STATE ESTIMATION VIA MULTIPLE OBSERVER. THE THREE TANK SYSTEM

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Abstract: In this paper, we consider a nonlinear system represented by a multiple model, where a part of its inputs is unknown. Our objective is to estimate the state variables of this system. For that, we propose the synthesis of a multiple observer based on the elimination of these unknown inputs. It is shown how to determine the gains of the local observers, these gains being solutions of a set of linear matrix inequalities (LMI). The model of an hydraulic system with three tanks is used to test the suggested procedure.

Key words: Nonlinear system, unknown inputs, multiple model approach, multiple observer, asymptotic stability, linear matrix inequality.

1. INTRODUCTION

A physical process is often subjected to disturbances which have as origin the noises due to its environment, uncertainties of measurements, faults of sensors and/or actuators. These disturbances have harmful effects on the normal behavior of the process and their estimation can be used to conceive a control strategy able to minimize their effects. The disturbances are called unknown inputs when they affect the input of the process and their presence can make difficult the state estimation.

Several works were achieved concerning the estimation of the state and the output in the presence of unknown inputs. They can be gathered into two categories. The first one supposes an a priori knowledge of information on these nonmeasurable inputs, in particular, Johnson proposes a polynomial approach (Johnson, 1975) and Meditch suggests approximating the unknown inputs by the response of a known dynamic system (Meditch, 1974). The second category proceeds either by estimation of the unknown inputs, or by their complete elimination from the equations of the system.

Among the techniques that do not require the elimination of the unknown inputs, that of (Wang, 1975) proposes an observer able to entirely reconstruct the state of a linear system in the presence of unknown inputs; in (Kobayashi, 1982) and (Lyubchik, 1993), the authors have used a method of model inversion to estimate the state.

Among the techniques which allow the elimination of the unknown inputs, that of (Kudva, 1980) establishes, in the case of linear systems, the existence conditions of the system observer with unknown inputs while being based on the technique of generalized inverse of matrix. Guan carried out the elimination of the unknown inputs of the state equations of a continuous linear system (Guan, 1991). Many of other alternatives exist, but most of them were developed principally for linear systems.

However, the real physical systems are often nonlinear. As it is delicate to synthesize an observer for a nonlinear system, we preferred to represent these systems with a multiple model. The idea of the multiple model approach is to apprehend the total behavior of a system by a set of local models (linear or affine), each local model characterizing the behavior of the system in a particular zone of operation. The local models are then aggregated by means of an interpolation mechanism.

The motivation of this approach rises owing to the fact that it is often difficult to design a model which takes into account all the complexity of the studied system. In (Takagi and Sugeno, 1985), the authors have presented their fuzzy model of a system described by a set of rules

if premise then consequence ", such as the consequence of a rule is an affine local model; the global model is obtained by the sum of the local models weighted by activation functions associated to each of them.
For state estimation, the suggested technique, consists in associating to each local model a local observer. The global observer (multiple observer), is the sum of the local observers weighted by their activation functions, which are the same than those associated with the local models (Patton, 1998). Our contribution lies in the design of this global observer by eliminating the unknown inputs from the system. The stabilization of the multiple observer is performed by the search of suitable Lyapunov matrices and the improvement of the performances of the multiple observer by pole assignment is formulated in a LMI context.

2. MULTIPLE OBSERVER OF A SYSTEM WITH UNKNOWN INPUTS

This section clarifies the construction of the observer. This last has an analytical form resulting from the aggregation of local observers and this form is particularly suitable for stability and convergence study of the estimation error. The numerical aspects concerning the determination of the gains of the observers will be also treated.

2.1 *Principle of the reconstruction*

Let us consider a system in multiple model (with r local models) form and dependent on unknown inputs:

$$\begin{cases} \dot{x}(t)=\sum_{i=1}^{r}\mu_i(\xi(t))\left(A_i x(t)+B_i u(t)+F\bar{u}(t)+D_i\right) \\ y(t)=Cx(t) \end{cases} \tag{1}$$

where $x(t)\in R^n$ is the state vector, $u(t)\in R^l$ is the input vector, $\bar{u}(t)\in R^q$ the vector of unknown inputs and $y(t)\in R^m$ the vector of measurable output. For the i^th^ model $A_i \in R^{n\times n}$ is the state matrix, $B_i \in R^{n\times l}$ is the matrix of input, $F\in R^{n\times q}$ is the matrix of influence of the unknown inputs and $D_i \in R^{n\times 1}$ is a matrix depending on the operating point. Finally, $C \in R^{m\times n}$ is the matrix of output and $\xi(t)$ represents the vector of decision depending on the input and/or the measurable state variables: the value of $\xi(t)$ allows to determine what are the active local models at time t. The procedure that allows to obtain this structure and to estimate its parameters is not developed here. Let us state that one can either uses techniques of parametric estimation (Gasso, 2001) or techniques of linearization (Johansen, 2000).
Let us consider the global functional state multiple observer, $\hat{x}(t)$, described as follows:

$$\dot{z}(t)=\sum_{i=1}^{r}\mu_i(\xi(t))\left(N_i z(t)+G_{i1}u(t)+G_{i2}+L_i y(t)\right) \tag{2a}$$

$$\hat{x}(t)=z(t)-Ey(t) \tag{2b}$$

$N_i\in R^{n\times n}$, $G_{i1}\in R^{n\times l}$, $L_i\in R^{n\times m}$ is the gain of the local observer, $G_{i2}\in R^{n\times 1}$ is a constant vector and E a matrix of transformation. All these matrices or vectors have to be defined so that the reconstructed state asymptotically converges to the actual state $x(t)$.
The reconstruction error of the state is given by:

$$\tilde{x}(t)=x(t)-\hat{x}(t) \tag{3}$$

that is while using (2b):

$$\tilde{x}(t)=(I+EC)x(t)-z(t)$$

Its time variation is :

$$\dot{\tilde{x}}(t)=\sum_{i=1}^{r}\mu_i(\xi(t))\begin{pmatrix} P\left(A_i x(t)+B_i u(t)+F\bar{u}(t)+D_i\right)- \\ N_i z(t)-G_{i1}u(t)-G_{i2}-L_i y(t) \end{pmatrix} \tag{4}$$

with

$$P=I+EC \tag{5}$$

The expression (4) can be rewritten :

$$\dot{\tilde{x}}(t)=\sum_{i=1}^{r}\mu_i(\xi(t))\begin{pmatrix} N_i\tilde{x}(t)+\left(PA_i-N_iP-L_iC\right)x(t)+ \\ \left(PB_i-G_{i1}\right)u(t)+ \\ \left(PD_i-G_{i2}\right)+PF\bar{u}(t) \end{pmatrix} \tag{6}$$

If the conditions (7) are satisfied (Maquin, 2000) and (Gaddouna, 1995):

$$\begin{aligned} & P=I+EC \\ & L_iC=PA_i-N_iP \\ & G_{i1}=PB_i \qquad i=1..r \\ & G_{i2}=PD_i \\ & PF=0 \\ & \sum_{i=1}^{r}\mu_i(\xi(t))N_i \text{ stable} \end{aligned} \tag{7}$$

then, the reconstruction error of the global state tends asymptotically towards zero, and (4) is reduced to:

$$\dot{\tilde{x}}(t)=\sum_{i=1}^{r}\mu_i(\xi(t))N_i\tilde{x}(t) \tag{8}$$

The numerical solution of the system (7) rises from the use of the generalized inverse $(CF)^-$ of (CF), the existence conditions being specified in (Kudva, 1980):

$$\begin{cases} E=-F(CF)^- \\ P=I-F(CF)^-C \\ G_{i1}=PB_i \\ G_{i2}=PD_i \\ N_i=PA_i-K_iC \\ L_i=K_i-N_iE \end{cases} \tag{9}$$

It is important to note that the stability of matrices N_i, $\forall\ i\in\{1,\ldots,r\}$ does not guarantee the stability of the matrix $\sum_{i=1}^{r}\mu_i(\xi(t))N_i$. This item is discussed in the next paragraph.

2.2 *Global convergence of the multiple observer*

In this part, we will develop the sufficient conditions of the asymptotic global convergence of the state estimation error.

The dynamic equation (8) is globally asymptotically stable if there exists a positive definite symmetrical matrix *X*, such that (Boyd, 1994):

$$N_i^T X + X N_i < 0,\ \forall i \in \{1,\dots,r\} \tag{10}$$

The search of a matrix X which is common to all matrices N_i in the preceding equations can be a rather conservative step. In what follows, another method, based on the existence of various local matrices $X_i > 0$, will be used.

Theorem 1 (Chadli, 2001): If there exists symmetrical and definite positive matrices X_i such as:

$$N_{ih} X_i + X_i N_{ih} < 0,\ \forall i \in \{1,\dots,r\},\ N_{ih} = \frac{1}{2}\left(N_i + N_i^T\right) \tag{11}$$

where N_{ih} is the hermitian part associated to the matrix N_i, then the multiple observer (2) is globally asymptotically convergent.

Proof

To solve (11), one uses $N_i = PA_i - K_i C$ from (9) and the inequality (11) becomes, $\forall\ i \in \{1,\dots,r\}$:

$$\begin{aligned}&\left((PA_i - K_i C) + (PA_i - K_i C)^T\right) X_i + \\ &X_i \left((PA_i - K_i C) + (PA_i - K_i C)^T\right) < 0\end{aligned} \tag{12}$$

It is noted unfortunately that the preceding inequalities present the disadvantage of being nonlinear with respect to the variables K_i and X_i (more precisely bilinear). A numerical procedure of resolution by linearization is presented in the following section.

2.3 *Method of resolution*

Methods of resolution were proposed to solve nonlinear and in particular bilinear inequalities (see (Chadli, 2001) and included references). The method that we adopted is known as local, because it is based on the linearization, with respect to variables K_i and X_i, well chosen around the initial values K_{0i} and X_{0i}. One poses:

$$K_i = K_{0i} + \partial K_i \ \ and\ \ X_i = X_{0i} + \partial X_i \tag{13}$$

The inequality (12) then becomes:

$$\begin{aligned}&\begin{pmatrix} \left(PA_i - (K_{0i} + \partial K_i) C\right) + \\ \left(PA_i - (K_{0i} + \partial K_i) C\right)^T \end{pmatrix} (X_{0i} + \partial X_i) + \\ &(X_{0i} + \partial X_i) \begin{pmatrix} \left(PA_i - (K_{0i} + \partial K_i) C\right) + \\ \left(PA_i - (K_{0i} + \partial K_i) C\right)^T \end{pmatrix} < 0 \\ &(X_{0i} + \partial X_i) > 0\end{aligned} \tag{14}$$

By neglecting the second order terms of the inequality (14), one obtains:

$$\begin{aligned}&\left((PA_i - K_{0i} C) + (PA_i - K_{0i} C)^T\right) \partial X_i + \\ &\partial X_i \left((PA_i - K_{0i} C) + (PA_i - K_{0i} C)^T\right) - \\ &\partial K_i C X_{0i} - (C X_{0i})^T \partial K_i^T - \\ &C^T \partial K_i^T X_{0i} - X_{0i} \partial K_i C + \\ &\left((PA_i - K_{0i} C) + (PA_i - K_{0i} C)^T\right) X_{0i} + \\ &X_{0i} \left((PA_i - K_{0i} C) + (PA_i - K_{0i} C)^T\right) < 0\end{aligned} \tag{15}$$

The system (15) is then of LMI type and its resolution is standard (Boyd, 1994). Let us note that the choice of initial values K_{0i} and X_{0i} remains the major disadvantage of the method and moreover convergence towards a solution is not always guaranteed. Unfortunately, from a practical point of view, one can be led to test various choices of initial values in order to obtain a solution.

Remark

The LMI system (15) is valid only in the vicinity of K_{0i} and X_{0i} ; this encouraged us, to improve the resolution, to propose the following additional constraints (in order to limit the variations of matrices K and X):

$$\begin{cases} \|\partial K_i\| < \varepsilon \|K_{0i}\| \\ \|\partial X_i\| < \varepsilon \|X_{0i}\| \end{cases},\ \ with\ \ 0 < \varepsilon << 1 \tag{16}$$

The LMI formulation of the constraints (16), are described in the following way:

$$\begin{cases} \begin{bmatrix} \varepsilon \|X_{0i}\| I_{n\times n} & \partial X_i \\ \partial X_i & \varepsilon \|X_{0i}\| I_{n\times n} \end{bmatrix} > 0 \\ \begin{bmatrix} \varepsilon \|K_{0i}\| I_{(n\times n)} & \partial K_i \\ \partial K_i^T & \varepsilon \|K_{0i}\| I_{(m\times m)} \end{bmatrix} > 0 \end{cases} \tag{17}$$

Indeed, if the LMI system (15) and (17) is realizable, then the multiple observer (2) globally asymptotically estimates the state of the multiple model (1).

3. POLE ASSIGNEMENT

In this part, we examine how to improve the performances of the multiple observer in particular with regard to the rate of convergence towards zero of the state error estimation. For better estimating the state variables of the multiple model, the dynamics of the multiple observer is selected in a manner which is appreciably faster than that of the multiple model.

Definition: The multiple observer with unknown inputs (2) is known as locally observable if the pairs (PA_i, C) are observable, $\forall\ i \in \{1,\dots,r\}$ (Patton, 1998).

To ensure a certain dynamics of convergence of the state estimation error, one defines, in the complex plane, an area $S(\alpha, \beta)$ built by the intersection between a circle, of center $(0,0)$ and of radius β, and the left half plane limited by a vertical line of X-coordinate equal to $(-\alpha)$ with α being a positive constant. The LMI formulation is proposed by the following corollary.

Corollary: The eigenvalues of the matrix $\sum_{i=1}^{r}\mu_i(\xi(t))N_i$ belong to the area $S(\alpha,\beta)$, if there exists matrices ∂X_i and ∂K_i such that:

$$\begin{bmatrix} -\beta(X_{0i}+\partial X_i) & N_{0i}^T X_i-(\partial K_i C)^T X_{0i} \\ X_i N_{0i}-X_{0i}(\partial K_i C) & -\beta(X_{0i}+\partial X_i) \end{bmatrix}<0$$

$$N_{0i}^T\partial X_i+\partial X_i N_{0i}-C^T\partial K_i^T X_{0i}-X_{0i}\partial K_i C \qquad (18)$$
$$+N_{0i}^T X_{0i}+X_{0i}N_{0i}+2\alpha(X_{0i}+\partial X_i)<0,\ \forall\ i\in\{1,...,r\}$$

with: $\begin{cases} N_{0i}=PA_i-K_{0i}C \\ X_i=X_{0i}+\partial X_i \end{cases}$

4. EXAMPLE

The selected nonlinear system is represented on figure 1. It results from a traditional benchmark (Zolghadri, 1996) and schematizes a hydraulic process made up of three tanks. These three tanks T_1, T_2, and T_3 with identical sections A, are connected to each others by cylindrical pipes of identical sections S_n. The output valve is located at the output of tank T_2; it ensures to empty the tank filled by the flow of pumps 1 and 2 with respectively flow rates $Q_1(t)$ and $Q_2(t)$. Combinations of the three water levels are measured. The pipes of communication between the tanks are equipped with manually adjustable ball valves, which allow the corresponding pump to be closed or open. The three levels x_1, x_2 and x_3 are governed by the constraint $x_1>x_3>x_2$; the process model is given by the equation (19).

Indeed, taking into account the fundamental laws of conservation of the fluid, one can describe the operating mode of each tank; one then obtains a nonlinear model expressed by the following state equations (Zolghadri, 1996):

$$\begin{cases} A\dfrac{dx_1(t)}{dt}=-\alpha_1 S_n\left(2g(x_1(t)-x_3(t))\right)^{1/2}+Q_1(t)+Qf_1\bar{u}(t) \\ A\dfrac{dx_2(t)}{dt}=\alpha_3 S_n\left(2g(x_3(t)-x_2(t))\right)^{1/2}-\alpha_2 S_n\left(2gx_2(t)\right)^{1/2} \\ \qquad +Q_2(t)+Qf_2\bar{u}(t) \\ A\dfrac{dx_3(t)}{dt}=\alpha_1 S_n\left(2g(x_1(t)-x_3(t))\right)^{1/2} \\ -\alpha_3 S_n\left(2g(x_3(t)-x_2(t))\right)^{1/2}+Qf_3\bar{u}(t) \end{cases} \qquad (19)$$

where α_1, α_2 and α_3 are constants, $\bar{u}(t)$ is regarded as an unknown input. $Qf_i\bar{u}(t), i\in\{1,...,3\}$ denote the additional mass flows into the tanks caused by leaks and g is the gravity constant.

The multiple model (1), with $\xi(t)=u(t)$, which approximates the nonlinear system (19), is described by:

$$\begin{cases} \dot{x}(t)=\sum_{i=1}^{4}\mu_i(u(t))(A_i x(t)+B_i u(t)+F\bar{u}(t)+D_i) \\ y(t)=Cx(t) \end{cases} \qquad (20)$$

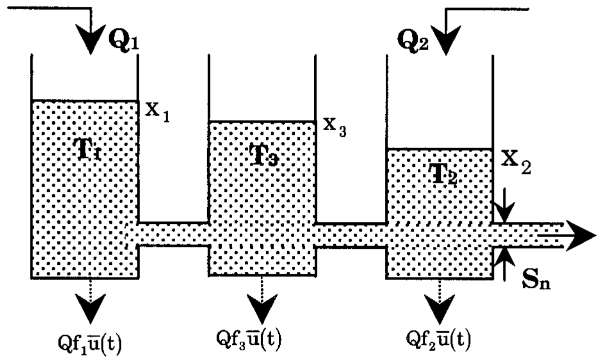

Figure 1. Three tank system

The matrices A_i, B_i and D_i are calculated by linearizing the initial system (19) around four points chosen in the operation range of the system. Four local models have been selected in an heuristic way. That number guarantees a good approximation of the state of the real system by the multiple model. The following numerical values were obtained:

$$A_1=\begin{bmatrix} -0.0109 & 0 & 0.0109 \\ 0 & -0.0206 & 0.0106 \\ 0.0109 & 0.0106 & -0.0215 \end{bmatrix},\ D_1=10^{-3}\begin{bmatrix} -2.86 \\ -0.38 \\ 0.11 \end{bmatrix}$$

$$A_2=\begin{bmatrix} -0.011 & 0 & 0.011 \\ 0 & -0.0205 & 0.01044 \\ 0.011 & 0.01044 & -0.0215 \end{bmatrix},\ D_2=10^{-3}\begin{bmatrix} -2.86 \\ -0.34 \\ 0.038 \end{bmatrix}$$

$$A_3=\begin{bmatrix} -0.0084 & 0 & 0.0084 \\ 0 & -0.0206 & 0.0095 \\ 0.0084 & 0.0095 & -0.018 \end{bmatrix},\ D_3=10^{-3}\begin{bmatrix} -3.70 \\ -0.14 \\ 0.69 \end{bmatrix}$$

$$A_4=\begin{bmatrix} -0.0085 & 0 & 0.0085 \\ 0 & -0.0205 & 0.0095 \\ 0.0085 & 0.0095 & -0.018 \end{bmatrix},\ D_4=10^{-3}\begin{bmatrix} -3.67 \\ -0.18 \\ 0.62 \end{bmatrix}$$

$$B_i=\frac{1}{A}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},\ C=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \text{ and } \forall\ i\in\{1,...,4\}$$

In the following, the functions Qf_1, Qf_2 and Qf_3 are constant, $\bar{u}(t)$ is a random sequence and the numerical application are performed with:

$$Qf_i=10^{-4},\ \forall\ i\in\{1,...,4\} \text{ and } t\in[0\ \ \infty[$$
$$\alpha_1=0.78,\ \alpha_2=0.78 \text{ and } \alpha_3=0.75$$
$$g=9.8,\ S_n=5\times10^{-5} \text{ and } A=0.0154$$

Determination of the multiple observer

The structure of the multiple observer is defined in (4). The matrix P is obtained by solving (9)

$$P=\frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

The observability of the pairs (PA_i,C) is checked. To obtain the matrices K_{0i} and X_{0i}, one can proceed in the following way.

1- One chooses as initial values:

$$X_{01}=\begin{bmatrix}1.24 & -0.003 & -0.005\\ -0.003 & 1.12 & 0.035\\ -0.005 & 0.035 & 1.17\end{bmatrix},\ K_{01}=10^{-2}\begin{bmatrix}3.2 & 4.4\\ 0.5 & -3.2\\ 2.4 & -0.06\end{bmatrix}$$

$$X_{02}=\begin{bmatrix}1.24 & -0.005 & -0.004\\ -0.005 & 1.13 & 0.036\\ -0.004 & 0.036 & 1.17\end{bmatrix},\ K_{02}=10^{-2}\begin{bmatrix}3.2 & 4.4\\ 0.46 & -3.2\\ 2.4 & -0.06\end{bmatrix}$$

$$X_{03}=\begin{bmatrix}1.24 & -0.006 & -0.005\\ -0.006 & 1.12 & 0.037\\ -0.005 & 0.037 & 1.17\end{bmatrix},\ K_{03}=10^{-2}\begin{bmatrix}3.5 & 4.7\\ 0.2 & -3.4\\ 0.26 & -0.16\end{bmatrix}$$

$$X_{04}=\begin{bmatrix}1.24 & -0.007 & 0.001\\ -0.007 & 1.12 & 0.037\\ 0.001 & 0.037 & 1.17\end{bmatrix},\ K_{04}=10^{-2}\begin{bmatrix}3.5 & 4.8\\ 0.16 & -3.5\\ 0.26 & -0.13\end{bmatrix}$$

2- For each local observer, the poles are placed in the area $S(\alpha,\beta)$, with $\alpha=0.01$ and $\beta=0.2$.

3- The matrices X_{0i} are then calculated ; starting from the equations (15), (17) and (18), one obtains:

$$\partial X_1=10^{-3}\begin{bmatrix}1.9 & 17 & -8.8\\ 17 & 4.3 & -2.5\\ -8.8 & -2.5 & 11\end{bmatrix},\ \partial K_1=10^{-4}\begin{bmatrix}-5.4 & -8.5\\ 5.6 & 6.4\\ -0.9 & -2.7\end{bmatrix}$$

$$\partial X_2=10^{-3}\begin{bmatrix}3.7 & 9.3 & -4.7\\ 9.3 & 5.3 & -1.5\\ -4.7 & -1.5 & 9.5\end{bmatrix},\ \partial K_2=10^{-4}\begin{bmatrix}-21 & -37\\ 35 & 37\\ -0.9 & -23\end{bmatrix}$$

$$\partial X_3=10^{-3}\begin{bmatrix}-4.4 & 38.9 & -19.6\\ 38.9 & -1.4 & -2.4\\ -19.6 & -2.4 & 12.3\end{bmatrix},\ \partial K_3=10^{-4}\begin{bmatrix}-9.6 & -13\\ 11.4 & 11.6\\ -3.7 & -5.7\end{bmatrix}$$

$$\partial X_4=10^{-3}\begin{bmatrix}-5.5 & 45.6 & -23\\ 45.6 & -3.4 & -2.5\\ -23 & -2.5 & 13\end{bmatrix},\ \partial K_4=10^{-4}\begin{bmatrix}-11 & -14.6\\ 13 & 13\\ -4.5 & -6\end{bmatrix}$$

The constraint on the proximity of the solution in the vicinity of K_{0i} and X_{0i} .is respected, while imposing:

$$\frac{\|\partial X_i\|}{\|X_{0i}\|}\approx 0.02 \text{ and } \frac{\|\partial K_i\|}{\|K_{0i}\|}\approx 0.03.$$

The obtained results show the effectiveness of the design algorithm of the multiple observer with unknown inputs. Figure 2 visualizes the two known inputs applied to the system as well as the unknown input. Figure 3 compares the states of the system reconstructed by the multiple observer with the corresponding states of the multiple model ; Figure 4 compares the states of the system reconstructed by the multiple observer with the corresponding states of nonlinear system ; the quality of the reconstruction is also highlighted on the figure 5 which visualizes the errors of the reconstructed states (the variations noted in the proximity of the time origin are due to the initial conditions of the observer which have been arbitrarily chosen). One notices however, the convergence of the multiple observer and the negligeable error between the real variables and the estimated variables.

Remark

A easy way to compute the initial values K_{0i} and X_{0i} may be proposed:

- Initial values of the gains K_{0i} are determined by a technique of pole assignment such as each gain is calculated independently of the others.
- The initial matrices X_{0i} are calculated by using the Lyapunov approach.

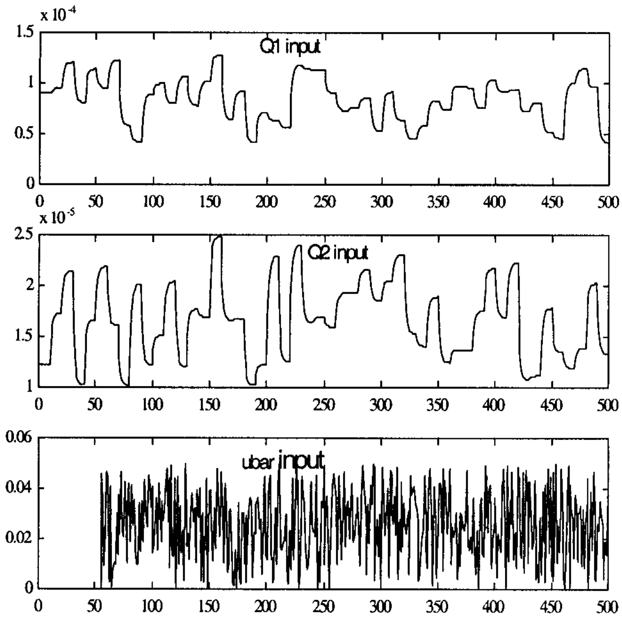

Figure 2. Inputs of the system

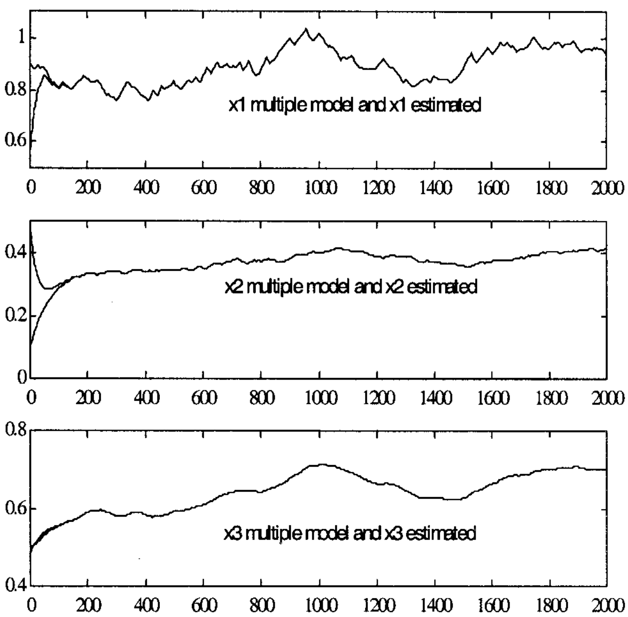

Figure 3. Multiple model (20) and multiple observer (2)

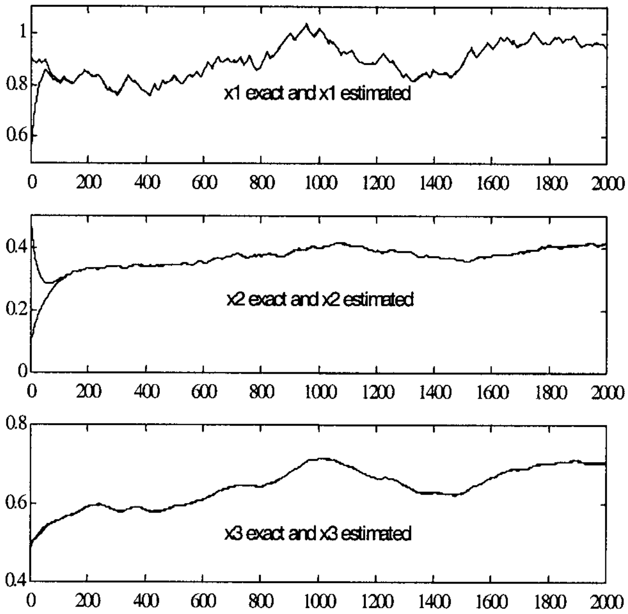

Figure 4. Nonlinear system (19) and multiple observer (2)

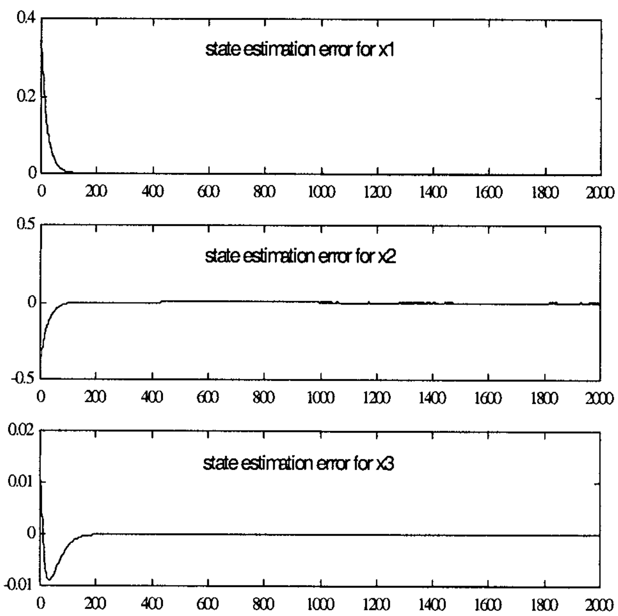

Figure 5 : states estimation error

5. CONCLUSION

Using a multiple model representation, we showed how to design a multiple observer using the principle of the interpolation of local observers. Moreover, one considered the case where some inputs of the system were unknown. The calculation of the gain of the global observer reduces to the calculation of the gains of the local observers ; the stability of the whole requires taking into account the coupling constraints between the local observers, which leads to the resolution of a problem of the BMI (Bilinear Matrix Inequalities) type. The resolution of these BMI constraints is carried out by linearization and the essential numerical disadvantage of this method only resides in the choice of the initial variables of matrices K_i and X_i.

The direct application of this observer could be, thanks to taking into account the unknown inputs, the base for the design of a detection procedure and localization of faults of actuators.

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