

Multiple Sensor Fault Detection in Heat Exchanger Systems

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Abstract

A two stage strategy for fault detection, isolation and estimation of multiple gross measurement errors (faults) in a class of systems described simultaneously by linear and bilinear equations is presented. The first stage consists in the detection and isolation of gross measurement errors intervening only in the linear equations. The second stage deals with the detection and isolation of gross measurement errors intervening on the bilinear equations by using corrected measurements of the preceding stage. A GLR method is employed to detect and to isolate the faults. An application of the proposed strategy is presented for a heat exchanger system for which the process model consists of a set of linear and bilinear equations. Simulations were performed to test the efficiency of the proposed method.

1. Introduction

Systems described simultaneously by linear and bilinear equations constitute an important potential area for data reconciliation techniques. This type of model structure is often used when describing a flow process (mineral or chemical industry, oil refinery) by material and/or energy balance equations, and can be found in heat exchangers.

The increasing importance of heat exchangers in the industry is not to be demonstrated. A set of interconnected heat exchangers may be described by a static linear-bilinear model obtained by the writing of mass and thermal balances of these exchangers.

We were interested in using fault detection and isolation techniques to validate the data provided by sensors before using them for controlling the process.

Classically, this data validation or data reconciliation is divided in the following steps : preliminary system decomposition

according to the observability, detection, isolation, estimation of faults and data reconciliation.

In the last two decades, various methods for detection and isolation of gross errors in process data have been proposed including generalized likelihood ratio method (GLR) [1], measurement test method [2], parity space method [3] and pseudo nodes method [4]. The aim of this paper is to extend the utilization of the GLR test to detect and isolate sensor faults for a class of systems described simultaneously by linear (mass balance) and bilinear equations (thermal balance). The chosen application is a heat exchanger system.

The proposed method comprises two stages. The first stage is the generation of linear residuals using a classical method like these cited previously. Afterwards, the detection, isolation, identification and correction of the measurements intervening only in linear equations are done.

The second stage involves the generation of bilinear residuals. The GLR test is applied to generate bilinear residuals by using validated measurements of the preceding stage. This test allows the detection of measurement faults that intervene only in bilinear equations. The phases of isolation, identification and correction can then be driven in analogous manner to the preceding stage.

This paper is organized as follows. In the second section, the hypotheses which are necessary for developing the proposed method are described. The third section is dedicated to the description of the used models. Afterwards, the residual generation is described in the following section. The fifth section remembers fault detection and isolation based on the use of GLR test and describes the proposed strategy. Then, a simple simulated example illustrates the method.

2. Hypotheses

The following hypotheses are assumed to be verified:

- the models (linear and bilinear) are exact,
- the measurement errors are not correlated and are normally distributed with zero mean,
- the variances of these errors are known or estimated.

In the following, we will denote by Q the vector of flow measurements and by T the vector of temperature measurement. The vector Q , the vector of measurement errors ε_Q and the vector of flow sensor faults d_Q are connected with the vector of true values Q^* by the model:

$$Q = Q^* + \varepsilon_Q + d_Q \quad (1)$$

The vector T , the vector of measurement errors ε_T and the vector of temperature sensor faults d_T are also connected with the vector of true values T^* by a similar model:

$$T = T^* + \varepsilon_T + d_T \quad (2)$$

All these vectors are of dimension $v \times 1$. Moreover, let us denote V_Q (respectively V_T) the variance-covariance matrix of ε_Q (respectively ε_T).

3. Representation of heat exchanger system

A heat exchanger unit can be modeled by a network represented in figure 1; four streams are considered with measurements of flow and temperature.

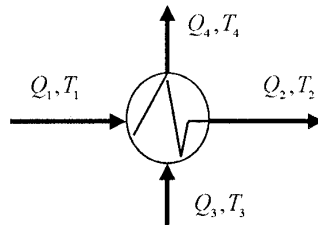


Figure 1 : heat exchanger unit

The incidence matrix M_Q of dimension $m \times v$ relative to linear equations can be written as:

$$M_Q = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

In this matrix, a "1" indicates an input variable and "-1" indicates an output variable. So the linear part of the mathematical model is written under the exact structural form:

$$M_Q Q^* = 0 \quad (3)$$

The true values of flows and temperatures are linked by the following bilinear constraints:

$$M_T D_{Q^*} T^* = 0 \quad (4)$$

where M_T is an incidence matrix of dimension $n \times v$ relative to bilinear equations and D_{Q^*} is a diagonal matrix whose diagonal is formed of the components of the vector of the true values of flows. This type of notation will be used throughout this paper.

In the case of a simple heat exchanger unit, M_T is given by:

$$M_T = \begin{pmatrix} 1 & -1 & 1 & -1 \end{pmatrix}$$

4. Residual generation

The vector of flow measurements Q does not verify the constraint equation (3). Let us define the vector of linear residuals R of dimension m :

$$R = M_Q Q \quad (5)$$

In the absence of flow faults and under the previous hypothesis relative to the measurement errors, the vector R is normally distributed with zero mean and a variance matrix:

$$H_R = M_Q V_Q M_Q^T \quad (6)$$

In the presence of flow faults, the vector R is normally distributed with a mean equal to $M_Q d_Q$ and a variance matrix H_R which is identical to the previous case.

The vector of flow measurements and the vector of temperature measurements does not verify the bilinear equation (4). In the same manner that in the linear case, a vector of bilinear residuals may be defined:

$$W = M_T D_Q T \quad (7)$$

where D_Q is a diagonal matrix whose diagonal is formed of components of vector Q .

In the absence of faults, the vector W is normally distributed with zero mean and a variance matrix:

$$H_W = M_T D_{Q^*} V_T D_{Q^*} M_T^T \quad (8)$$

while in the presence of faults, its mean and variance matrices become:

$$\text{Esp}(W) = M_T \left(D_T \cdot d_Q + D_{(Q^* + d_Q)} d_T \right) \quad (9)$$

$$\text{Var}(W) = M_T A M_T^T$$

with $A = D_Q \cdot V_T D_Q + D_T \cdot V_Q D_T + V_Q V_T$

Any diagonal matrix D_\bullet is constituted by the entries of the vector \bullet .

As we have observed, the expected value of the vector W depends simultaneously on the magnitude of flow and temperature measurement faults and true magnitudes of these two variables. In the presence of flow and temperature measurement faults, it will be practically impossible to isolate the flow or the temperature that is faulty directly. In addition the matrix of variance-covariance of residuals depends on unknown true magnitudes. To detect flow and temperature measurement faults, it is therefore necessary to estimate this matrix of variance-covariance.

The aim of the following is to generate a new residual of bilinear constraint which is insensitive to flow measurement faults. The flow measurements are then replaced by their estimation and a new residual vector \hat{W} may be written as:

$$\hat{W} = M_T D_{\hat{Q}} T \quad (10)$$

With this transformation, the detection-isolation of temperature measurement faults may be done as for linear systems as the matrices M_T and $D_{\hat{Q}}$ are perfectly known.

5. Detection by GLR method

Thereafter, we consider first the case where only a single temperature sensor is faulty and next, we will extend the method to the case of multiple faults.

In the case of a single temperature fault, equation (2) becomes:

$$T = T^* + \varepsilon_T + b_T e_j \quad (11)$$

where e_j is a vector of dimension v containing 1 in position j and zero elsewhere and b_T is the unknown magnitude of temperature fault.

The problem of detection, isolation and correction of this fault consists in determining the faulty measurement j and the magnitude of this fault.

The mathematical expectation of \hat{W} is:

$$\text{Esp}(\hat{W}) = b_T f_j \quad (12)$$

with

$$f_j = M_T D_{\hat{Q}} e_j$$

Its variance matrix is:

$$\hat{H}_W = M_T D_{\hat{Q}} V_T D_{\hat{Q}} M_T^T \quad (13)$$

If we consider that flow estimations are closed to their true values, the matrix of variance-covariance (13) may be considered as a good estimation of (8).

So as to detect sensor faults, a test of hypotheses may be implemented. The two following hypotheses focus on the value of the expected value of bilinear constraint residuals.

H_0 : $\text{Esp}(\hat{W}) = 0$ no temperature fault is present.

H_1 : $\text{Esp}(\hat{W}) = b_T f_j$ a temperature fault is present.

In order to test these hypotheses, using the normal probability density of \hat{W} , the likelihood ratio of these two hypotheses is calculated:

$$\eta = \frac{\exp\left(-\frac{1}{2}(\hat{W} - b_T f_j)^T \hat{H}_W^{-1} (\hat{W} - b_T f_j)\right)}{\exp\left(-\frac{1}{2}\hat{W}^T \hat{H}_W^{-1} \hat{W}\right)} \quad (14)$$

As η is positive, maximizing this likelihood ratio is equivalent to maximize, with regard to b_T , the logarithm of η , for a given fault j , the following quantity [1]:

$$P_j = b_T (2 f_j^T \hat{H}_W^{-1} \hat{W} - b_T f_j^T \hat{H}_W^{-1} f_j) \quad (15)$$

The estimate \hat{b}_T of b_T is obtained by maximizing P_j :

$$\hat{b}_T = (f_j^T \hat{H}_W^{-1} f_j)^{-1} (f_j^T \hat{H}_W^{-1} \hat{W}) \quad (16)$$

By replacing \hat{b}_T in the expression (15), we obtain:

$$P_j = \frac{(f_j^T \hat{H}_W^{-1} \hat{W})^2}{f_j^T \hat{H}_W^{-1} f_j} \quad (17)$$

The variable P_j , sum of squares of n normally distributed variables follows a Chi 2 law with n degrees of freedom. To detect and isolate the presence of a fault, we use the next statistic:

$$P = \sup_j P_j > \chi_{n,1-\alpha}^2 \quad (18)$$

The value of P is compared to a threshold indexed by a confidence level α . If P is less than this threshold, measurement magnitudes are accepted and no temperature fault are detected. On the other hand, if P is greater than this threshold, the variables belonging to an equation which normalized residual is greater than a threshold (fixed by the choice of the probability of error of type I) are declared faulty. These normalized residuals are computed as follows:

$$\hat{W}_N = (D_{\hat{H}_W})^{1/2} \hat{W} \quad (19)$$

where $D_{\hat{H}_w}$ represents the diagonal matrix containing diagonal terms of the matrix of variance-covariance \hat{H}_w .

The procedure of detection and isolation of flow and temperature measurement faults can be summarized by the following steps:

- **step 1** - detection, isolation and estimation of flow measurement faults

Once we have isolated all flow measurement faults and estimated their magnitude by a well adapted technique for linear systems (method of parity space, method of measurement test, GLR method or others) which produce the same results, we proceed to the estimation of flow measurements. The estimation is given by the classical expression:

$$\hat{Q} = (I_v - V_Q M_Q^T (M_Q V_Q M_Q^T)^{-1} M_Q) (Q - \hat{d}_Q) \quad (20)$$

where I_v is the identity matrix of dimension $v \times v$.

- **step 2** - detection of suspicious temperature measurements

The normalized residual W_N given by equation (19) is calculated. If no component of this vector exceeds the fixed threshold, the procedure is stopped. If the normalized residual of the i th equation exceeds the threshold, the variables intervening in this equation are suspected to be faulty.

- **step 3** - application of the GLR test to isolate suspicious temperature measurements

All the variables P_j (equation (15)) associated to each suspected variables is therefore calculated. The variable P given by the expression (18) is evaluated and compared to a threshold. If P is greater than this value, the corresponding temperature measurement is declared faulty.

- **step 4** - estimation of the magnitude of temperature fault

The magnitude of the temperature measurement fault is computed by using equation (16); the corresponding measurement is compensated and the residuals of bilinear constraint are evaluated again, this procedure is repeated from step 2 to detect eventual new temperature measurement faults.

- **step 5** - data reconciliation of temperature measurements

After having determined temperatures that are faulty and estimated magnitude of their faults, we proceed to the estimation of temperature measurements.

The data reconciliation problem consists in determining the estimation \hat{T} of the temperatures which minimizes the criterion:

$$\Phi = \frac{1}{2} \|\hat{T} + \hat{d}_T - T\|_{V_T^{-1}}^2 \quad (21)$$

$$\text{subject to: } M_T D_{\hat{Q}} \hat{T} = 0 \quad (22)$$

This problem can be solved by classical search of the stationary point of the associated Lagrangian. The estimation of temperature measurements is given by:

$$\hat{T} = (I_v - V_T D_{\hat{Q}} M_T^T \hat{H}_w^{-1} M_T D_{\hat{Q}}) (T - \hat{d}_T) \quad (23)$$

6. Application

A cold flow Q_c at a temperature T_c is heated by a hot flow Q_h at a temperature T_h through heat exchangers.

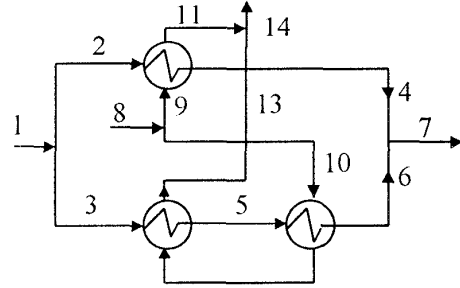


Figure 2: heat exchanger network

The incidence matrix of flow measurements is equal to:

$$M = \begin{pmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -1 \end{pmatrix}$$

and the incidence matrix M_T is given by the following matrix:

$$M_T = \begin{pmatrix} 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -1 & 0 \end{pmatrix}$$

One thousand observations whose measurement errors are normally distributed with zero mean and a known variance have been simulated. Afterwards, we have biased simultaneously the magnitude of the flow Q_1 of +20 %, that of Q_{11} of +30% and that of the temperature T_{11} of +25 % of their initial values.

The flow variables Q_1 , Q_4 , Q_6 , Q_{11} and Q_{13} as well as the temperature variables T_1 , T_4 , T_5 , T_6 , T_8 , T_{11} , and T_{13} are measured. Notice that all the unmeasured variables can be deduced as the system is observable. The table 1 shows simulated measurements of flow and temperature:

stream number	flow measurement t/day	temperature measurement °C
1	117.6	81.8
2	40.3	80.2
3	61.8	81.1
4	40.6	162.8
5	59.4	131.1
6	61.1	158.9
7	99.5	153.4
8	128.1	226.2
9	79.3	217.8
10	49.4	230.7
11	103.4	232.5
12	50.6	192.4
13	48.9	137.2
14	130.9	171.2

Table 1: values of flow and temperature measurements

The different steps of detection, isolation and estimation are the following. In the first stage, the proposed strategy has been applied to detect and isolate the two faulty flow sensors. For a given threshold, the suspicious residuals due to the presence, in the flow data, of gross measurement errors can be detected. We can also estimate the magnitude of faults of these measurements and deduce the estimation of flow measurements 1 and 11. The vectors of estimated magnitudes of faults and corrected flow measurements are:

$$\hat{d}_Q = (24.1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 16.7 \ 0 \ 0 \ 0)$$

$$\hat{Q} = (100.9 \ 40.3 \ 60.6 \ 40.3 \ 60.6 \ 60.6 \ 100.9 \ 129.2 \ 79.5 \ 49.6 \ 79.5 \ 49.6 \ 49.6 \ 129.2)^T.$$

The second stage consists in detecting and isolating the faulty temperature sensors. Applying the proposed strategy, we obtain:

$$\hat{d}_T = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 48.8 \ 0 \ 0 \ 0)$$

$$\hat{T} = (81.4 \ 80.8 \ 81.8 \ 159.2 \ 129.3 \ 156.9 \ 157.8 \ 225.6 \ 229.1 \ 183.7 \ 195.6 \ 137.5 \ 165.9)^T.$$

7. Conclusions

In this communication, we have extended the method of fault detection and localization based on the GLR test in the case of bilinear systems. The method has been applied to detect and isolate faults as well as to estimate flow and temperature measurements of heat exchanger systems.

In the first stage, the variables intervening in linear equations only are analyzed. The measurement faults of these variables are detected, isolated and their magnitudes are estimated. Thereafter, a classical method of estimation using projection matrix allows the corrected measurements to be obtained. After this step, a set of corrected flow measurements fitting the linear equations perfectly is obtained.

In the second stage, the GLR test is applied to bilinear constraint residuals computed using the previous corrected set of flow measurements. The test allows the gross measurement errors of temperature to be detected from the bilinear equations.

The results obtained from the heat exchanger example shows that the procedure simultaneously detects flow gross errors and temperature gross errors issued from the same stream. Analyzing the obtained results, we can say that this strategy is much more reliable than the pseudo node method [4] which was applied essentially in mineral processes. This method is based on the generation and analysis of all the residuals that can be obtained by elimination of common variables between two equations. Applied on the proposed example, the method of pseudo node was not able to detect and isolate correctly the measurement faults. In particular, the measurement faults on stream 11 (flow and temperature faults) were not correctly isolated. However, the pseudo node method is numerically more efficient because it needs less computation time.

8. References

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