Estimation of unknown inputs in linear systems

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Abstract

A systematic design procedure is developed for determining unknown inputs of a dynamic system. The results presented in this paper can be used directly for the design of fault detection and isolation observers for systems with unknown inputs or for systems with faulty inputs.

1. Introduction

In the last several years, considerable attention has been focused on the design of observer for linear systems with unknown inputs. Johnson (1975) and Hostetter (1973) proposed methods which assume some a priori knowledge about the unmeasurable inputs; this may impose some limitations. Kobayashi (1982) suggested an approach which follows Silverman's system inverse method. Using the simplest matrix generalised inverse, a procedure has been established by Miller (1982) for the construction of a reduced-order Luenberger observer. Necessary and sufficient conditions for the existence of an observer for linear multivariable systems with unknown inputs have been presented by Kudva (1980) and Kurek (1983). Fairman (1984) proposed a method of designing a Luenberger observer capable of determining the state of a system having some inputs which are completely unknown; this method is based on singular value decomposition technique. Yang (1988) developed a direct design procedure of a full order observer using straightforward matrix calculation. The case of singular systems with unknown inputs has been discussed by Yang (1989). Recently, Hou (1992) suggested a design of reduced-order observer with some comparison within the framework of descriptor system observers design principles.

In that recent paper, Hou presents a design of reduced-order observers for linear systems with unknown inputs. He proposes to decompose the state equation of the system into two subsystems; the first one depends on the unknown inputs and, in the second one, the unknown inputs may be dropped. Under the assumption that the state of the second subsystem may be obtained through the measurement equation, he designs a reduced-order observer for the unknown inputs. However, they are many situations in which this hypothesis does not hold and therefore we suggest an alternative method.

In this paper, the procedure proposed by Hou is briefly presented with some modification, then a straightforward treatment allowing the unknown input estimation is proposed.

2. The basic procedure

Consider the linear time-invariant system

$$x(k+1) = A\,x(k) + B\,u(k) + D\,d(k) \quad (1a)$$
$$y(k) = C\,x(k) \quad (1b)$$

where $x \in R^n$, $u \in R^p$, $y \in R^m$, $d \in R^s$ are respectively the state vector, the input vector, the measurement vector and the unknown input vector. The matrices A, B, C and D are constant with appropriate dimensions. Without loss of generality, we assume that C is a full row rank matrix and that D is a full column rank matrix. This last assumption yields the decomposition:

$$D = H \begin{pmatrix} R \\ 0 \end{pmatrix} K^T \quad (2)$$

where $H \in R^{n.n}$, $R \in R^{s.s}$, $K \in R^{s.s}$.

The system (1) is restricted system equivalent to:

$$\bar{x}(k+1) = \bar{A}\,\bar{x}(k) + \bar{B}\,u(k) + \begin{pmatrix} R \\ 0 \end{pmatrix} \bar{d}(k) \quad (3a)$$
$$y(k) = \bar{C}\,\bar{x}(k) \quad (3b)$$

where:

$$\bar{A} = H^T A H \qquad \bar{B} = H^T B \qquad \bar{C} = C H \quad (4a)$$
$$\bar{x}(k) = H^T x(k) \quad (4b)$$
$$\bar{d}(k) = K^T d(k) \quad (4c)$$

The partitioning of $\bar{x}$ into $(\bar{x}_1 \;\; \bar{x}_2)^T$ where $\bar{x}_1 \in R^s$ and $\bar{x}_2 \in R^{n-s}$, points out the decomposition of the system (3) into an unknown-input depending subsystem and an unknown-input-free subsystem:

$$\bar{x}_1(k+1) = \bar{A}_{11}\,\bar{x}_1(k) + \bar{A}_{12}\,\bar{x}_2(k) + \bar{B}_1\,u(k) + R\,\bar{d}(k) \quad (5a)$$
$$\bar{x}_2(k+1) = \bar{A}_{21}\,\bar{x}_1(k) + \bar{A}_{22}\,\bar{x}_2(k) + \bar{B}_2\,u(k) \quad (5b)$$
$$y(k) = \bar{C}_1\,\bar{x}_1(k) + \bar{C}_2\,\bar{x}_2(k) \quad (5c)$$

with $C_1 \in R^{m.s}$ and $C_2 \in R^{m.(n-s)}$.

As R is a regular matrix, the equation (5a) may be used to estimate the unknown input d(k) (or equivalently d(k)) . Under the assumption that the state $\bar{x}_1(k)$ may be obtained from y(k), equations (5b) and (5c) describe a dynamic linear system which depends on $\bar{x}_2(k)$ only. Thus, following Hou and Müller works, $\bar{C}_1$ is supposed to be a full column rank matrix and may be expanded as:

$$\bar{C}_1 = H_1 \begin{pmatrix} R_1 \\ 0 \end{pmatrix} K_1^T \quad (6)$$

with $H_1 \in R^{m.m}$, $R_1 \in R^{s.s}$ and $K_1 \in R^{s.s}$. Let us consider the partition $H_1 = (H_{11}\ H_{12})$ where $H_{11} \in R^{m.s}$, $H_{12} \in R^{m.(m-s)}$ and the transformed output $\bar{y}(k) = H_1^T\ y(k)$. Then, the measurement equation may be decomposed:

$$\bar{y}_1(k) = R_1 K_1^T \bar{x}_1(k) + H_{11}^T \bar{C}_2 \bar{x}_2(k) \quad (7a)$$

$$\bar{y}_2(k) = H_{12}^T \bar{C}_2 \bar{x}_2(k) = C_2 \bar{x}_2(k) \quad (7b)$$

It should be noticed that:

$$\bar{y}_1(k) = G_s^T\ \bar{y}(k) \quad (8)$$

with: $G_s^T = (I_s \quad 0_{s.(m-s)}) \in R^{s.m}$

As R_1 and K_1^T are regular, one can deduce from (7a):

$$\bar{x}_1(k) = K_1 R_1^{-1}\ G_s^T\ H_1^T (y(k) - \bar{C}_2 \bar{x}_2(k)) \quad (9)$$

Sustituting (9) into (5b) leads to:

$$\bar{x}_2(k+1) = A_2 \bar{x}_2(k) + B_2 u(k) + D_2 y(k) \quad (10)$$

with:

$$A_2 = \bar{A}_{22} - \bar{A}_{21} K_1 R_1^{-1}\ G_s^T\ H_1^T \bar{C}_2 \quad (11a)$$

$$B_2 = \bar{B}_2 \quad (11b)$$

$$D_2 = \bar{A}_{21} K_1 R_1^{-1}\ G_s^T\ H_1^T \quad (11c)$$

After the elimination of the unknown inputs, the system is described by the state equation (10) and the measurement equation (7b). As suggested by Hou, following the conventional Luenberger observer design procedure, one can design a reduced-order observer for the unknown-input free system (10)-(7b). However, this design is conditioned by the detectability of the pair $\{A_2, C_2\}$. A full order observer can be written as:

$$\hat{\bar{x}}_2(k+1) = (A_2 - LC_2)\ \hat{\bar{x}}_2(k) + B_2 u(k) + (D_2 + LH_{12}^T)\ y(k) \quad (12)$$

In order to obtain an estimation of the unknown inputs, one uses the definition (4c) and the equation (5a), in which the state $\bar{x}_1(k)$ has to be eliminated:

$$\hat{d}(k) = K R^{-1} ((q I - \bar{A}_{11})\ \bar{x}_1(k) - \bar{A}_{12} \bar{x}_2(k) - \bar{B}_1 u(k)) \quad (13)$$

$$\hat{d}(k) = \alpha(q)\ y(k) + \beta(q)\ \bar{x}_2(k) + \gamma(q)\ u(k) \quad (14)$$

with:

$$\alpha(q) = K R^{-1} (q I - \bar{A}_{11})\ K_1 R_1^{-1}\ G_s^T\ H_1^T \quad (15a)$$

$$\beta(q) = - K R^{-1} (\bar{A}_{12} + (q I - \bar{A}_{11})\ K_1 R_1^{-1}\ G_s^T\ H_1^T \bar{C}_2) \quad (15b)$$

$$\gamma(q) = - K R^{-1} \bar{B}_1 \quad (15c)$$

3. Examples

Let us consider the following discrete system:

$$A = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 0.5 \\ 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad D = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

Following (2), it could be verified that:

$$H = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad K = 1 \qquad R = 1$$

The change of variables (4) yields:

$$\bar{A} = \begin{pmatrix} -1 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & -1 \end{pmatrix} \quad \bar{B} = \begin{pmatrix} -1 \\ 0.5 \\ 1 \end{pmatrix} \quad \bar{C} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and the state equations (5) are expressed:

$$\bar{x}_1(k+1) = - \bar{x}_1(k) + (-1 \quad 0)\ \bar{x}_2(k) - u(k) + d(k)$$

$$\bar{x}_2(k+1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \bar{x}_1(k) + \begin{pmatrix} 0 & 0 \\ -1 & -1 \end{pmatrix} \bar{x}_2(k) + \begin{pmatrix} 0.5 \\ 1 \end{pmatrix} u(k)$$

$$y(k) = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \bar{x}_1(k) + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bar{x}_2(k)$$

The unknown input is estimated by:

$$\hat{d}(k) = - (q + 1)\ y_{.1}(k) + \hat{\bar{x}}_{2.1}(k) + u(k)$$

where $z_{.i}$ denotes the i^{th} entry of the vector z.

In order to illustrate the non-applicability of the method proposed by Hou, let us consider the following fourth-order dynamic system described by:

$$A = \begin{pmatrix} -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \quad B = \begin{pmatrix} -1 \\ 0 \\ 0.5 \\ 0 \end{pmatrix} \quad C = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 0 \\ 0 & 0.5 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

For this example, it is clear that:

$$\bar{C}_1 = \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}$$

As $\bar{C}_1$ is not a full column rank matrix the decomposition (6) does not hold and the deduction of the sub-state $\bar{x}_2(k)$ from the measurement equation is not possible. However, the estimation of the unknown input may be achieved. Indeed from the decomposition (5) one obtains:

$$\bar{x}_1(k+1) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \bar{x}_1(k) + \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \bar{x}_2(k) + \begin{pmatrix} -1 \\ 0 \end{pmatrix} u(k) + \begin{pmatrix} 1 & 0 \\ 0 & 0.5 \end{pmatrix} \bar{d}(k) \quad (16a)$$

$$\bar{x}_2(k+1) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \bar{x}_1(k) + \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bar{x}_2(k) + \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} u(k) \quad (16b)$$

$$y(k) = \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} \bar{x}_1(k) + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bar{x}_2(k) \quad (16c)$$

Then, equation (16b) allows $\bar{x}_1(k)$ to be expressed as a function of $\bar{x}_2(k)$ and u(k); therefore, from (16c), $\bar{x}_2(k)$ may be deduced.

$$\bar{x}_2(k) = - \frac{1}{q} \begin{pmatrix} 1 & -(q+1) \\ 0 & -q \end{pmatrix} y(k) + \frac{1}{q} \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} u(k) \quad (17a)$$

$$\bar{x}_1(k) = \begin{pmatrix} -1 & q+1 \\ 0 & q+1 \end{pmatrix} y(k) \qquad (17b)$$

Substituting the state variables in (16a), the expression of the unknown input is obtained as:

$$\bar{d}(k) = \begin{pmatrix} -(q+1+\frac{1}{q}) & \frac{(q+1)(q^2+q+1)}{q} \\ 0 & 2(q^2-2) \end{pmatrix} y(k) + \begin{pmatrix} 1+\frac{0.5}{q} \\ 0 \end{pmatrix} u(k) \qquad (18)$$

4. The proposed solution

In the case where the matrix $\bar{C}_1$ is not of full row rank, the decomposition (6) is not available. In order to be independent of this hypothesis, consider once again the equations (5). They form a linear system in respect with the states $\bar{x}_1(k)$, $\bar{x}_2(k)$ and the unknown inputs $\bar{d}(k)$. As R is a regular matrix and due to the fact that $\bar{d}(k)$ is only present in equation (5a), this equation may be used to estimate $\bar{d}(k)$. The remaining equations (5b) and (5c) are then used to determine the states $\bar{x}_1(k)$ and $\bar{x}_2(k)$. A global approach for this estimation may be achieved by rewriting the system (5) as:

$$N\begin{pmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \end{pmatrix} = N\begin{pmatrix} u(k) \\ y(k) \end{pmatrix} + P\bar{d}(k) \qquad (19)$$

with:

$$M = \begin{pmatrix} -\bar{A}_{21} & qI - \bar{A}_{22} \\ \bar{C}_1 & \bar{C}_2 \\ qI - \bar{A}_{11} & -\bar{A}_{12} \end{pmatrix}, \quad N = \begin{pmatrix} \bar{B}_2 & 0 \\ 0 & I \\ \bar{B}_1 & 0 \end{pmatrix}, \quad P = \begin{pmatrix} 0 \\ 0 \\ R \end{pmatrix} \qquad (20)$$

If $\Omega(q) = (\omega_1(q) \;\; \omega_2(q) \;\; \omega_3(q))$ is a matrix orthogonal to M (this is not a tedious task because M is a binomial matrix (Gantmacher, 1959)), one may deduce from (19) the relationship between the unknown inputs $\bar{d}(k)$ and the known inputs and outputs u(k) and y(k):

$$\Omega(q)\left(N\begin{pmatrix} u(k) \\ y(k) \end{pmatrix} + P\bar{d}(k) \right) = 0 \qquad (21)$$

or with an appropriate partitioning of Ω:

$$\left(\omega_1(q)\bar{B}_2 + \omega_3(q)\bar{B}_1\right) u(k) + \omega_2(q)y(k) + \omega_3(q)R\,\bar{d}(k) = 0 \qquad (22)$$

The unknown inputs are then estimated after analysing the rank of the polynomial matrix $\omega_3(q)R$. For the proposed example, one obtains:

$$M = \begin{pmatrix} -1 & 0 & q & 0 \\ 0 & -1 & 0 & q+1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ q+1 & 0 & 1 & 0 \\ 0 & q-1 & 0 & -1 \end{pmatrix} \quad N = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad P = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0.5 \end{pmatrix}$$

For example, the following matrix Ω is orthogonal to M:

$$\Omega = \begin{pmatrix} 0 & q-1 & 0 & 2-q^2 & 0 & 1 \\ 1 & -(1+q+q^2) & -(1+q+q^2) & (1+q)(1+q+q^2) & -q & 0 \end{pmatrix}$$

With:

$$N\begin{pmatrix} u(k) \\ y(k) \end{pmatrix} + P\bar{d}(k) = \begin{pmatrix} 0.5u(k) \\ 0 \\ y_{.1}(k) \\ y_{.2}(k) \\ -u(k) + \bar{d}_{.1}(k) \\ 0.5\bar{d}_{.2}(k) \end{pmatrix}$$

one may verify that the equation (21) allows one to obtain directly the estimations (18) of d(k) founded from the state equations of the system.

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