FAULT DETECTION WITH MODEL PARAMETER STRUCTURED UNCERTAINTIES

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ABSTRACT

This paper focuses on an original fault detection method, able to take into account parameter uncertainties described by bounded variables. Intervals offer an attractive alternative for uncertain modelling and allow a straightforward generation of adaptive thresholds. This method is based on the parity space approach. The original point treated herein concerns the choice of the parity matrix, according to uncertain matrices of state representations. After expressing external forms of parity relations, interval arithmetic is used to evaluate internal forms and to deduce residual enclosures delimiting normal operation field. Then, we stress on the problems caused by dependencies between interval variables.

1. INTRODUCTION

Fault detection schemes often use the concept of analytical redundancy. They are based on consistency tests where sensory observations of a physical system are confronted with the information embodied in its model. Then residuals are generated and a normal operation procedure is usually made. An inconsistency is revealed when at least one residual becomes non-zero or one estimated parameter abnormally deviates. Reviews of model-based fault detection and isolation (F.D.I.) have been published for twenty years: [13], [3], [4] for state estimation; [5] for parameter identification.

A problem met in the field of F.D.I. schemes lies in the fact that a model only defines an approximate behaviour of a physical system. This is caused by modelling errors when a model is made linear or when some physical phenomena are not taken into account. However, a modelling error has not to be identified with a fault. Some methods generate robust residuals using uncertainties de-coupling techniques. These are considered as disturbances whose distribution matrices are well-known. Some investigators have applied the unknown input observer (U.I.O.) or have built observers with the assignment of eigenstructure scheme [11]. The major drawback is that diagnosis is only made on the certain part of the model, without taking into account the information embodied in the eliminated uncertainties. In some other methods, uncertainties receive a stochastic description and are described as an additive random noise. If statistical hypotheses about parameters are made (multiplicative uncertainties), the problem becomes insoluble because of operations on probability density functions.

This paper focuses on a fault detection method taking into account the structure and the amplitude of uncertainties. Each uncertain parameter is described by a bounded variable (or interval variable). This representation is used in parameter identification [8] but is not widely-known in diagnosis [1]. A major drawback of interval arithmetic being its explosive nature in the case of recursive systems (because of dependencies between interval variables), a parity space approach has been chosen. In this way, the dynamic equations of a model can be formulated in the form of algebraic relations. Sensory observations are stacked on a finite time horizon and a static representation is obtained. A procedure allows to optimise the parity matrix W to obtain a total or partial de-coupling of residuals from state variables.

Then, the enclosures of residuals are built with the help of interval arithmetic. These take into account the ranges of uncertain parameters and define the bounds of the normal operation field. This approach is comparable with stochastic methods, where thresholds depend on the no-detection and false alarm rates as well as the statistic characteristics of the measured signals. Nevertheless, interval variables are interesting because they allow to straightforwardly describe parameter uncertainties in a model and then thresholds are built more naturally.

Thus, this paper has the following structure. The general expressions of the static forms are defined in the next section. In section 3, a procedure allowing to choose the parity matrix W is presented. Internal forms using this result are given with respect to the uncertain matrix of the state representation in section 4. At last, interval arithmetic is presented to evaluate previous internal forms and build the enclosures of residuals.

2. STATIC FORMS

2.a. First static form

Structured uncertain models allow to take into account the lack of knowledge on a physical system by indicating which parameters are uncertain. We only consider dynamic systems given by linear discrete state equations. The structured uncertainties are described by additive terms, which modify the matrices A, B, C of the next model:

$$\begin{cases} X(k+1) = A(\theta_A)X(k) + B(\theta_B)U(k) & X(k) \in \mathbb{R}^n \\ Y(k) = C(\theta_C)X(k) & Y(k) \in \mathbb{R}^m, U(k) \in \mathbb{R}^q \end{cases}, \quad (1)$$

where: $G(\theta_G) = G_0 + \sum \theta_{G,i} G_i$, $G \in \{A,B,C\}$.

U(k), X(k) and Y(k) respectively define the state, actuator input and sensor output vectors at time k. The certain part of G is described by G_0 and the time invariant scalar $\theta_{G,i}$ is the i^{th} uncertain parameter associated with the matrix G_i. Its value is unknown but its bounds are well-known, so, interval formalism is used to represent this notion [9], [10]. A real interval is a closed, connected and bounded subset of $\mathbb{R}$. If x is a real and scalar variable, then the associated interval is defined by a letter in italics *x*:

$$x = \{x \in \mathbb{R} \,/\, \underline{x} \le x \le \overline{x}\} = [\underline{x}, \overline{x}],$$

where $\underline{x}$ and $\overline{x}$ respectively denote the lower and upper bounds of *x*. To boil down to the case all interval variables have the same bounds -α, α and the same midpoint 0, the next modification is made:

$$x = \mathrm{mid}(x) + [-\alpha, \alpha]\frac{\mathrm{width}(x)}{2\alpha},$$

where mid and width are the midpoint and the width of *x*:

$$\mathrm{mid}(x) = \frac{\underline{x} + \overline{x}}{2}, \quad \mathrm{width}(x) = \overline{x} - \underline{x}.$$

That is the reason why the case all intervals have the same bounds -α and α, remains general.

From (1), we can easily express the equality which links up the state vector X(k+j) (j∈ $\mathbb{N}^*$) to X(k) and inputs:

$$X(k+j) = A^j(\theta_A)X(k) + \sum_{i=0}^{j-1} A^{j-i-1}(\theta_A)B(\theta_B)U(k+i). \quad (2)$$

After multiplying (2) by $C(\theta_C)$ and piling measures on an observation window [k,k+s], the first static form is deduced:

$$\mathbb{Y}(k,s) = \mathbb{C}_s(\theta_A,\theta_C)X(k) + \mathbb{H}_s(\theta_A,\theta_B,\theta_C)\mathbb{U}(k,s-1), \quad (3)$$

$$\mathbb{Y}(k,s) = \left[Y^T(k) \cdots Y^T(k+s)\right]^T, \mathbb{Y}(k,s) \in \mathbb{R}^{s_t}, s_t = (s+1)m,$$

$$\mathbb{U}(k,s-1) = \left[U^T(k) \quad \cdots \quad U^T(k+s-1)\right]^T, \mathbb{U}(k,s-1) \in \mathbb{R}^{sq},$$

$$\mathbb{C}_s(\theta_A,\theta_C) = \left[(C(\theta_C))^T \; (C(\theta_C)A(\theta_A))^T \cdots (C(\theta_C)A^s(\theta_A))^T\right]^T,$$

$$\mathbb{H}_s(\theta_A,\theta_B,\theta_C) = \begin{bmatrix} 0 & \cdots & 0 \\ C(\theta_C)B(\theta_B) & & 0 \\ \vdots & \ddots & \vdots \\ C(\theta_C)A^{s-1}(\theta_A)B(\theta_B) & \cdots & C(\theta_C)B(\theta_B) \end{bmatrix},$$

$\mathbb{C}_s \in \mathbb{R}^{s_t \times n}$, $\mathbb{H}_s \in \mathbb{R}^{s_t \times sq}$.

Generalised parity space methods are based on this static form (3) obtained for a certain model [2], [6]. Then, external and internal forms of parity relations are defined and allow to respectively calculate residuals and their enclosures. Since the first only contains measured variables, uncertain parameters and terms which depend on the state vector, are put together in the second form.

To obtain a parity relation p(k+s), (3) is multiplied by a suitable row vector Ω^T (whose choice will be later specified):

$$p(k+s) = \Omega^T\left(\mathbb{Y}(k,s) - \mathbb{H}_{s,0}\mathbb{U}(k,s-1)\right), \; \mathbb{H}_{s,0} = \mathbb{H}_s(0,0,0)$$
$$= \underbrace{\Omega^T\mathbb{C}_s(\theta_A,\theta_C)X(k)}_{\text{residual terms}} + \underbrace{\Omega^T\mathbb{H}_{s,\theta}(\theta_A,\theta_B,\theta_C)\mathbb{U}(k,s-1)}_{\text{uncertain terms}}. \quad (4)$$

Remarks. Since the external form is independent of uncertainties, it is valid whatever the uncertain matrices of the model. $\mathbb{H}_{s,0}$ and $\mathbb{H}_{s,\theta}$ respectively denote the certain and uncertain part of $\mathbb{H}_s(\theta_A,\theta_B,\theta_C)$. Under ideal circumstances (no noise and no fault), residuals are non-zero since they depend on uncertain parameters. Residual terms describe the coupling between residuals and X(k), which has to be minimised by the choice of Ω^T.

2.b. Second static form

Instead of expressing X(k+j) according to the initial state vector X(k) as in (2), it is deduced from X(k+j-1):

$$\begin{bmatrix} A(\theta_A) & -I \end{bmatrix}\begin{bmatrix} X(k+j-1) \\ X(k+j) \end{bmatrix} = -B(\theta_B)U(k+j-1).$$

From this relation, state and input vectors are stacked on a time horizon [k,k+s] which leads to the next equation:

$$M(\theta_A)\mathbb{X}(k,s) = N(\theta_B)\mathbb{U}(k,s-1), \text{ with:} \quad (5)$$

$$\mathbb{X}(k,s) = \left[X^T(k) \cdots X^T(k+s)\right]^T, \mathbb{X}(k,s) \in \mathbb{R}^{s_n}, s_n = (s+1)n,$$

$$M(\theta_A) = \begin{bmatrix} A(\theta_A) & -I_n & 0 & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & 0 & A(\theta_A) & -I_n \end{bmatrix}, \; M \in \mathbb{R}^{sn \times s_n},$$

$N(\theta_B) = -\mathrm{diag}_s(B(\theta_B))$, $N \in \mathbb{R}^{sn \times sq}$, where $\mathrm{diag}_s()$ is a bloc diagonal matrix and I_n the n-order identity matrix.

Using the relationship between measures and state variables:

$$\mathbb{Y}(k,s) = L(\theta_C)\mathbb{X}(k,s), L(\theta_C) = \mathrm{diag}_{s+1}(C(\theta_C)), L \in \mathbb{R}^{s_t \times s_n},$$

and the relation (5), a new static form is finally deduced:

$$\tilde{\mathbb{C}}_s(\theta_A,\theta_C)\mathbb{X}(k,s) = \tilde{\mathbb{H}}_s(\theta_B)\begin{bmatrix} \mathbb{U}(k,s-1) \\ \mathbb{Y}(k,s) \end{bmatrix}, \quad (6)$$

with: $\tilde{\mathbb{C}}_s(\theta_A,\theta_C) = \begin{bmatrix} L(\theta_C) \\ M(\theta_A) \end{bmatrix}$, $\tilde{\mathbb{H}}_s(\theta_B) = \begin{bmatrix} 0 & I_{st} \\ N(\theta_B) & 0 \end{bmatrix}$,

$\tilde{\mathbb{C}}_s \in \mathbb{R}^{\{sn+s_t\} \times s_n}$, $\tilde{\mathbb{H}}_s \in \mathbb{R}^{\{sn+s_t\} \times \{sq+s_t\}}$.

The expression (6) is general and valid whatever the uncertain matrices of the model. As the first static form, a parity relation $\tilde{p}(k+s)$ is obtained by multiplying (6) by a suited row vector $\tilde{\Omega}^T$:

$$\tilde{p}(k+s) = \tilde{\Omega}^T \tilde{\mathbb{H}}_{s,0} \begin{bmatrix} \mathbb{U}(k,s-1) \\ \mathbb{Y}(k,s) \end{bmatrix} \quad \text{where } \tilde{\mathbb{H}}_{s,0} = \tilde{\mathbb{H}}_s(0)$$

$$= \underbrace{\tilde{\Omega}^T \tilde{\mathbb{C}}_s(\theta_A, \theta_C)\mathbb{X}(k,s)}_{\text{residual terms}} - \underbrace{\tilde{\Omega}^T \tilde{\mathbb{H}}_{s,\theta}(\theta_B) \begin{bmatrix} \mathbb{U}(k,s-1) \\ \mathbb{Y}(k,s) \end{bmatrix}}_{\text{uncertain terms}} \quad (7)$$

Both equalities (7) respectively define external and internal forms of the second static form. $\tilde{\Omega}^T$ is chosen in order to minimise the influence of residual terms.

3. CHOICE OF THE PARITY MATRIX

3.a. Optimisation procedure

The aim of the method treated herein is the search of a row vector Ω^T orthogonal to $\mathbb{C}_s$. Then, there are two possibilities: the matrix $\mathbb{C}_s$ may be well-known or depend on parameter uncertainties. If the actuator matrix is uncertain, the classic generalised parity space method is used to determinate Ω^T [2], [6], [7]. Under present conditions, a total de-coupling with respect to state variables is made. Nevertheless, if the state or observation matrices are uncertain, the quantity $\Omega^T\mathbb{C}_s(\theta)$ has to be minimised with respect to the vector θ containing uncertain parameters θ_i, $i \in \{1,...,r\}$. Therefore, the row vector Ω^T is determined by minimising the criterion:

$$J = \int_{-\alpha}^{\alpha} \cdots \int_{-\alpha}^{\alpha} \left\| \Omega^T \mathbb{C}_s(\theta) \right\|^2 d\theta_1 \cdots d\theta_r$$

$$= \Omega^T \underbrace{\left(\int_{-\alpha}^{\alpha} \cdots \int_{-\alpha}^{\alpha} \mathbb{C}_s(\theta)\mathbb{C}_s(\theta)^T d\theta_1 \cdots d\theta_r \right)}_{S} \Omega, \quad (8)$$

under the constraint $\Omega^T\Omega = 1$ (in order to eliminate the trivial solution). The orthogonality between Ω^T and $\mathbb{C}_s(\theta)$ is measured by this criterion J, which describes the influence of the state vector X(k) on residuals. It is minimised by taking Ω equal to the eigenvector associated with the smallest eigenvalue of S. The value of the criterion J is well-known for a chosen eigenvalue λ and its eigenvector ω: $J = \lambda\omega^T\omega = \lambda$.

But, the analytical expression of S must be known to use the previous criterion. To do it, two cases must be studied according to the uncertain matrix.

3.b. Uncertain observation matrix $C(\theta_c)$

Let us assume the matrix $C(\theta_c)$ depends on r uncertain parameters θ_i, then, it is written as:

$$C(\theta_c) = C_0 + \sum_{i=1}^{r} \theta_i C_i , \quad (9)$$

where C_0 is the nominal (certain) part of $C(\theta_c)$.

The matrix $\mathbb{C}_s(\theta_c)$ defined in (3) is straightforwardly expressed from the equality (9):

$$\mathbb{C}_s(\theta_c) = \mathbb{C}_{s,0} + \sum_{i=1}^{r} \theta_i \mathbb{C}_{s,i} , \quad \mathbb{C}_{s,i} = \left[C_i^T \cdots \left(C_i A^s \right)^T \right]^T . \quad (10)$$

Using the properties: $\int_{-\alpha}^{\alpha} \theta_i^2 d\theta_i = \frac{2\alpha^3}{3}$ and $\int_{-\alpha}^{\alpha} \theta_i d\theta_i = 0$, the expression of the matrix S defined in (8) can be written as:

$$S = (2\alpha)^r \left(\mathbb{C}_{s,0}\mathbb{C}_{s,0}^T + \frac{\alpha^2}{3} \left(\mathbb{C}_{s,1}\mathbb{C}_{s,1}^T + \cdots + \mathbb{C}_{s,r}\mathbb{C}_{s,r}^T \right) \right).$$

Since the analytical expression of S is known when the observation matrix $C(\theta_c)$ is uncertain, Ω^T can be determined by the minimisation of the criterion J.

3.c. Uncertain state matrix $A(\theta_A)$

Some difficulties exist if the previous method is used when the state matrix $A(\theta_A)$ is uncertain:

$$A(\theta_A) = A_0 + \sum_{i=1}^{r} \theta_i A_i \quad (11)$$

where A_0 is the nominal part of the matrix $A(\theta_A)$.

Since the structure of the matrix $\mathbb{C}_s(\theta_A)$ (3) is not additive as (10) and is highly non linear in respect to θ_A, it is very difficult to determine the analytical expression of S. That is why the second static form (6) is used to solve this problem. The structure of the matrix $\tilde{\mathbb{C}}_s(\theta_A)$ is deduced from (11):

$$M(\theta_A) = M_0 + \sum_{i=1}^{r} \theta_i M_i ,$$

$$M_0 = \begin{bmatrix} A_0 & -I_n & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ 0 & 0 & A_0 & -I_n \end{bmatrix} \text{ and } M_i = \begin{bmatrix} A_i & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ 0 & 0 & A_i & 0 \end{bmatrix},$$

which leads to: $\tilde{\mathbb{C}}_s(\theta_A) = \tilde{\mathbb{C}}_{s,0} + \sum_{i=1}^{r} \theta_i \tilde{\mathbb{C}}_{s,i}$, (12)

with: $\tilde{\mathbb{C}}_{s,0} = \left[L^T \quad M_0^T \right]^T$, $\tilde{\mathbb{C}}_{s,i} = \left[0 \quad M_i^T \right]^T$.

So, the most orthogonal row vector $\tilde{\Omega}^T$ to the uncertain matrix $\tilde{\mathbb{C}}_s(\theta_A)$ is obtained when the quantity $\tilde{\Omega}^T \tilde{\mathbb{C}}_s(\theta_A)$ is minimised with respect to the range of θ_A. $\tilde{\Omega}^T$ is obtained by minimising the following criterion:

$$\tilde{J} = \int_{-\alpha}^{\alpha} \cdots \int_{-\alpha}^{\alpha} \left\| \tilde{\Omega}^T \tilde{\mathbb{C}}_s(\theta_A) \right\|^2 d\theta_1 \cdots d\theta_r$$

$$= \tilde{\Omega}^T \underbrace{\left(\int_{-\alpha}^{\alpha} \cdots \int_{-\alpha}^{\alpha} \tilde{\mathbb{C}}_s(\theta_A)\tilde{\mathbb{C}}_s(\theta_A)^T d\theta_1 \cdots d\theta_r \right)}_{\tilde{S}} \tilde{\Omega},$$

under the constraint $\tilde{\Omega}^T \tilde{\Omega} = 1$.

Since the analytical expressions of $\tilde{\mathbb{C}}_s(\theta_A)$ and $\tilde{S}$ have the same structures as $\mathbb{C}_s(\theta_c)$ and S, $\tilde{S}$ is easily deduced:

$$\tilde{S} = (2\alpha)^r \left(\tilde{\mathbb{C}}_{s,0}\tilde{\mathbb{C}}_{s,0}^T + \frac{\alpha^2}{3}\left(\tilde{\mathbb{C}}_{s,1}\tilde{\mathbb{C}}_{s,1}^T + \cdots + \tilde{\mathbb{C}}_{s,r}\tilde{\mathbb{C}}_{s,r}^T\right)\right).$$

$\tilde{\Omega}$ is logically given by the eigenvector associated with the smallest eigenvalue of the matrix $\tilde{S}$.

4. INTERNAL FORMS OF PARITY RELATIONS

In this section, internal forms (4) or (7) and the procedure allowing to calculate the parity matrix are expressed with respect to the uncertain matrix of the state representation.

4.a. Uncertain actuator matrix $B(\theta_B)$

If the actuator matrix is uncertain:

$$B(\theta_B) = B_0 + \sum_{i=1}^{r} \theta_i B_i ,$$

where B_0 is the nominal part of $B(\theta_B)$, then the matrix $\mathbb{H}_s(\theta_B)$ is defined by the additive expression:

$$\mathbb{H}_s(\theta_B) = \mathbb{H}_{s,0} + \sum_{i=1}^{r} \theta_i \mathbb{H}_{s,i} ,$$

$$\text{with: } \mathbb{H}_{s,i} = \begin{bmatrix} 0 & \cdots & 0 \\ CB_i & \cdots & 0 \\ \vdots & \ddots & \vdots \\ CA^{s-1}B_i & \cdots & CB_i \end{bmatrix}, \ i \in \{0,1,...,r\}.$$

The parity relation (4) becomes:

$$\begin{aligned} p(k+s) &= \Omega^T\left(\mathbb{Y}(k,s) - \mathbb{H}_{s,0}\mathbb{U}(k,s-1)\right) \\ &= \Omega^T\left(\mathbb{C}_s X(k) + \sum_{i=1}^{r}\theta_i \mathbb{H}_{s,i}\mathbb{U}(k,s-1)\right). \end{aligned} \qquad (13)$$

Since the matrix $\mathbb{C}_s$ is certain, a total de-coupling with respect to state variables can be obtained. Generalised parity space method is applied to determine a vector Ω^T orthogonal to the matrix $\mathbb{C}_s$: $\Omega^T\mathbb{C}_s = 0$. Therefore, residual terms are equal to zero and the previous internal form (13) becomes:

$$p(k+s) = \sum_{i=1}^{r}\theta_i\left(\Omega^T\mathbb{H}_{s,i}\right)\mathbb{U}(k,s-1) .$$

4.b. Uncertain observation matrix $C(\theta_C)$

From (9), the additive structure of $\mathbb{H}_s(\theta_C)$ is deduced:

$$\mathbb{H}_s(\theta_c) = \mathbb{H}_{s,0} + \sum_{i=1}^{r} \theta_i \mathbb{H}_{s,i} ,$$

$$\text{with: } \mathbb{H}_{s,i} = \begin{bmatrix} 0 & \cdots & 0 \\ C_iB & \cdots & 0 \\ \vdots & \ddots & \vdots \\ C_iA^{s-1}B & \cdots & C_iB \end{bmatrix}, \ i \in \{0,1,...,r\}.$$

Using (4) and the expression of $\mathbb{C}_s(\theta_C)$ (10), the following parity relation is obtained:

$$\begin{aligned} p(k+s) &= \Omega^T\left(\mathbb{Y}(k,s) - \mathbb{H}_{s,0}\mathbb{U}(k,s-1)\right) \\ &= \Omega^T\left\{\left(\mathbb{C}_{s,0} + \sum_{i=1}^{r}\theta_i\mathbb{C}_{s,i}\right)X(k) + \sum_{i=1}^{r}\theta_i\mathbb{H}_{s,i}\mathbb{U}(k,s-1)\right\}. \end{aligned}$$

The calculus of Ω^T using the criterion J (8) is interesting because if the chosen eigenvalue is equal to zero, then the residual is de-coupled from unknown state variables.

The term $\Omega^T\mathbb{C}_s(\theta_C)$ is equal to zero for all values of the r uncertain parameters θ_i if the following properties hold:

$\Omega^T\mathbb{C}_{S,0} = 0$ and $\Omega^T\mathbb{C}_{S,i} = 0$, $i \in \{1,...,r\}$, $\theta_i \in [-\alpha,\alpha]$. (14)

If the criterion J is equal to zero (as the associated eigenvalue), then the following equality holds:

$$\Omega^T S\Omega = (2\alpha)^r \left\| \begin{bmatrix} \mathbb{C}_{s,0}^T & \frac{\alpha\mathbb{C}_{s,1}^T}{\sqrt{3}} & \cdots & \frac{\alpha\mathbb{C}_{s,r}^T}{\sqrt{3}} \end{bmatrix}\Omega \right\|^2 = 0 . \qquad (15)$$

If (14) is verified, so does (15), and the reverse is always true. Then, a minimal observation window size always exists for which the matrix $\mathbb{C}_S(\theta_C)$ defined in (10) is full row rank, which allows a total de-coupling. To show that, the Cayley-Hamilton theorem is applied on the state matrix A. Thus, it is proved there is at least one non-zero row vector Ω^T which satisfies the following property:

$$\Omega^T\left[C_j^T \cdots \left(C_jA^n\right)^T\right]^T = \Omega^T\mathbb{C}_{n,j} = 0, \ \forall\, j \in \{0,\cdots,r\} .$$

For an observation window size n, the product on the left hand size of $\mathbb{C}_n(\theta_C)$ by Ω^T is equal to zero and so does $\Omega^T S$. These relations hold for an integer greater than n. In conclusion, several row vectors Ω^T orthogonal to $\mathbb{C}_S(\theta_C)$ are obtained for an observation window size s greater or equal to n. The vectors Ω are defined by the eigenvectors associated with the eigenvalues of S equal to zero. The observation window size s has to be sufficient to obtain rank(C) eigenvalues equal to zero or smaller than a fixed tolerance.

4.c. Uncertain state matrix $A(\theta_A)$

Only the second form static is interesting because the determination of Ω^T is otherwise very problematic. From (7), the expression of the parity relation becomes:

$$\begin{aligned} \tilde{p}(k+s) &= \tilde{\Omega}^T\tilde{\mathbb{H}}_s\begin{bmatrix} \mathbb{U}(k,s-1) \\ \mathbb{Y}(k,s) \end{bmatrix}, \\ &= \tilde{\Omega}^T\tilde{\mathbb{C}}_s(\theta_A)\mathbb{X}(k,s) \end{aligned}$$

where the matrix $\tilde{\mathbb{C}}_s(\theta_A)$ is defined in (12). Since the matrix $\tilde{\mathbb{H}}_s$ is certain, uncertain terms are thus equal to zero. Like the previous paragraph, the eigenvectors associated with the eigenvalues of $\tilde{S}$ equal to zero allow a total de-coupling. Nevertheless, this situation is not always possible. Then, the internal form becomes:

$$\tilde{p}(k+s) = \tilde{\Omega}^T\tilde{\mathbb{C}}_{s,0}\mathbb{X}(k,s) + \sum_{i=1}^{r}\theta_i\tilde{\Omega}^T\tilde{\mathbb{C}}_{s,i}\mathbb{X}(k,s) .$$

Since it depends on the state vector expressed on an observation window [k,k+s], the evaluation of its enclosure is problematic when the observation matrix C is not full column rank. Otherwise, the state variables are estimated:

$$\hat{X}(k,s) = \mathrm{diag}_{s+1}\left\{\left(C^T C\right)^{-1} C^T\right\} Y(k,s) .$$

5. ENCLOSURES EVALUATION

5.a. Interval arithmetic

Uncertain parameters are defined by bounded variables intervening in the previous internal forms. To evaluate their bounds at any time, interval formalism is used [9], [10], [12]. This procedure determines enclosures which represent the normal operation field. A fault is detected if a residual calculated from its external form, goes out its enclosure.

The arithmetic operations (+, –, ×, /) on real variables can be reformulated in the case of independent interval variables.

Operation	Interval obtained
Addition	$x+y=\left[\underline{x}+\underline{y},\overline{x}+\overline{y}\right]$
Subtraction	$x-y=\left[\underline{x}-\overline{y},\overline{x}-\underline{y}\right]$
Product	$xy=\left[\min\left(\underline{x}\underline{y},\underline{x}\overline{y},\overline{x}\underline{y},\overline{x}\overline{y}\right),\max\left(\underline{x}\underline{y},\underline{x}\overline{y},\overline{x}\underline{y},\overline{x}\overline{y}\right)\right]$
Division if $0 \notin y$	$\frac{x}{y}=\left[\min\left(\frac{\underline{x}}{\underline{y}},\frac{\underline{x}}{\overline{y}},\frac{\overline{x}}{\underline{y}},\frac{\overline{x}}{\overline{y}}\right),\max\left(\frac{\underline{x}}{\underline{y}},\frac{\underline{x}}{\overline{y}},\frac{\overline{x}}{\underline{y}},\frac{\overline{x}}{\overline{y}}\right)\right]$

Table 1. Interval arithmetic operations

An elementary function g is a real function, continuous on every closed interval on which it is defined (absolute value, square, square root, exponential, logarithm, sine, cosine, arc tangent,...). Then, it can be reformulated in the case of interval variables: $g(x)=\{g(x)/x\in x\}$.

An interval extension *f* of a real function f is an interval function with the following property: *f*(x) = f(x), x ∈ *x*. If this extension is inclusion monotonic [9], it satisfies:
if $x \subset y$ then $f(x) \subset f(y)$.

For any given real function f, there are infinitely many extensions. One of them, called united extension, satisfies:

$$\tilde{f}(x) = \bigcup_{x\in x} f(x)$$

The global extrema of an elementary function g are often known on a range *x*, then the bounds of the interval function $g(x)$ (and the united extension of g), are defined by:

$$g(x)=\left[\inf_{x\in x} g(x), \sup_{x\in x} g(x)\right].$$

But, for more complicated functions, this would require solving two global optimisation problems, which often exceeds time consuming. Then, if f results from the composition of real operators or elementary functions, it is impossible to compute the united extension. So, natural extensions *f* are used. They are obtained by replacing, in the expression of f, its real argument x by *x*, and the elementary functions or operators by the associated united extensions. An inclusion monotonic and natural extension always includes the united extension but they are different in general.

5.b. Dependence between interval variables

In the case of dependence, where some interval variables occur repeatedly in an expression, interval arithmetic leads to an overestimation of the minimal range associated with an united extension. To show that, the following interval extension is considered: $f(x) = x - x$. From definition, the united extension is given by: $\tilde{f}(x) = \{x - x / x \in x\}$. So, for every value of the real variable x, this expression is equal to {0}. Nevertheless, when the formula giving the subtraction is used, $f(x) = \left[\underline{x}-\overline{x},\overline{x}-\underline{x}\right]$ is deduced. If *x* is not reduced to a real number, *f* leads to an overestimation which depends on the width of *x*. In fact, the dependence is not taken into account, because the following operation is made: *x - z*, where *z* is an independent interval variable with the same bounds as *x*. This problem leads to the subdistributivity property:
$x(y \pm z) \subseteq xy \pm xz$ or $(z \pm y)\, x \subseteq zx \pm yx$.

So, the natural extensions of two equivalent functions in the arithmetic of real numbers are not necessary equivalent in the interval arithmetic. Nevertheless, if each interval variable appears at most once in the expression of an natural extension, then its evaluation by interval arithmetic leads to the united extension. Otherwise, for determining it without searching the extrema of the real function by non linear computing (Newton-Raphson, simplex,...), some recursive algorithms based on Mean Value or Centered Forms as the Monoticity Test Form can be used [9], [10].

6. EXAMPLE

Let us consider the following system where the state matrix depends on an uncertain parameter $\theta_1 \in [-0.5,0.5]$:

$$A_0 = \begin{bmatrix} 0.8 & 1 \\ 0 & 0.2 \end{bmatrix}, A_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The study of $\mathbb{C}_1(\theta_1)$ (θ_1 is time invariant):

$$\mathbb{C}_1(\theta_1) = \begin{bmatrix} C \\ CA(\theta_1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0.8+0.1\theta_1 & 1 \\ 0 & 0.2 \end{bmatrix},$$

shows this matrix is always of rank 2 for every possible value of θ_1. So, the observation window size is chosen equal to 1.
Since $\tilde{\mathbb{C}}_1(\theta_1)$ is uncertain, the matrix $\tilde{S}$ is built. It is not full rank and one of its eigenvalues is equal to zero. The others are equal to: $2.30.10^{-4}$, 1, 1, 2.02 and 3.67.

In case the chosen eigenvalue is equal to zero, the enclosure of the residual is reduced to 0. In the other cases, interval arithmetic is used to evaluate the internal form, which is very simple in that example:

$\tilde{p}(k,\theta_1) = a_k + b_k\theta_1$,

where the parameters a_k and b_k depend on measures. Since there is no dependency between interval variables in a same residual, its bounds are easily deduced:

$\tilde{p}(k,\theta_1) \in [a_k - 0.5|b_k|, a_k + 0.5|b_k|]$.

The model is simulated during one second. The input is piece wise constant with a magnitude comprised between -1 and 1. Its value changes every 0.2s. The measures y_1 and y_2 obtained when θ_1 is a constant and is successively equal to -0.5 and 0.5 are represented below.

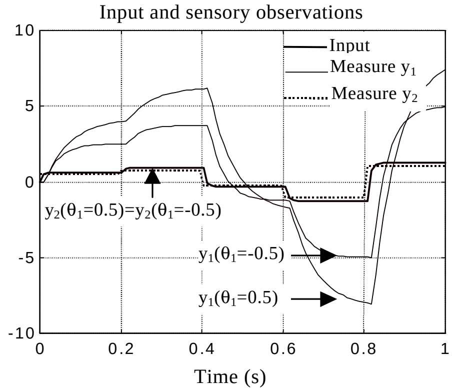

Figure 1. Input and sensory observations

The residual and its enclosure obtained for $\lambda = 2.30.10^{-4}$ are presented below. Both situations where θ_1 is equal to $-\alpha$ or α are examined. At time t = 0.5s, a bias equal to 0.625 occurs on the first sensor. It represents 10 per cent of the maximal magnitude of the measure y_1 obtained for an input equal to 1.

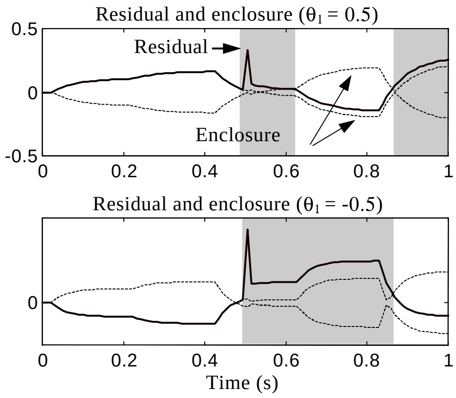

Figure 2. Bias on the first sensor

During a normal operation, the residual stays inside its enclosure (t < 5s). But, when the fault is present, the detection depends on the operating point and the value of the uncertain parameter.

7. CONCLUSION

This paper deals with a fault detection method suited to structured uncertain models. According to the uncertain matrix of the state representation, different techniques are proposed to choose the parity matrix. In the case of an uncertain observation matrix, the choice of eigenvalues equal to zero during the minimisation of the criterion J allows to solve the problem of state estimation. Otherwise, the second static form allows to treat the case of an uncertain state matrix. Once the external forms of parity relations were expressed, the internal forms are evaluated by means of interval arithmetic. Then the enclosures, which define the normal operation field, are obtained. If a residual leaves its enclosure, a fault is detected.

8. REFERENCES

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