On the existence of strong solutions to a fluid structure interaction problem with Navier boundary conditions

Imene Aicha DJEBOUR Takéo TAKAHASHI

Université de lorraine
IECL
Equipe SPHINX

1. Motivation

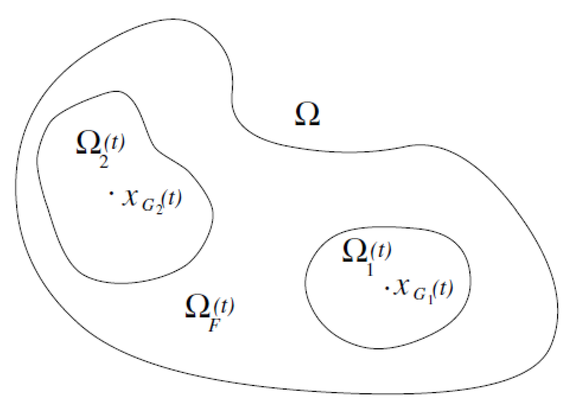
Fluid-structure interaction problems (FSI) are coupled systems that describe the interaction of some movable or deformable structure with a fluid flow. Fluid-structure interactions are a crucial consideration in the design of many engineering systems, e.g: aircraft, spacecraft, engines and bridges. Failing to consider the effects of such interaction can be catastrophic, especially in structures comprising materials. For instance, the first Tacoma Narrows Bridge is probably one of the most infamous examples of large scale-failure.

The collapse of the Tacoma Narrows Bridge on November 7, 1940

Fluid-structure interaction problems play aslo a major role in appropriate modeling of blood flow. Blood vessels act as compliant tubes that change size dynamically when there are variations of blood pressure and flow velocity.

In this work, we study a nonlinear moving boundary fluid-structure interaction problem between a viscous, incompressible Newtonian fluid, modeled by the Navier-Stokes equations, and an elastic plate. The fluid and the structure are coupled through two coupling conditions: the **Navier slip boundary condition** and the continuity of contact forces at the fluid-structure interface. The Navier slip boundary conditions state that the slip between the tangential components of the fluid and structure velocities is proportional to the tangential components of the fluid normal stress evaluated at the fluid-structure interface, while the normal components of the fluid and structure velocities are continuous. The main motivation for using the Navier slip boundary condition comes from studying FSI problems involving elastic structures with "rough" boundaries, for example when studying FSI between blood flow and cardiovascular tissue that involve cells on tissue which interact with blood flow at the boundary. Another motivation for using the Navier slip boundary condition comes from studying FSI problems near a **contact** or a **collision** of the structure at the fluid boundary.

Fluid-solid interaction schema



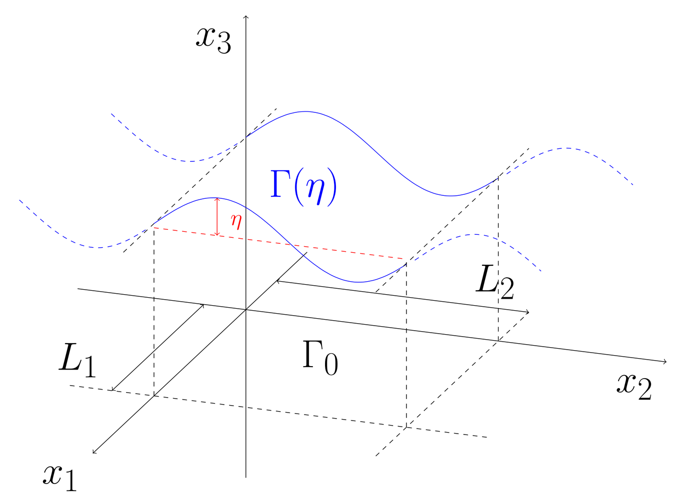
It has been recently shown that the no-slip condition or Dirichlet condition is not a realistic physical condition. In fact, if we consider no-slip boundary condition, it was shown that smooth rigid bodies can not touch each other. An explanation for the no-collision paradox is that the no-slip boundary condition does not describe near-contact dynamics well.

2. Problem setting

We suppose that an incompressible fluid fills a 3D periodic container whose upper boundary is made of an elastic structure denoted $\Gamma(\eta)$ where η stands for the transversal deformation of the structure, while we assume that the domain cavity is fixed.

The fluid domain denoted $\Omega(\eta(t))$ depends on time since it depends on the structure displacement η.

The domain's configuration at time t:



The fluid's equation

The velocity-field of the fluid denoted by U and its internal pressure P which satisfy

$$\begin{cases} \partial_t U + (U\cdot\nabla)U - \nu\Delta U + \nabla P = 0, \ t>0, \ x\in\Omega(\eta(t)), \\ \nabla\cdot U = 0, \ t>0, \ x\in\Omega(\eta(t)). \end{cases}$$

The positive constant ν represents the viscosity of the fluid.

The structure's equation

The structure motion is given by a linear damped plate equation

$$\partial_{tt}\eta + \Delta^2\eta - \Delta\partial_t\eta = \mathbb{H}_\eta(U,P), \ t>0$$

The fluid and the structure equations are coupled through the source term $\mathbb{H}_\eta(U,P)$, which corresponds to the surface force exerted by the fluid on the plate.

Navier boundary conditions

The fluid and the plate are coupled also through the following boundary conditions

$$\begin{cases} U_n = 0 \ t>0, \ x\in\Gamma_0, \\ [2\nu D(U)n + \beta_1 U]_\tau = 0 \ t>0, \ x\in\Gamma_0, \\ (U - \partial_t\eta e_3)_n = 0 \ t>0, \ x\in\Gamma(\eta(t)), \\ [2\nu D(U)n + \beta_2(U - \partial_t\eta e_3)]_\tau = 0 \ t>0, \ x\in\Gamma(\eta(t)), \end{cases}$$

where $\beta_i \geq 0$, $i=1,2$ are the friction coefficients.

3. Main result

We obtained results on existence and uniqueness of **strong solutions** to the system described above up to collision.

References

[1] David Gérard-Varet, Matthieu Hillairet, and Chao Wang. The influence of boundary conditions on the contact problem in a 3D Navier-Stokes flow. *J. Math. Pures Appl. (9)*, 103(1):1–38, 2015.

[2] Céline Grandmont and Matthieu Hillairet. Existence of global strong solutions to a beam-fluid interaction system. *Arch. Ration. Mech. Anal.*, 220(3):1283–1333, 2016.

[3] Matthieu Hillairet and Takéo Takahashi. Collisions in three-dimensional fluid structure interaction problems. *SIAM J. Math. Anal.*, 40(6):2451–2477, 2009.