UNKNOWN INPUT MULTIPLE OBSERVER BASED-APPROACH - APPLICATION TO SECURE COMMUNICATIONS

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Abstract: This paper presents multiple observer design for nonlinear chaotic systems with unknown inputs in multiple model approach. The considered unknown inputs influences the states and the outputs of the system. The main objective is to estimate the state variables as well as the unknown inputs of this system. For that, we propose the synthesis of a multiple observer based on the elimination of these unknown inputs. The synthesis conditions of the proposed multiple observer are derived in linear matrix inequalities (LMI) terms. The proposed method is applied to secure communication. An simulation example is given to illustrate the effectiveness of the proposed synthesis conditions.

Keywords: Multiple model, unknown input observer, state estimation, multiple observer, secure communication, linear matrix inequalities (LMI).

1. INTRODUCTION

Synchronisation in chaotic systems and its potential application to secure communication have received a large attention over the last decade (Carroll and Pecora, 1991), (Cuomo *et al.*, 1993), (Darouach and Boutayeb, 1995), (Nijmeijer and Mareels, 1997), (Hasler, 1998), (Boutayeb *et al.*, 2002). The idea of secure communication is to encrypt a plain text at the transmitter and decrypt the cipher text at the receiver. The transmission channels are public in general. Therefore, it is advisable to mask or modulate the information within a chaotic signal and retrieve it from the received signal. Pecora and Carroll, in their pioneering work (Carroll and Pecora, 1991), proposed some stable subsystems of the given chaotic systems for constructing unidirectionally coupled synchronization systems. After that, vast amounts of research of chaos synchronization and its application to secure communication have been presented in the literature. Recently, in (Lin *et al.*, 2005) an adaptive robust observer-based scheme for the synchronization of unidirectional coupled chaotic systems with unknown channel time-delay and system uncertainties was proposed. Liao and Tsai (Liao and Tsai, 2000) addressed an adaptive observer to estimate the unknown parameter and disturbance of a chaotic system with output feedback term. Feki (Feki and Robert, 2003) designed complete adaptive observer-based response system to synchronize chaos with parameter uncertainties. The above research works are essentially based on classical methods to analyze and design the synchronization of continuous-time or discrete-time chaotic system. In this work, the synchronization by multiple model approach is proposed.

The basic idea of the multiple model approach is to apprehend the total behavior of nonlinear model by a set of LTI models (linear or affine). The local models are then interpolated with convex functions (Murray-Smith, 1997). The motivation of this approach is related to the fact that it is often difficult to design a model which takes into account all the complexity of the studied system. This approach which includes the Takagi-Sugeno (T-S) models (Takagi and Sugeno, 1985) and Polytopic Linear Differential Inclusions (PLDI) (Boyd *et al.*, 1994) has been extensively considered in the last decade(see among others (Patton *et al.*, 1998), (Tanaka *et al.*, 1998), (Chadli *et al.*, 2003) and references therein). However there is few studies concerning the secure communication using the multiple model approach (Ting, 2005)(Li *et al.*, 2005), (Chen *et al.*, 2005). For example in (Ting, 2005), authors are used an adaptive fuzzy observer design to synchronize chaotic systems. The chaotic system is expressed in the form of T–S model.

In this paper, we consider firstly the state and unknown input estimation of chaotic system in multiple model representation. The design of the unknown input multiple observer is obtained by eliminating the unknown inputs. The synthesis conditions of the proposed structure of multiple observer are derived in LMI terms.

The rest of this paper is organized as follows. In section 2, the general structure of multiple model is presented. In section 3, the considered structure of multiple observer is given and the main results are presented. The derived conditions ensuring the global asymptotic convergence of estimation error are given as a set of LMI with additional equality constraints. A method allowing to estimate the unknown input ends this section. The last section gives a numerical example to illustrate the effectiveness of the proposed results in secure communication domain.

Notation: throughout the paper, the following useful notation is used: X^T denotes the transpose of the matrix X, $X > 0$ means that X is a symmetric positive definite matrix and $\mathbb{I}_M = \{1, 2, ..., M\}$.

2. GENERAL STRUCTURE OF MULTIPLE MODEL

Let us consider a class of nonlinear systems subject to unknown inputs and represented by a multiple model as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i\left(\xi(t)\right) \Big(A_i x(t) + B_i u(t) + R_i \bar{u}(t) \Big) \\ y(t) = Cx(t) + F\bar{u}(t) \end{cases} \quad (1)$$

with:

$$\begin{cases} \sum_{i=1}^{M} \mu_i(\xi(t)) = 1 \\ 0 \leq \mu_i(\xi(t)) \leq 1 \;\; \forall\, i \in \mathbb{I}_M \end{cases} \quad (2)$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}^m$ the input vector, $\bar{u}(t) \in \mathbf{R}^q$, $q < n$, contains the unknown input and $y \in \mathbf{R}^p$ the measured outputs. Matrices $A_i \in \mathbf{R}^{n\times n}$ and $B_i \in \mathbf{R}^{n\times m}$ denote the state matrix and the input matrix associated to the ith local model. The matrices $R_i \in \mathbf{R}^{n\times q}$ and $F \in \mathbf{R}^{p\times q}$, with $rank(F) = q < p$ are the distribution matrices of unknown inputs and $C \in \mathbf{R}^{p\times n}$ is the output matrix. In this paper, the so-called decision variables $\xi(t)$ depend on measurable variables (known inputs and/or measured output).

The choice of the variable $\xi(t)$ leads to different classes of models. It can depend on the measurable state variables, be a function of the measurable outputs of the system and possibly on the input. In this case, the multiple model describes a class of nonlinear system or a T-S model (Takagi and Sugeno, 1985). It can also be an unknown constant value, the multiple model then represents a PLDI (Boyd *et al.*, 1994).

In the following the considered problem concerns both the reconstruction of the state variable $x(t)$ and the unknown input $\bar{u}(t)$, using only the available information namely the known input $u(t)$ and the measured output $y(t)$.

Remark: in the following, to simplify the expression of equations, time variable (t) will be omitted.

3. MULTIPLE OBSERVER DESIGN

Multiple observer is obtained by convex interpolation of numerous Luenberger observers (Patton *et al.*, 1998), (Tanaka *et al.*, 1998), (Chadli *et al.*, 2003) (Akhenak *et al.*, 2004). In this work, we consider the case of continuous-time multiple model with unknown inputs. Our goal is to estimate the state and the unknown inputs of the bellow structure of multiple model. The considered structure of multiple observer has the following form:

$$\begin{cases} \dot{z} = \sum_{i=1}^{M} \mu_i(\xi) \Big(N_i z + G_i u + L_i y \Big) \\ \hat{x} = z - Ey \end{cases} \quad (3)$$

where $N_i \in \mathbf{R}^{n\times n}$, $G_i \in \mathbf{R}^{n\times m}$, $L_i \in \mathbf{R}^{n\times p}$ are the local observer gains, E is a transformation matrix to be determined. This set of matrices has to be properly defined to ensure the convergence of the estimated state towards the true state. For that purpose, let us define the state estimation error:

$$\tilde{x} = x - \hat{x} \quad (4)$$

From this definition and using the expression of $\hat{x}$ given by equation (3), the state estimation error can be written:

$$\tilde{x} = (I + EC)x - z + EF\bar{u}$$

Thus, the dynamic of the state estimation error is given as follows:

$$\dot{\tilde{x}} = \sum_{i=1}^{M} \mu_i(\xi) \Big(P\left(A_i x + B_i u + R_i \bar{u}\right) - N_i z - G_i u - L_i y \Big) + EF\dot{\bar{u}} \quad (5)$$

with:

$$P = I + EC \quad (6)$$

Replacing y and z by their respective expressions given by (1) and (3), the state error takes the form:

$$\dot{\tilde{x}} = \sum_{i=1}^{M} \mu_i(\xi) \Big(N_i \tilde{x} + \big(PA_i - K_i C - N_i\big)x + \left(PB_i - G_i\right)u + \big(PR_i - K_i F\big)\bar{u} \Big) + EF\dot{\bar{u}} \quad (7)$$

with:

$$K_i = N_i E + L_i \quad (8)$$

If the following conditions are fulfilled:

$$PR_i = K_i F \quad (9a)$$
$$G_i = PB_i \quad (9b)$$
$$N_i = PA_i - K_i C \quad (9c)$$
$$EF = 0 \quad (9d)$$

where P and K_i are defined in (6) and (8) respectively, the equation (7) is reduced to:

$$\dot{\tilde{x}} = \sum_{i=1}^{M} \mu_i(\xi) N_i \tilde{x} \quad (10)$$

Then the state estimation error tends asymptotically towards zero if the following conditions hold $\forall\, i \in \mathbb{I}_M$:

$$\exists X > 0, N_i^T X + X N_i < 0 \quad (11)$$

Thus, the constraints (9) and (11) allow to complete synthesis of the multiple observer (3) for the multiple model with unknown inputs (1). We recall that the matrix F must be full column rank and $rank(F) < p$.

3.1 Global convergence of the multiple observer

In this section, sufficient conditions for global asymptotic convergence of state estimation error (7) are established in LMI term with additional structural constraints.

Theorem 1. The state estimation error between unknown input multiple model (1) and multiple observer (3) converges globally asymptotically towards zero, if there exists matrices $X > 0$, S and W_i such that the following conditions hold $\forall\, i \in \mathbb{I}_M$:

$$A_i^T X + X A_i + A_i^T C^T S^T + SCA_i - W_i C - C^T W_i^T < 0 \quad (12a)$$

$$(X + SC)R_i = W_i F \quad (12b)$$
$$SF = 0 \quad (12c)$$

Then multiple observer (3) is completely defined by:

$$E = X^{-1} S \quad (13a)$$
$$G_i = (I + X^{-1}SC)B_i \quad (13b)$$
$$N_i = (I + X^{-1}SC)A_i - X^{-1}W_i C \quad (13c)$$
$$L_i = X^{-1}W_i - N_i E \quad (13d)$$

Proof: We have shown that the constraints (9) and (11) guarantee the global asymptotic convergence of the state estimation error (7). However these constraints are nonlinear in the synthesis variables. In order to convert these conditions into an LMI formulation, we consider the following change of variables:

$$W_i = XK_i \quad (14a)$$
$$S = XE \quad (14b)$$

Taking into account the change of variable (14), the expression (6) we get from equation (11) equation (12a).

The two equality constraints (12b) and (12c) are obtained by pre-multiplying constraints (9a) and (9d) by $X > 0$ with the change of variables (14).

Therefore classical numerical tools may be used for solving the LMI problem subject to linear equality constraints (12). After having solving this problem and based on the definitions (9), the different matrices defining the proposed observer can be deduced from the knowledge of X, S

and W_i as mentioned in (13). This completes the proof.

4. UNKNOWN INPUT ESTIMATION

Several works were realized for the unknown input estimation within the framework of linear dynamic systems (see for e.g. (Stotsky and Kolmanovsky, 2001), (Edwards and Spurgeon, 2000)). For example Edwards et al are proposed two methods for detecting and reconstructing sensor faults using sliding mode observers (Edwards and Spurgeon, 2000). In (Liu and Peng, 2002) a method to simultaneously estimate unknown states and disturbances of linear time invariant systems are presented; the state is estimated using a Luenberger-like observer while the disturbance signals are estimated based on an inverse-dynamics motivated algorithm. In this part, the proposed method is based on the hypothesis of the good estimation of the state variables.

We have previously shown that the convergence of the multiple observer (3) is guaranteed if the conditions of theorem 1 are satisfied. In steady state regime, the state estimation error tends towards zero; by replacing x by $\hat{x}$ in the equation (1) we obtain the following approximation:

$$\begin{cases} \dot{\hat{x}} = \sum_{i=1}^{M} \mu_i(\xi) \left(A_i \hat{x} + B_i u + R_i \hat{\bar{u}} \right) \\ \hat{y} = C\hat{x} + F\hat{\bar{u}} \end{cases} \tag{15}$$

An estimation of unknown input is obtained as follows:

$$\hat{\bar{u}} = (W^T W)^{-1} W^T \begin{pmatrix} \dot{\hat{x}} - \sum_{i=1}^{M} \mu_i(\xi)(A_i \hat{x} + B_i u) \\ \hat{y} - C\hat{x} \end{pmatrix} \tag{16}$$

with:

$$W = \begin{pmatrix} \sum_{i=1}^{M} \mu_i(\xi) R_i \\ F \end{pmatrix} \tag{17}$$

W must be of full column rank.

Remark: if the matrix F is of full column rank, the calculation of the unknown input estimation can be carried out in a simpler way:

$$\hat{\bar{u}} = (F^T F)^{-1} F^T (y - \hat{y}) \tag{18}$$

5. SIMULATION EXAMPLE: APPLICATION TO SECURE COMMUNICATION

The approaches developed in sections 3 and 4 can be applied to synthesize a secure communication system. The problem we are faced with consists of transmitting some coded message with a signal broadcasted by a communication channel. At the receiver side, the hidden signal is recovered by a decoding system. In this section, the proposed multiple observer is used to design a secure communication scheme. For this purpose we consider chaotic multiple model (1) with two LTI local models:

$$\begin{cases} \dot{x} = \sum_{i=1}^{2} \mu_i(y_1) \left(A_i x + R_i \bar{u} \right) \\ y = Cx + F\bar{u} \end{cases} \tag{19}$$

with:

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \text{ and } y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

where x_1 is limited by: $x_1 \in \left[-30 \quad 30 \right]$.

$$A_1 = \begin{pmatrix} -10 & 10 & 0 \\ 28 & -1 & -30 \\ 0 & 30 & -8/3 \end{pmatrix} \quad A_2 = \begin{pmatrix} -10 & 10 & 0 \\ 28 & -1 & 30 \\ 0 & -30 & -8/3 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad B_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad F = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$R_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad R_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

The activation functions are the following form:

$$\mu_1(y_1) = \frac{1}{2}\left(1 + \frac{y_1}{30}\right) \text{ and } \mu_2(y_1) = \frac{1}{2}\left(1 - \frac{y_1}{30}\right)$$

where the message is modulated into the chaotic system (Lorenz's equation) via the previously designed vectors R_i (Lian *et al.*, 2001); using vector F, the transmitted signal y is embedded with the message $\bar{u}$.

The simulation of multiple model without the unknown input $\bar{u}$ and with the initial value $x_0 = (1\ 1\ 1)^T$ shows the chaotic behavior of the example (see figure (1) plotted in the phase plan of the system).

In the following and in the context of secure communication, the unknown input represents the hidden message to be transmitted. Thus the transmitted signal y is embedded with the hidden message $\bar{u}$.

The considered multiple observer for this application is given by the following equation:

$$\begin{cases} \dot{z} = \sum_{i=1}^{2} \mu_i(y_1) \left(N_i z + L_i y \right) \\ \hat{x} = z - Ey \end{cases} \tag{20}$$

The resolution of the conditions of theorem 1 with $B_1 = B_2 = (0, 0, 0)^T$ lead to the following result:

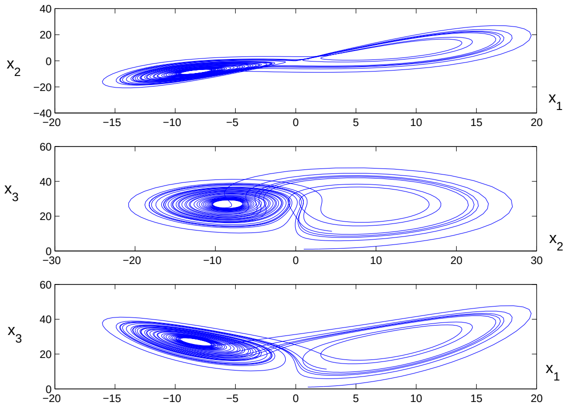

Figure 1. Phase plan of the chaotic multiple model (19)

$$X = \begin{pmatrix} 1.750 & 1.650 & -0.003 \\ 1.650 & 1.750 & -0.003 \\ -0.003 & -0.003 & 0.195 \end{pmatrix} \quad E = \begin{pmatrix} -3.05 & 3.05 \\ 3.99 & -3.99 \\ -0.004 & 0.004 \end{pmatrix}$$

$$N_1 = \begin{pmatrix} 33.66 & 47.89 & -91.72 \\ -36.18 & -45.78 & 89.96 \\ 61.12 & -32 & -2.79 \end{pmatrix} \quad L_1 = \begin{pmatrix} -16.44 & 17.44 \\ -14.94 & 15.94 \\ 253.9 & -252.9 \end{pmatrix}$$

$$N_2 = \begin{pmatrix} 35.06 & 49.54 & 91.72 \\ -37.82 & -47.14 & -89.96 \\ -62.08 & 31.20 & -2.53 \end{pmatrix} \quad L_2 = \begin{pmatrix} -19.38 & 17.32 \\ -13.67 & 17.67 \\ -252.38 & 253.37 \end{pmatrix}$$

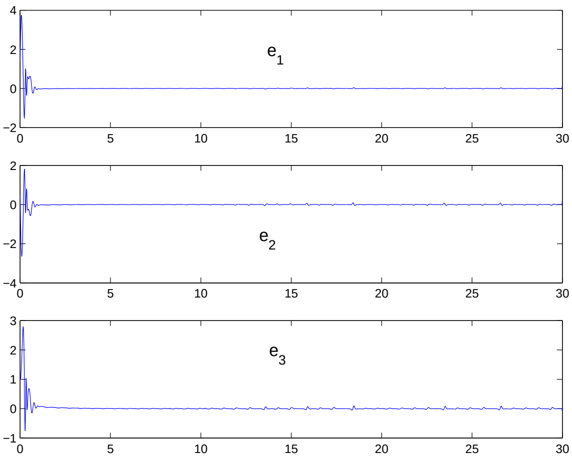

Figure 2. Estimation errors $e_i = x_i - \hat{x}_i$, $i \in \{1, 2, 3\}$

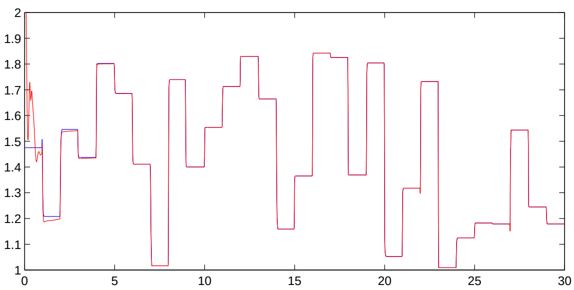

Figure 3. Hidden message $\bar{u}$ and its estimate

Figure (2) represent the state estimation error with the initial conditions $x_0 = (1\ 1\ 1)^T$ et $\hat{x}_0 = (0\ 0\ 0)^T$. Figure (3) displays the hidden transmitted message and its estimate. Excepted around the time origin, the estimated message perfectly matches the true one.

6. CONCLUSION

Using multiple model representation, We have showed how to design a multiple observer for synchronization of chaotic multiple models with unknown inputs. Sufficient conditions to design such observer is given in LMI formulation with additional equality constraints easy to compute with classical numerical tools. Under some assumption, we have showed that the state and unknown input estimation are possible. A numerical example representing an application to secure communication is given to illustrate the effectiveness of the derived synthesis conditions. The simulation results show that the synchronization in chaotic multiple models and the retrieve of the hidden transmitted signal are very satisfactory.

REFERENCES

Akhenak, A., M. Chadli, J. Ragot and D. Maquin (2004). State estimation of uncertain multiple model with unknown inputs. *43rd IEEE Conference on Decision and Control, Atlantic, Paradise Island, Bahamas.*

Boutayeb, M., M. Darouach and H. Rafaralahy (2002). Generalized state-space observers for chaotic synchronization and secure communication. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications* **49(3)**, 345–349.

Boyd, S., L. El Ghaoui, E. Feron and V. Balakrishnan (1994). *Linear Matrix Inequalities in System and Control Theory.* Philadelphia: SIAM.

Carroll, TL. and LM. Pecora (1991). Synchronizing chaotic systems. *IEEE Transactions on Circuits and Systems, I: Fundamental Theory and Applications* **38**, 453 – 459.

Chadli, M., D. Maquin and J. Ragot (2003). Multiple observer for discrete-time multiple model. *IFAC Congress, Safeprocess* pp. 801 – 806.

Chen, Maoyen, Donghua Zhou and Yun Shang (2005). A new observer-based synchronization scheme for private communication. *Chaos, Solitons and Fractals* **24**, 1025–1030.

Cuomo, KM., AV. Oppenheim and Strogatz SH. (1993). Synchronisation of lorenz-based chaotic circuits with application to communications. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Application* **40**, 626 – 659.

Darouach, M. and M. Boutayeb (1995). Design of observers for descriptor systems. *IEEE Trznsactions on Automatic Control* **40**, 1323 – 1327.

Edwards, C. and S. K. Spurgeon (2000). Sliding mode observers for fault detection and isolation. *Automatica* **36(4)**, 541–553.

Feki, Moez and Bruno Robert (2003). Observer-based chaotic synchronization in the presence of unknown inputs. *Chaos, Solitons & Fractals* **15(5)**, 831–840.

Hasler, M. (1998). Synchronization of chaotic systems and transmission of information. *Intrnational Journal of Bifurcation Chaos* **8**, 647 – 661.

Li, Chuandong, Xiaofeng Liao and Kwok wo Wong (2005). Lag synchronization of hyperchaos with application to secure communications. *Chaos, Solitons and Fractals* **23**, 183–193.

Lian, Kuang-Yow, Chian-Song Chiu, Tung-Sheng Chiang and Peter Liu (2001). Secure communications of chaotic systems with robust performance via fuzzy observer-based design. *IEEE Transactions on Fuzzy Systems* **9(1)**, 212–220.

Liao, T.L. and S.H. Tsai (2000). Adaptive synchronization of chaotic systems and its application to secure communications. *Chaos, Solitons and Fractals* **11**, 1387 – 1396.

Lin, Jui-Sheng, Teh-Lu Liao, Jun-Juh Yan and Her-Terng Yau (2005). Synchronization of unidirectional coupled chaotic systems with unknown channel time-delay: Adaptive robust observer-based approach. *Chaos, Solitons and Fractals* **26**, 971 – 978.

Liu, C. S. and H. Peng (2002). Inverse-dynamics based state and disturbance observers for linear time-invariant systems. *Journal of Dynamic System, Measurement and Control* **124**, 375 – 381.

Murray-Smith, R. (1997). Multiple model approaches to modelling and control. *Taylor and Francis.*

Nijmeijer, H. and I. Mareels (1997). An observer looks at synchronization. *IEEE Transactions Ciscuits system I* **44**, 882 – 889.

Patton, R. J., J. Chen and C. J. Lopez-Toribio (1998). Fuzzy observer for nonlinear dynamic systems fault diagnosis. *IEEE Conference on Decision and Control* **1**, 84 – 89.

Stotsky, A. and I. Kolmanovsky (2001). Simple unknown input estimation techniques for automotive applications. *American Control Conference* pp. 3312–3317.

Takagi, J; and M. Sugeno (1985). Fuzzy identification of systems and its application to modelling and control. *IEEE Trans. on Systems Man and Cybernetics* **15**, 116 – 132.

Tanaka, K., T. Ikeda and Y. Y. He (1998). Fuzzy regulators and fuzzy observers: relaxed stability conditions and lmi-based design. *IEEE Trans. Fuzzy Systems* **6(1)**, 250 – 256.

Ting, Chen-Sheng (2005). An adaptive fuzzy observer-based approach for chaotic synchronization. *International Journal of Approximate reasoning* **39**, 97–114.