Design of observers for Takagi-Sugeno fuzzy models for Fault Detection and Isolation

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Abstract: This paper addresses the design of an unknown input fuzzy observer for Takagi-Sugeno (T-S) fuzzy model subject to unknown inputs. The main contribution of the paper is the development of a robust fuzzy observer in presence of disturbances. Based on Lyapunov function, it is shown how to determine observers gains in linear matrix inequalities (LMI). The proposed T-S observer is used for detection and isolation of faults which can affect a T-S model. The proposed methodology is illustrated by estimating the yaw rate and the fault of automatic steering vehicle.

1. INTRODUCTION

A system for automatic fault detection and isolation (FDI) in a technical process can bring several benefits to the operator of the plant. Indeed, it provides him with information on the state of health of the supervised system at each moment. This allows the operator to follow the evolution of possible system degradations, and to predict when maintenance will be needed (see for example Patton et al. [1998], Kinnaert [1999]). A typical system for FDI is made of three parts: a residual generator, a residual evaluation system, a decision-making part.

Many standard observer-based techniques exist in the literature providing different solutions to both the theoretical and practical aspects of FDI problem for linear and nonlinear system (Frank [1996], Frank et al. [2000]). Many of these procedures are based on the design of an unknown input observer robust with respect to the disturbances (Staroswiecki and Varga [2001], Kinnaert [2003], Yan and Edwards [2007]). If the disturbances and modelling errors are not properly taken into account in the estimation process, it is then likely that any attempt in monitoring the system's health based on the observer leads to numerous false alarms.

Over the past decades, many researchers have paid attention to the problem of state estimation of dynamic linear systems subjected to both known and unknown inputs (Darouach et al. [1994], Wang et al. [1995], Sename [1997]). Many approaches have developed full and/or reduced order unknown input observers to estimate the state of linear time-invariant dynamical system driven by both known and unknown inputs (Xiong and Saif [2003], Edwards [2004]).

However, the real physical systems are often nonlinear. As it is delicate to synthesize an observer for an unspecified nonlinear system, it is preferable to represent this system with the Takagi-Sugeno (T-S) fuzzy model (Takagi and Sugeno [1985]). The idea of the T-S model approach is to apprehend the global behaviour of a system by a set of local models (linear or affine), each of them characterizing the behaviour of the system in a particular zone of operation. The local models are then aggregated by means of an interpolation mechanism. This approach includes the multiple model approach (Murray-Smith [1997], Chadli et al. [2003]) and Polytopic Linear Differential Inclusions (Boyd et al. [1994]) .

In this paper, for state and unknown input estimation, the suggested technique consists in associating to each local model a local unknown input high gain observer. The considered observer is then a convex interpolation of these local observers. This interpolation is obtained throughout the same activation functions as the T-S fuzzy model. Our contribution here lies in the design of the unknown input fuzzy observer by eliminating the unknown inputs from the dynamics of the state estimation error. The synthesis conditions of fuzzy observer are expressed in LMI terms. The designed observer is then applied for sensors and actuators FDI.

The paper is organized as follows. In Section 2, the general structure of the considered T-S fuzzy model is presented. In Section 3, the proposed structure of unknown input fuzzy observer is described and the main result is presented. The derived conditions ensuring the global asymptotic convergence of the estimation error are given as a set of LMI terms. The proposed T-S observer is used for detection and isolation of faults which can affect a nonlinear models in T-S representation. The validity of the proposed methodology is illustrated by estimating the yaw rate and faults of automatic steering vehicle.

Notation: Throughout the paper, the following useful notation is used: X^T denotes the transpose of the matrix X, $X > 0$ means that X is a symmetric positive definite matrix, $\mathbb{I}_N = \{1, 2, ..., N\}$ and $\|.\|$ represents the Euclidean norm for vectors and the spectral norm for matrices.

2. TAKAGI-SUGENO FUZZY MODEL REPRESENTATION

Many physical systems are very complex in practice so that rigorous mathematical model can be very difficult to obtain, if not impossible. However, many of these systems can be expressed in some form of mathematical model locally or as an aggregation of a set of mathematical models. Here, using the T-S fuzzy dynamic model (Takagi and Sugeno [1985]), we consider a complex nonlinear system with unknown inputs. Then, the following T-S fuzzy model is adopted :

$$\begin{cases} \dot{x} = \sum_{i=1}^{N} \mu_i(\xi)\Big(A_i x + B_i u + R_i \bar{u} + D_i\Big) \\ y = Cx \end{cases} \tag{1}$$

with

$$\sum_{i=1}^{N} \mu_i(\xi) = 1, \quad 0 \le \mu_i(\xi) \le 1 \quad \forall i \in \mathbb{I}_N$$

where $x \in \mathbf{R}^n$ is the state vector, $u \in \mathbf{R}^m$ the input vector, $\bar{u} \in \mathbf{R}^q$, $q < n$, contains the unknown inputs and $y \in \mathbf{R}^p$ the measured outputs. Matrices $A_i \in \mathbf{R}^{n\times n}$ and $B_i \in \mathbf{R}^{n\times m}$ denote respectively the state matrix, the input matrix associated with the *ith* local model. Matrices $R_i \in \mathbf{R}^{n\times q}$ are the distribution matrices of unknown inputs. $D_i \in \mathbf{R}^n$ is introduced to take into account the operating point of the system. At last, ξ is the so-called decision vector which may depend on some subset of the known inputs and/or measured variables to define the operating regimes.

Matrices A_i, B_i, R_i and C can be obtained by using the direct linearization of an a priori nonlinear model around operating points, or alternatively by using an identification procedure (Gasso et al. [2001], Johansen and Babuska [2003], Angelov and Filev [2004], Murray-Smith [1997]). In the following it is assumed that the vector ξ depends on measurable variables.

3. STRUCTURE OF THE UNKNOWN INPUT FUZZY OBSERVER

In this paper, we consider the state and unknown input estimation of a T-S fuzzy model perturbed by unknown inputs. The proposed unknown input fuzzy observer is based on a nonlinear combination of local unknown input observer. The proposed structure of the T-S fuzzy observer has the following form:

$$\dot{\hat{x}} = \sum_{i=1}^{N} \mu_i(\xi)\Big(A_i\hat{x} + B_i u + D_i + G_i(y - C\hat{x}) + R_i\hat{\bar{u}}_i\Big) \tag{2a}$$

$$\hat{\bar{u}}_i = \gamma W_i\big(y - C\hat{x}\big) \tag{2b}$$

$$\hat{\bar{u}} = \sum_{i=1}^{M} \mu_i(\xi)\,\hat{\bar{u}}_i \tag{2c}$$

The aim of the design is to determine gain matrices $G_i \in \mathbf{R}^{n\times p}$, $W_i \in \mathbf{R}^{q\times p}$ and the positive scalar γ, that guarantee the asymptotic convergence of $\hat{x}$ towards x. Let us note that $\hat{\bar{u}}_i$ can be considered as variables which compensate the errors due to the unknown inputs.

Assumption: In this paper, we consider that the inputs $\bar{u}(t)$ are bounded:

$$\|\bar{u}\| < \rho \tag{3}$$

where ρ is a positive scalar.

The set of residuals is defined as follows:

$$r = y - \hat{y} \tag{4a}$$

$$= Ce \tag{4b}$$

where e represents the state estimation error:

$$e = x - \hat{x} \tag{5}$$

The dynamic of the state estimation error is given as follows:

$$\dot{e} = \sum_{i=1}^{N} \mu_i(\xi)\Big((A_i - G_iC)e + R_i\bar{u} - R_i\hat{\bar{u}}_i\Big) \tag{6}$$

Replacing $\hat{\bar{u}}_i$ by its expression (2b) we get

$$\dot{e} = \sum_{i=1}^{N} \mu_i(\xi)\Big((A_i - G_iC)e + R_i\bar{u} - \gamma R_iW_iCe\Big) \tag{7}$$

The following result proposes a solution to design the gain parameters $G_i \in \mathbf{R}^{n\times p}$, $W_i \in \mathbf{R}^{q\times p}$ and the positive scalar γ of the proposed observer (2).

Theorem 1: the state estimation of the unknown input fuzzy observer (2) can asymptotically estimate with any desired degree of accuracy $\varepsilon > 0$, the state of the T-S fuzzy model (1), if there exist a symmetric positive definite matrices P and Q and the gain matrices G_i and W_i which satisfies the following constraints $\forall\, i \in \mathbb{I}_N$:

$$(A_i - G_iC)^TP + P(A_i - G_iC) < -Q \tag{8a}$$

$$W_iC = R_i^TP \tag{8b}$$

The fuzzy observer (2) is then completely defined by choosing:

$$\gamma \ge \frac{1}{2}\Big(\lambda_{\min}(P^{-1}Q)\,\lambda_{\min}(P)\,\varepsilon^2\Big)^{-1}\rho^2 \tag{9}$$

and the input estimation is given by

$$\hat{\bar{u}} = \gamma\sum_{i=1}^{M} \mu_i(\xi)\,W_i\big(y - C\hat{x}\big) \tag{10}$$

where ρ is defined in (3).

Proof: Consider the Lyapunov function

$$V(e) = e^TPe \tag{11}$$

Its derivative with respect to time is

$$\dot{V} = \sum_{i=1}^{N} \mu_i(\xi)\Big(e^T\Big((A_i - G_iC)^TP + P(A_i - G_iC)\Big)e + 2e^TPR_i\bar{u} - 2\gamma e^TPR_iW_iCe\Big)$$

$$\le \sum_{i=1}^{N} \mu_i(\xi)\Big(e^T\Big((A_i - G_iC)^TP + P(A_i - G_iC)\Big)e + 2\rho\|(PR_i)^Te\| - 2\gamma e^TPR_iR_i^TPe\Big)$$

For any positive scalar β, we have the following inequality:

$$2\rho\|(PR_i)^Te(t)\| \le \beta^{-1}\rho^2\|(PR_i)^Te(t)\|^2 + \beta \tag{12}$$

Then

$$\dot{V} \leq \sum_{i=1}^{N} \mu_i(\xi) \Big(e^T \left((A_i - G_i C)^T P + P(A_i - G_i C) \right) e + \beta - [2\gamma - \beta^{-1}\rho^2] \| (PR_i)^T e \|^2 \Big)$$

Choosing

$$\gamma \geq \frac{1}{2}\beta^{-1}\rho^2 \tag{13}$$

$$\dot{V} \leq \sum_{i=1}^{N} \mu_i(\xi) \Big(e^T \left((A_i - G_i C)^T P + P(A_i - G_i C) \right) e + \beta \Big) \tag{14}$$

Using the inequality (8a), the derivative of the Lyapunov function becomes as follows

$$\dot{V}(e) \leq -e^T Q e + \beta \tag{15}$$

with $\alpha = \frac{1}{2}\lambda_{\min}(P^{-1}Q) > 0$, we can easily deduce

$$\dot{V}(e) \leq -2\,\alpha\, V(e) + \beta \tag{16}$$

Tacking account the expression of $V(e)$ (11) and the fact that $\alpha > 0$, we get when $t \to \infty$ the following inequality

$$\|e\| \leq \sqrt{\frac{1}{\lambda_{\min}(P)} \frac{\beta}{2\alpha}} \tag{17}$$

To guarantee for any desired $\epsilon > 0$ that $\|e\| < \epsilon$, it suffices to choose

$$\gamma \geq \frac{1}{2}\left(\lambda_{\min}(P^{-1}Q)\,\lambda_{\min}(P)\,\varepsilon^2\right)^{-1}\rho^2 \tag{18}$$

which end the proof.

Remark 1: The inequalities (8a) are nonlinear in P and G_i. To linearize these inequalities, the following change of variables is used

$$K_i = PG_i \tag{19}$$

We obtain a linear matrix inequalities in P and K_i that can be easily solved by the means of LMI tools:

$$A_i^T P + PA_i - C^T K_i^T - K_i C \,<\, -Q \tag{20a}$$

$$W_i C \,=\, R_i^T P \tag{20b}$$

Finally, the matrix gains G_i are computed as follows

$$G_i = P^{-1} K_i \tag{21}$$

Remark 2: In case of presence of noise measurement on the system output, choosing a large gain γ leads to the amplification of noise and thus a bad estimation state and unknown inputs.

4. APPLICATION TO AUTOMATIC STEERING OF VEHICLE

4.1 Representation of the vehicle model by a T-S fuzzy model

Different models related to automatic steering of vehicle have been studied in the literature (see for example Zhang and Xu [2002], El Hajjaji and Bentalba [2003], Moriwaki [2005], Chadli et al. [2008], Oudghiri et al. [2008]). Here, we have chosen to consider the coupling model of longitudinal and lateral motions of a vehicle. This model, already used in Zhang and Xu [2002], is strongly nonlinear and is given by the following equations:

$$\dot{u} \,=\, vr - fg + \frac{(fk_1 - k_2)}{M}u^2 + c_f\frac{v + ar}{Mu}\delta + \frac{T}{M} \tag{22a}$$

$$\dot{v} \,=\, -ur - \frac{\left(c_f + c_r\right)}{Mu}v + \frac{\left(bc_r - ac_f\right)}{Mu}r + \frac{c_f\delta + T\delta}{M} \tag{22b}$$

$$\dot{r} \,=\, \frac{\left(bc_r - ac_f\right)}{I_z u}v - \frac{\left(b^2c_r + a^2c_f\right)}{I_z u}r + \frac{aT\delta + ac_f\delta}{I_z} \tag{22c}$$

where, u, v and r are the longitudinal velocity, the lateral velocity and the yaw rate, respectively, δ is the steering angle, T is the traction and/or braking force. Table 1 lists the parameters of the above vehicle model.

	Parameters of the vehicle system	
M	Mass of the full vehicle	1480 kg
I_z	Moment of inertia	2350 $kg.m^2$
g	Acceleration of gravity force	9.81 m/s^2
f	Rotating friction coefficient	0.02
a	Distance from front axle to CG [1]	1.05 m
b	Distance from rear axle to CG	1.63 m
c_f	Cornering stiffness of front tyres	135000 N/rad
c_r	Cornering stiffness of rear tyres	95000 N/rad
k_1	Lift parameter from aerodynamics	0.005 Ns^2/m^2
k_2	Drag parameter from aerodynamics	0.41 Ns^2/m^2

The nonlinear vehicle dynamics can be written as follows:

$$\dot{x} \,=\, F(x,\, w) \tag{23a}$$

$$y \,=\, Cx \tag{23b}$$

where $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, F is a nonlinear function of the state vector $x = [u \;\; v \;\; r]$, w gathers the two inputs δ and T and $y(t)$ gathers the two inputs $y_1 = u$ and $y_2 = r$. As it is delicate to synthesize an observer for a nonlinear system, we preferred to represent this system with a T-S fuzzy model. Then, we propose to linearize the nonlinear model (23) around some operating points $[x^{(i)} \;\; w^{(i)}]$. Next, we integrate the set of the linear models in a T-S fuzzy model. The proposed T-S model is described as follows (Akhenak et al. [2007]):

$$\dot{x} \,=\, \sum_{i=1}^{N} \mu_i(y_1)\,(A_i x + B_i w + D_i) \tag{24a}$$

$$A_i \,=\, \left.\frac{\partial F}{\partial x}\right|_{\substack{x=x^{(i)}\\ w=w^{(i)}}} \quad B_i \,=\, \left.\frac{\partial F}{\partial w}\right|_{\substack{x=x^{(i)}\\ w=w^{(i)}}} \tag{24b}$$

$$D_i \,=\, F(x^{(i)},\, w^{(i)}) - A_i x^{(i)} - B_i w^{(i)} \tag{24c}$$

with $\displaystyle\sum_{i=1}^{N} \mu_i(u) = 1 \quad \text{and} \quad \mu_i(u) \geq 0 \quad \forall i \in \mathbb{I}_M$

$$A_i = \begin{bmatrix} A_{11i} & A_{12i} & A_{13i} \\ A_{21i} & A_{22i} & A_{23i} \\ A_{31i} & A_{32i} & A_{33i} \end{bmatrix}$$

$$A_{11i} = \frac{2\,(fk_1 - k_2)\,u_i}{M} - \frac{c_f\,(v_i + ar_i)}{Mu_i^2}\delta_i$$

$$A_{12i} = r_i + \frac{c_f\delta_i}{Mu_i} \quad A_{13i} = v_i + \frac{ac_f\delta_i}{Mu_i}$$

$$A_{21i} = -r_i + \frac{(c_f + c_r)}{Mu_i^2}v_i - \frac{(bc_r - ac_f)}{Mu_i^2}r_i$$

$$A_{22i} = -\frac{(c_f + c_r)}{Mu_i} \quad A_{23i} = -u_i + \frac{(bc_r - ac_f)}{Mu_i}$$

$$A_{31i} = -\frac{(bc_r - ac_f)}{I_z u_i^2} v_i + \frac{\left(b^2 c_r + a^2 c_f\right)}{I_z u_i^2} r_i$$

$$A_{32i} = \frac{(bc_r - ac_f)}{I_z u_i} \quad A_{33i} = -\frac{\left(b^2 c_r + a^2 c_f\right)}{I_z u_i}$$

$$B_i = \begin{bmatrix} c_f \dfrac{v_i + ar_i}{Mu_i} & \dfrac{1}{M} \\ \dfrac{c_f + T_i}{M} & \dfrac{\delta_i}{M} \\ \dfrac{aT_i + ac_f}{I_z} & \dfrac{a\delta_i}{I_z} \end{bmatrix}$$

$$D_i = F(x_i,\ \delta_i,\ T_i) - A_i x_i - B_i \begin{bmatrix} \delta_i \\ T_i \end{bmatrix}$$

where $x_i = (u_i \;\; v_i \;\; r_i)^T$, with $i = \{1,\ 2,\ 3\}$, x_i, δ_i and T_i are the operating points. Three local models are used to approximate the nonlinear model (22). Using a quadratic criterion of the error between the state variables of the nonlinear model (22) and state variables of the T-S fuzzy model (24), we obtain the following operating points:

Three local models were chosen for this application. This number gives a good compromise between the quality of the obtained model and its complexity. The activation functions given in figure 2 depend only on the longitudinal velocity $u(t)$. The numerical values of the different matrices A_i, B_i, D_i are:

$$A_1 = \begin{bmatrix} 0.052 & 0.403 & 0.239 \\ -0.366 & -10.82 & -13.743 \\ 0.728 & 0.388 & -11.890 \end{bmatrix} \quad B_1 = \begin{bmatrix} 10.99 & 7 \times 10^{-4} \\ 91.216 & -10^{-4} \\ 60.319 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.085 & 2.895 & 1.925 \\ -0.989 & -9.282 & -16.213 \\ 0.507 & 0.333 & -10.198 \end{bmatrix} \quad B_2 = \begin{bmatrix} 3.359 & 7 \times 10^{-4} \\ 91.216 & 3 \times 10^{-4} \\ 60.319 & 2 \times 10^{-4} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.031 & 2.065 & 0.693 \\ -1.141 & -8.468 & -17.870 \\ 0.441 & 0.303 & -9.303 \end{bmatrix} \quad B_3 = \begin{bmatrix} 1.548 & 7 \times 10^{-4} \\ 91.216 & 2 \times 10^{-4} \\ 60.319 & 1 \times 10^{-4} \end{bmatrix}$$

$$D_1 = \begin{bmatrix} -0.832 \\ 5.259 \\ -10.46 \end{bmatrix} \quad D_2 = \begin{bmatrix} 0.087 \\ 16.562 \\ -8.496 \end{bmatrix} \quad D_3 = \begin{bmatrix} 0.392 \\ 20.951 \\ -8.092 \end{bmatrix}$$

The membership functions $\mu_1(u)$, $\mu_2(u)$ and $\mu_3(u)$ are given in figure 2.

The system (22) is simulated using the steering angle δ and attraction force T depicted in figure 1. Figures 2 and 3show that the state variables of the nonlinear model (22) and its approximate by T-S fuzzy model (24) are superimposed.

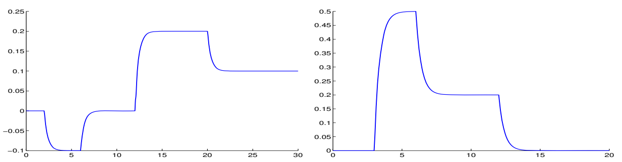

Fig. 1. δ: steering angle $\qquad$ T: attraction force

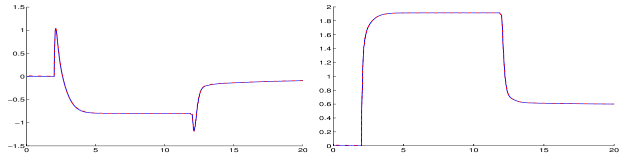

Fig. 2. Membership functions $\qquad$ u and its approximate by (22a)

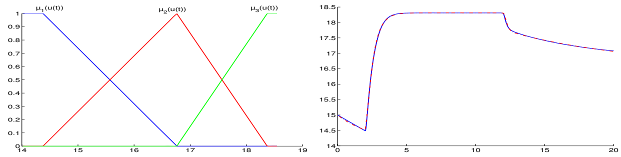

Fig. 3. v and its approximate by (22b $\qquad$ r and its approximate by (22a)

In order to check the good accuracy of the T-S fuzzy model (24), its behaviour and that of the nonlinear model (22) have been simulated. Figure 1 shows the time evolution of the two inputs δ, the steering angle and T the traction-braking force.

Figures 2 and 3 show the superposition of the state vector of the nonlinear model (22) and their approximation by the T-S fuzzy model (24). Clearly the T-S fuzzy model is able to represent the nonlinear behaviour of the considered system.

The following section exploits the obtained T-S model and the observer designed above to propose a diagnosis method for the steering vehicle model (22).

5. FAULT DETECTION AND ISOLATION

The objective of this part is to generate residuals that reflect the faults acting on the system (24). An ideal residual signal should remain zero in the fault-free case and non-zero when fault occurs. Once a fault has been detected, it must be estimated. The fault estimation will specify the type of fault, its duration, its amplitude and eventually its probable evolution. In the literature, there are several fault detection techniques. They are generally based on the change detection of the average and the variance. In this FDI study, we will not deal with the detection thresholds of residuals. We will confine ourselves only to the detection and localization of sensor (subsection *A*) and actuator faults (subsection *B*) taking into account the uncertainties modeling.

5.1 Sensor fault detection and isolation

In order to identify the sensor fault, we consider that the actuators are faultless ($\bar{w} = 0$) while the output vector y is corrupted by the sensor fault Δy. Then the system (24) becomes:

$$\begin{cases} \dot{x} = \sum_{i=1}^{M} \mu_i(y_1) \Big(A_i x + B_i w + D_i \Big) \\ y = Cx + \Delta y + \nu \end{cases} \tag{25}$$

Firstly we can easily checked that the following necessary conditions are satisfied: $\forall i \in \{1, 2, 3\}$, $j \in \{1, 2\}$ rank$(A_i, C(j,:)) = 3$

Three T-S observers are designed, the first is based on the longitudinal velocity $y_1 = u$, the second is based on the yaw rate $y_2 = r$ and the last is based on the two outputs u and r.

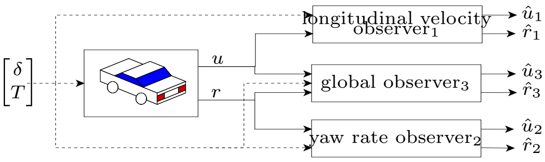

Fig. 4. Block diagram of the banc observer-based FDI

The sensor fault detection and localization is based on the analysis of the residuals $r_{y_{ik}} = y_i - \hat{y}_{i,k}$, $k \in \{1, 2, 3\}$, $i \in \{1, 2\}$ generated by three observers (figure 4) which depend on two inputs δ and T applied to the system (22). The longitudinal velocity observer$_1$ and the yaw rate observer$_2$ use respectively only one output u and r. The global observer$_3$ uses two outputs u and r.

Figures 5 shows the additive signal that represents sensor failure, the fault has been added to sensor 2 output $y_2 = r$ between 5 and 10s.

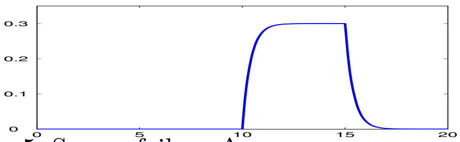

Fig. 5. Sensor failure Δy_2

FDI using global observer$_3$ and yaw rate observer$_2$. The simulation results of the fault detection and isolation based on the observer$_3$ and the observer$_2$ are illustrated by the figures 6 and 7 with the initial conditions $(u_0 \;\; v_0 \;\; r_0) = (15 \;\; 0 \;\; 0)$ and $(\hat{x}_1(0) \;\; \hat{x}_2(0) \;\; \hat{x}_3(0)) = (16 \;\; 1 \;\; 1)$. The residuals $(u - \hat{u}_i)$ and $(r - \hat{r}_i)$, $i = 1, 2$ show that there are sensor failures without being able to locate them since the corresponding observers depend on the faulty output $y_2 = r$.

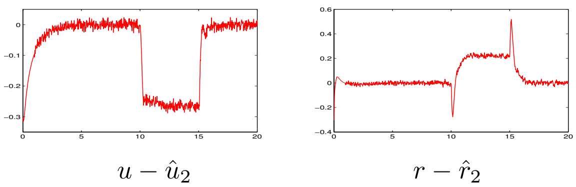

Fig. 6. Sensor fault detection and isolation using observer$_2$

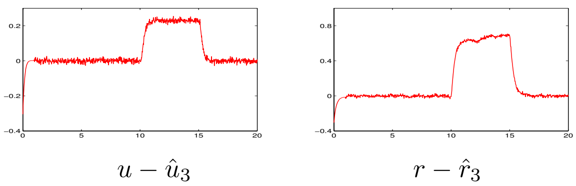

Fig. 7. Sensor fault detection and isolation using observer$_3$

FDI using longitudinal velocity observer$_1$. The simulation results of the fault detection and isolation based on the observer$_1$ are illustrated by the figure 8. The residuals $(u - \hat{u}_1)$ and $(r - \hat{r}_1)$ generated by the observer$_1$ allow to detect and locate the fault sensor on the yaw rate output r. Thus the fault detection and localization is possible by these three observers.

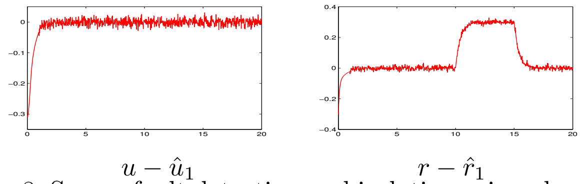

Fig. 8. Sensor fault detection and isolation using observer$_1$

5.2 *Fault detection using unknown input fuzzy observer*

The problem is to detect the occurrence of two fault signals m_1 and m_2 such as the wind force, which disturb the motion of the vehicle. To this end, we suppose the existence of a fault signal on the system (nonlinear model (22)). For that, we use the unknown input fuzzy observer (2) developed previously in this paper. In this case, we consider the nonlinear model with defect describes as follows:

$$\dot{x} = F\,(x,\ \delta,\ T) + Rm \tag{26}$$

$$\text{with } \; m = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 0 \end{bmatrix}$$

The considered unknown input fuzzy observer that estimates the state and the defect (regarded as an unknown input) of the nonlinear model (22) is described by:

$$\dot{\hat{x}} = \sum_{i=1}^{2} \mu_i\,(u) \Big(A_i\hat{x} + B_i w + D_i + G_i(y - C\hat{x}) + R\hat{m}_i \Big) \tag{27a}$$

$$\hat{m}_i = \gamma W_i \big(y - C\hat{x} \big) \tag{27b}$$

$$\hat{m} = \sum_{i=1}^{2} \mu_i\,(u)\,\hat{m}_i \tag{27c}$$

For simulation the following initial conditions are considered: $(u_0 \;\; v_0 \;\; r_0) = (15 \;\; 0 \;\; 0)$ and $(\hat{x}_1(0) \;\; \hat{x}_2(0) \;\; \hat{x}_3(0)) = (16 \;\; 1 \;\; 1)$. The gain matrices G_1, G_2, G_3, W_1, W_2 and W_3 are obtained by solving the constraints (20). We get

$$G_1 = \begin{bmatrix} 9.22 & -3.88 \\ 0.45 & -1.02 \\ 22.51 & -11.92 \end{bmatrix} \quad G_2 = \begin{bmatrix} 10.78 & -4.10 \\ 6.64 & 0.55 \\ 27.38 & -16.19 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 8.49 & -4.83 \\ 4.40 & 1.36 \\ 20.27 & -17.07 \end{bmatrix} \quad \gamma = 78.12$$

$$W_1 = W_2 = W_3 = \begin{bmatrix} 34.14 & 0 \\ 0 & -10 \end{bmatrix}$$

Figures 9 and 10 represent the comparison between the fault signals m_1 and m_2 affecting the nonlinear model (22) and their estimates by the unknown input fuzzy observer (27). These figures clearly show the occurrence of the fault signals m_1 and m_2. Figures 11 and 12 present the comparison between the output of the nonlinear model with the faults m_1 and m_2 (26) and their estimates with the designed unknown input observer. The output variables and their estimates are superimposed except in the vicinity of the origin (choice of the initial conditions).

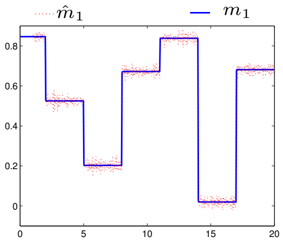

Fig. 9. Fault m_1 and its estimate $\hat{m}_1$ by (27)

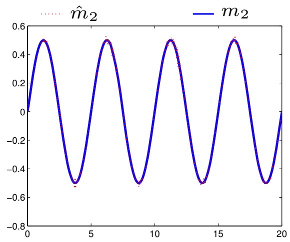

Fig. 10. Fault m_2 and its estimate $\hat{m}_2$ by (27)

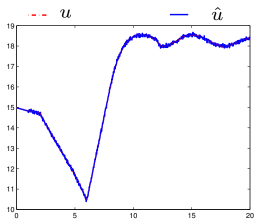

Fig. 11. longitudinal velocity u and its estimate by (27)

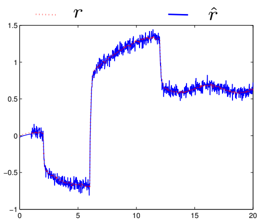

Fig. 12. lateral velocity v and its estimate by (27)

6. CONCLUSION

In this paper, based on a T-S fuzzy model representation, the design of an unknown input T-S observer is proposed. The synthesis conditions lead to the resolution of an LMI problem. Moreover, the estimation of unknown inputs of the system is considered. The proposed observer is then used for state estimation and for detection and isolation of faults which can affect nonlinear models. The effectiveness of the proposed methodology is illustrated by estimating the yaw rate and faults of automatic steering vehicle. The considered structure of unknown input T-S observer can be also benefic for fault detection and isolation of a fault actuator.

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