(\frac{\lambda_{\min}\left(\left((R_{ij}+R_{ji})^T P_1 (L_iC_j + L_jC_i)\right)^T \left(\frac{(R_{ij}+R_{ji})^T}{2} P_1 \frac{(R_{ij}+R_{ji})}{2} - P_1 - \frac{Q_1}{2}\right)^{-1} \left((R_{ij}+R_{ji})^T P_1 (L_iC_j + L_jC_i)\right) - (L_iC_j+L_jC_i)^T P_1 (L_iC_j+L_jC_i)\right)}{\lambda_{\max}\left(\frac{(\Theta_{ij}+\Theta_{ji})^T}{2} P_2 \frac{(\Theta_{ij}+\Theta_{ji})}{2} - P_2 - \frac{Q_2}{2}\right)}\right)$$

such that $\mu_i(z(k))\mu_j(z(k)) \neq 0$ (31b) with $\sigma$ satisfies (30) and $(\sigma_1, \sigma_2) = 10^4(9.35,\ 5.10)$ obtained from (31), allow us to show the global exponential stability of the augmented system of (32) and prove that the design of multiple controllers and multiple observers can be done separately for the stability conditions derived above.

The simulation of the augmented multiple model of (32) with the control law (24) is presented in figure 1.

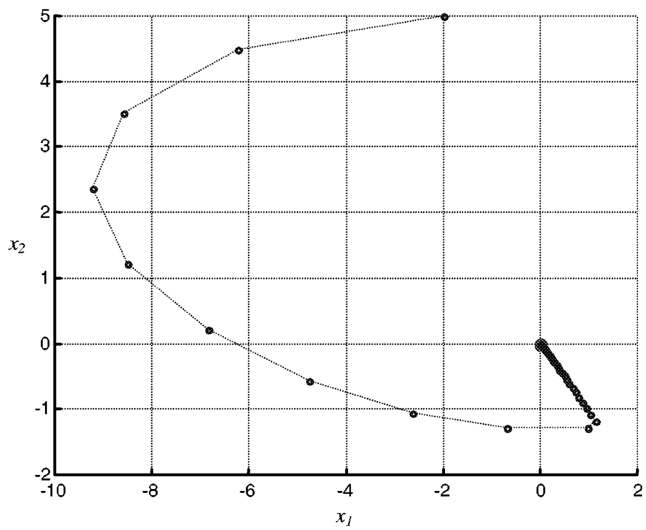


Figure 1. Example of simulation of the augmented multiple model of (32) with the control law (24).

## 6. CONCLUSION

In this paper, the separation principle for discrete nonlinear systems in multiple model representation is considered. First, the global exponential convergence of discrete multiple observer are derived in LMIs form which are dual to those for the stability of multiple controller. Finally in case the decision variables depend only on the measurable state variables and possibly on the input, it is proved that the combination of the global exponential convergent multiple observer and the global exponential stabilising multiple controller, designed separately, guarantees the stability of the overall system. By the same way, a parameterised Lyapunov function is computed for the augmented system. It remains to prove that this property holds when the decision variables depend on the state variables estimated by the multiple observer.

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