Hybrid Formation Control for Heterogeneous Uncertain Linear Two-time-scale Systems ⋆

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Abstract

This paper investigates time-varying output formation of interconnected heterogeneous linear two-time-scale systems with model uncertainty. Unlike previous works on the cooperation of interconnected two-time-scale systems, we consider nonidentical dynamics for each agent that may be characterized by different dimensions and time-scaling factors. Additionally, the systems are interconnected through a switching graph with a disconnected topology, rendering more challenging the analysis and the controller design. To address these challenges, a hybrid two-layer hierarchical control protocol is proposed. The upper layer utilizes an impulsive cooperative control strategy to generate local references, enabling discrete-time interactions among agents and thereby reducing the communication burden. The lower layer implements an internal model-based controller for the two-time-scale dynamics to track the generated references, demonstrating robustness against small model uncertainties. Closed-loop analysis is based on input-to-state stability (ISS) results for hybrid systems. Furthermore, the obtained result is extended to achieve the output consensus. Finally, two examples are presented to illustrate the effectiveness of the results.

Key words: Two-time-scale, robust output regulation, input saturation, internal model.

1 Introduction

Most of the results for analysis and control design within framework of multi-agent systems (MAS) are developed under the hypothesis that the agents have identical dynamics. However, this assumption is often unrealistic as slight variations typically exist even among devices aimed to be identical [1]. For instance, two engines or batteries with identical characteristic usually exhibit different degrees of fatigue/deterioration, leading to slightly different dynamics. Recently, cooperative control of interconnected heterogeneous systems has gained significant interest due to its widespread application across many industrial fields [2–4]. Formation control is a classical and practical topic, aimed at guiding interconnected systems to achieve predefined static or time-varying formation patterns. Many valuable works have been done on the formation control of interconnected systems with identical dynamics [5–8]. However, applying these cooperative algorithms becomes challenging when the dynamics and dimension of each system are different. To address this heterogeneity, the two-layer hierarchical control framework has been extensively employed, and yielded numerous excellent results in both static and time-varying formation control [9–11]. Within the two-layer hierarchical control framework, the upper layer orchestrates the generation of necessary local references through the cooperation of the designed interconnected virtual systems, and the lower layer is responsible for tracking these local references for each systems, facilitating the desired cooperative behavior.

Note that the aforementioned results only pertain to single time scale systems, and are challenging to apply to systems with dynamics evolving on two time scales, typically modeled as singularly perturbed systems [12–17]. For instance, consider the cooperation of interconnected heterogeneous inverted pendulums or robotics via DC motor control, where the angular speed and armature

⋆ This work is supported by the National Natural Science Foundation of China under Grants 62303189, 62433018, 62403394 and U24B20158. The work of I.-C. Morărescu was supported by the ANR through the grant COMMITS ANR-23-CE25-0005 and the PNRR project DECIDE No. 760069. Corresponding author Z. Wang.

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current of controlled DC motor evolve on slow and fast time scales [14,15]. Unlike the heterogeneity in interconnected single time scale systems, two-time-scale systems (TTSSs) also encompasses differences in the time-scaling factor. Moreover, the tracking algorithms design often encounters challenges related to numerically ill-conditioning and high dimensionality. Thus, the existing control design for single time scale systems cannot be applied on the shelf. Although significant progress has been achieved on the consensus problem for identical interconnected TTSSs [18–21], only few results address the formation problem [22,23], and no effective solutions exist for the consensus or formation control of interconnected heterogeneous two-time-scale systems (HTTSSs). In [22,23], cooperative control laws are developed using singular perturbation techniques to achieve the formation of quadrotor unmanned aerial vehicles (UAVs) and autonomous underwater vehicles (AUVs). Notably, in these studies, the multi-time-scale feature arises from high gain controller rather than the original systems dynamics. Besides, these results overlook the inherent heterogeneity of the interconnected TTSSs. Although event-triggered consensus has been achieved for interconnected linear TTSSs with slightly different dynamics due to structured uncertainties [19], significant gaps and challenges remain in extending these findings to interconnected HTTSSs, especially when system dimensions and time-scaling factors differ.

Furthermore, in practical applications, communication resources are valuable yet limited for interconnected systems, making it essential to develop methods with a low communication burden. Impulsive control techniques, which built on the time-triggered mechanism allowing data transmission at discrete-time instants, are effective and popular [24]. Recently, impulsive control has found widespread application in the formation of heterogeneous interconnected single time scale systems [9,25,26], primarily due to its advantages including robustness, simplicity and lower communication burden. However, it remains challenging and interesting to explore how to integrate impulsive control into the cooperation of interconnected HTTSSs. Moreover, given the dynamic nature of the external environment, model uncertainty emerges as a significant concern [27,28]. It's also imperative to enhance the robustness of the control design to effectively handle this uncertainty.

In this context, we focus on interconnected heterogeneous linear TTSSs subject to model uncertainty, and investigate the time-varying output formation (TVOF) problem. Each system may have the nonidentical dimensions and time-scale factors, and they are interconnected through a switching direct disconnected topology. To adapt to the two-time-scale heterogeneity and reduce the communication losses, an impulsive two-layer hierarchical control framework is employed. This problem presents three main challenges. First, unlike in [22,23,9,25,26], we consider a switching directed topology where each sub-graph may be disconnected. This poses challenges for the impulsive cooperative control design in the upper layer to generate the reference trajectories that adapt to time-varying formation patterns. Second, we address the output formation problem related to the fast states when each agent dynamics evolves on two time-scales under unmatched model uncertainty. Therefore, solving regulation equation may encounter numerical issues due to the small time-scaling factor, which prevents the direct variable transformation in [9] from converting the formation problem to output regulation problem in the lower layer. Achieving tracking of the hybrid local reference under these conditions is particularly challenging. Third, the hybrid dynamics arises not only from the switching topology but also from the impulsive control signals. While these impulses aim to generate suitable trajectories for formation realization, they also introduce disturbances affecting dynamics on both slow and fast time scales in the lower layer. Consequently, ensuring the stability of the overall two-time-scale hybrid closed-loop systems requires a particular attention and tailored tools. To overcome these challenges, we introduce a virtual leader aligned with the TVOF patterns. Accordingly, we propose an internal model based robust hybrid two-layer hierarchical control protocol that incorporates two-time-scale feature. In the upper layer, an impulsive leader-following cooperative control scheme is developed to generate appropriate reference trajectories with discrete-time communication, while accommodating a switching disconnected topology. In the lower layer, an internal-model based control scheme is designed for robust output tracking on two-time-scale with model uncertainty. Under the proposed controller, TVOF is achieved for interconnected uncertain linear HTTSSs. Additionally, for cases where the system state is not measurable, an output feedback control law is designed and extended to solve the output consensus problem. This article presents three main contributions.

1) We further considered the two-time-scale feature for the TVOF of interconnected heterogeneous systems, where each linear dynamics can be characterized by different dimensions and time-scaling factors.
2) A robust impulsive two-layer hierarchical control protocol is proposed to address the challenges of heterogeneous uncertain two-time-scale dynamics under switching directed disconnected topology, enhance control design flexibility. Additionally, this approach reduces communication burdens, and ensures robustness to small model uncertainties while adapting to cases with singular fast dynamic matrices.
3) The overall system is modeled using a hybrid formalism and a tailored ISS analysis method is employed to ensure asymptotic convergence. Additionally, a scheme for estimating the available region of the time-scaling factor is provided.

The rest of this paper is organized as follows. Section

II formulates the problem. Section III presents the proposed solution. Section IV gives two illustrative examples. Section V draws the conclusions.

Notation. The function $f : [0,\infty)^2 \to \mathbb{R}^{m\times n}$ is $O(\varepsilon)$ if there exist constants $\bar{\varepsilon} > 0$ and $k > 0$ so that for all $\varepsilon \in [0, \varepsilon^*]$ and $\forall t \in [0,\infty)$, $\|f(t,\varepsilon)\| \le k\varepsilon$. $\mathcal{K}_\infty$ and $\mathcal{KL}$ functions are considered to be defined in [13, Chapter 4].

2 Preliminaries and Problem Statement

2.1 Graph theory

A graph $\mathcal{G} = (\bar{\mathcal{V}}, \mathcal{E}, \mathcal{A})$ is described by a set of nodes $\bar{\mathcal{V}} = \{0, 1, 2, \ldots, N\}$, a set of edges $\mathcal{E} \in \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ and a weighted adjacency matrix $\mathcal{A} = (a_{ij})_{i,j=0}^N$ with non-negative adjacency elements. Note that, $a_{ij} > 0$ if and only if the i-th agent can obtain the information from the j-th agent. The matrix $\bar{\mathcal{L}} = (l_{ij})_{i,j=1}^N$ is defined as $l_{ij} = -a_{ij}$ if $i \ne j$, and $l_{ij} = \sum_{k=1,k\ne i}^N a_{ik}$ otherwise. We assume that the interaction between agents takes place at specific time instants define by a strictly increasing sequence $\mathcal{T} = \{t_0, t_1, t_2, \ldots\}$ with $t_0 = 0$ and $0 < \underline{\tau} < t_{k+1} - t_k \le \overline{\tau}$ for $k \in \mathbb{N}$. For the sake of convenience, denote $\mathcal{G}_k = (\bar{\mathcal{V}}, \mathcal{E}_k, \mathcal{A}_k)$ as the graph describing the information flow when $t = t_k$, $k \in \mathbb{N}_+$.

Definition 1 *A sequence of graphs $\{\mathcal{G}_k\}_{i=1}^m$ is jointly connected if the union graph $\bigcup_{i=1}^m \mathcal{G}_i$ contains a spanning tree with node 0 as the root.*

Definition 2 *A sequence of graphs $\{\mathcal{G}_k\}_{i=1}^m$ is sequentially connected if there exist $\mathcal{V}_k \subseteq \bar{\mathcal{V}}$, $k = 1, \ldots, m+1$ with $\mathcal{V}_1 = \{0\}$ and $\mathcal{V}_{k+1} \subseteq \mathcal{V}_k \cup \mathcal{N}(\mathcal{G}_k, \mathcal{V}_k)$ satisfying $\mathcal{V}_{m+1} = \bar{\mathcal{V}}$, where $\mathcal{N}(\mathcal{G}_k, \mathcal{V}_k) = \{j : i \in \mathcal{V}_k, (j,i) \in \mathcal{E}_k\}$.*

Assumption 1 ([29]) *There exists $\alpha \in (0,1)$ such that, for any $k \in \mathbb{N}_+$, $\sum_{j=0}^N a_{ij}(t_k) = 1$, with $a_{ii}(t_k) \ge \alpha$ for $i \in \mathcal{V}$; $a_{ij}(t_k) \ge \alpha$ when $a_{ij} \ne 0$ for $i \in \mathcal{V}$, $j \in \bar{\mathcal{V}}$.*

Assumption 1 is standard and has also been used in [29] to describe the conditions on the coupling coefficients.

Assumption 2 ([29]) *There is $T \in \mathbb{N}_+$ such that the sequence of graphs $\{\mathcal{G}_k\}_{k=rT+1}^{(r+1)T}, \forall r \in \mathbb{N}$ is sequentially connected.*

Assumption 3 ([29]) *There is $T \in \mathbb{N}_+$ such that the sequence of graphs $\{\mathcal{G}_k\}_{k=rT+1}^{(r+1)T}, \forall r \in \mathbb{N}$ is jointly connected.*

Assumptions 2 and 3 are mild conditions on the topology that allow each subgraph to be disconnected, and encompass both the fixed and switching topologies that are connected at all times as special cases.

2.2 Problem statement

Consider a group of N HTTSSs, labelled with $i \in \mathcal{V}$, respectively. The dynamic of the i-th TTSS is

$$\begin{cases} \dot{x}_i = A_{i,11}(w_i)x_i + A_{i,12}(w_i)z_i + B_{i,1}(w_i)u_i, \\ \varepsilon_i \dot{z}_i = A_{i,21}(w_i)x_i + A_{i,22}(w_i)z_i + B_{i,2}(w_i)u_i, \\ y_i = C_{i,1}(w_i)x_i + C_{i,2}(w_i)z_i, \end{cases} \quad (1)$$

where $x_i \in \mathbb{R}^{n_{x_i}}$ and $z_i \in \mathbb{R}^{n_{z_i}}$ are the slow and fast states, $\varepsilon_i \ll 1$ is positive parameter that governs the separation between the slow and fast dynamics, $u_i \in \mathbb{R}^{p_i}$ and $y_i \in \mathbb{R}^q$ are the control input and measurement output, $w_i \in \mathbb{R}^{n_w}$ is an uncertain parameter vector. Suppose that $A_{i,mn}(w_i)$, $B_{i,m}(w_i)$, $C_{i,m}(w_i)$, $m, n = 1, 2$, are all continuous matrix functions of $w_i \in \mathbb{W}_i$ with appropriate dimensions, where $\mathbb{W}_i$ is an open neighborhood of the origin. For convenience, the matrices $A_{i,mn}(0)$, $B_{i,m}(0)$, $C_{i,m}(0)$, $m, n = 1, 2$, are denoted by $A_{i,mn}$, $B_{i,m}$, $C_{i,m}$ respectively, which are all known constant matrices.

The formation of agents is described via $h_i(t) \in \mathbb{R}^n$, which adheres to the feasible formation condition $\dot{h}_i(t) = A_0 h_i(t)$, for $i \in \mathcal{V}$. The desired displacement between two agents output is denoted as $C_0 h_{ij}(t)$, where $h_{ij}(t) = h_i(t) - h_j(t)$. Both A_0 and C_0 are known constant matrices. Regarding the formation patterns, the virtual leader is introduced in the form

$$\begin{cases} \dot{x}_0 = A_0 x_0, \\ y_0 = C_0 x_0, \end{cases} \quad (2)$$

where $x_0 \in \mathbb{R}^n$ and $y_0 \in \mathbb{R}^p$ are the system state and output state of the leader. Let the leader be labeled with 0. Denote $h_0(t) = 0$, the TVOF is formulated as follows.

Definition 3 (TVOF) *The interconnected TTSSs (1) are said to achieve TVOF asymptotically if for any initial states, $\lim_{t\to\infty} \|y_i(t) - C_0 h_i(t) - y_0(t)\| = 0$, $i \in \mathcal{V}$.*

In this paper, our objective is to design a distributed controller over the set of discrete-time sequences, enabling interconnected TTSSs to achieve TVOF. To achieve this goal, the next assumptions are presented.

Assumption 4 ([27]) *The eigenvalues of matrix A_0 are semi-simple with zero real parts.*

Assumption 4 is standard and common for guaranteeing the anti-Hurwitz stability of the virtual leader and the boundedness of relative displacements h_i.

Assumption 5 ([27]) *For the i-th TTSS, it holds that, for each λ_{A_0} being an eigenvalue of A_0,*

$$\mathit{rank}\begin{pmatrix} A_{i,\varepsilon_i} - \lambda_{A_0} I & B_{i,\varepsilon_i} \\ C_i & 0 \end{pmatrix} = n_{x_i} + n_{z_i} + q,$$

where $A_{i,\varepsilon_i} = E_i A_i$, $B_{i,\varepsilon_i} = E_i B_i$, $E_i = \mathit{diag}\{I_{n_{x_i}}, I_{n_{z_i}}\}$,

$$A_i = \begin{pmatrix} A_{i,11} & A_{i,12} \\ A_{i,21} & A_{i,22} \end{pmatrix}, B_i = \begin{pmatrix} B_{i,1} \\ B_{i,2} \end{pmatrix}, C_i = \begin{pmatrix} C_{i,1}^\top \\ C_{i,2}^\top \end{pmatrix}^\top.$$

Assumption 5 is standard and essential for ensuring the existence of the solution to the regulator equations, thus facilitating the solvability of the problem.

Lemma 1 ([20]) *For the symmetric matrix $M_0 \in \mathbb{R}^{n\times n}$ and matrices $M_1 \in \mathbb{R}^{m\times n}$ and $M_2 \in \mathbb{R}^{m\times n}$, the following statements are equivalents:*

(1) $M_0 + He\{M_1^\top M_2\} < 0$;
(2) $\exists\, Q_1 \in \mathbb{R}^{n\times m}$ *and* $Q_2 \in \mathbb{R}^{n\times m}$:

$$\begin{pmatrix} M_0 + He\{Q_1 M_1\} & \star \\ M_2 - Q_1^\top + Q_2 M_1 & -Q_2 - Q_2^\top \end{pmatrix} < 0.$$

3 Main Results

In this section, the state feedback two-layer hierarchical control design and the hybrid model of the closed-loop system are given. Then, the convergence property is ensured. Then, the output feedback controller is further proposed and extended to achieve the output consensus.

3.1 Two-layer hierarchical control design

In this subsection, we present a two-layer hierarchical control scheme for TVOF of HTTSSs (1), following a two-step design procedure as follows.

Step 1 (Virtual systems design in the upper layer): The upper layer comprises N virtual systems, responsible for generating the required local references. Each virtual system is connected to a corresponding agent node in a one-to-one manner. In this context, the dynamic of i-th virtual systems are design as follows:

$$\dot{\zeta}_i(t) = A_0\zeta_i(t) - \sum\nolimits_{k\in\mathbb{N}_+} \delta(t-t_k)\psi_i(t), i \in \mathcal{V}, \quad (3)$$

where $\zeta_i \in \mathbb{R}^n$ is the states, $\delta(\cdot)$ is the Dirac delta distribution defined over the time sequence $\mathcal{T}$, i.e., $t_k \in \mathcal{T}$, and

$$\psi_i(t) = \sum\nolimits_{j=0}^{N} a_{ij}(t)(\zeta_i(t) - \zeta_j(t) - h_{ij}(t)). \quad (4)$$

Note that, according to the dynamic form specified in (3), each system i is required to transmit data ζ_i only at the discrete time t_k. Additionally, the time clocks of systems are assumed to be synchronous.

Step 2 (Design of controller for TTSSs in the lower layer): The lower layer consists of N HTTSSs, tasked with tracking the generated references. Motivated by [28], the post-processing internal model based state feedback controller for i-th TTSS is designed as follows:

$$\begin{cases} \dot{\eta}_i(t) = \Phi_{i,c}\eta_i(t) + \Gamma_{i,c}\hat{y}_i(t), \\ u_i(t) = K_{i,1}x_i(t) + K_{i,2}z_i(t) + G_i(x_i(t), z_i(t), \eta_i(t)), \end{cases} \quad (5)$$

where $\hat{y}_i = y_i - C_0\zeta_i$, the dynamic unit of $\eta_i \in \mathbb{R}^{r\times q}$ is so-called linear internal model with post processing structure, $\Gamma_{i,c} = \Gamma_i \otimes I_q$, $\Phi_{i,c} = \Phi_i \otimes I_q$ with the minimal polynomial of $\bar{\Phi}_i$ coinciding with that of A_0, and the pair (Φ_i, Γ_i) being controllable. The control matrices $K_{i,1}$, $K_{i,2}$ are designed such that $\Lambda_{i,22}$ and $\Lambda_{i,0} = \Lambda_{i,11} - \Lambda_{i,12}\Lambda_{i,22}^{-1}\Lambda_{i,21}$ are both Hurwitz with $\Lambda_{i,mn} = A_{i,mn} + B_{i,m}K_{i,n}$, $m, n = 1, 2$. In this case, there exist positive definite matrices $P_{i,1}$, $P_{i,2}$, $P_{i,3}$ such that

$$\Lambda_{i,0}^\top P_{i,1} + P_{i,1}\Lambda_{i,0} < 0, \quad (6)$$

$$\Lambda_{i,22}^\top P_{i,2} + P_{i,2}\Lambda_{i,22} < 0, \quad (7)$$

$$\Phi_{i,c}^\top P_{i,3} + P_{i,3}\Phi_{i,c} \le 0. \quad (8)$$

Then, the function $G_i(x_i, z_i, \eta)$ is designed based on the forwarding technique inspired by [28] with

$$G_i(x_i, z_i, \eta) = -G_{i,c}\xi_i + B_i^\top M_i^\top P_{i,3}(\eta_i - M_i\bar{E}_i\xi_i),$$

where $\xi_i = (x_i, z_i)$, $G_{i,c} = (B_{i,1}^\top P_{i,1} - B_{i,2}^\top(\Lambda_{i,12}\Lambda_{i,22}^{-1})^\top P_{i,1} + B_{i,2}^\top P_{i,2}\Lambda_{i,22}^{-1}\Lambda_{i,21}, B_{i,2}^\top P_{i,2})$ and M_i satisfies

$$M_i\Lambda_i = \Phi_{i,c}M_i\bar{E}_i + \Gamma_{i,c}C_i, \quad (9)$$

where $\bar{E}_i = \mathrm{diag}\{I_{n_{x_i}}, 0\}$, $\Lambda_i := \begin{pmatrix} \Lambda_{i,11} & \Lambda_{i,12} \\ \Lambda_{i,21} & \Lambda_{i,22} \end{pmatrix}$. Note that, the existence of M_i can be guaranteed when ε_i is small enough, Assumptions 4-5 hold, and $\Lambda_{i,22}$ and $\Lambda_{i,0}$ are both Hurwitz, see [16, Lemma 3] for more detail. Denote $\xi_{i,v} = (\eta_i, \xi_i)$. Then, the overall systems is

$$\dot{\xi}_{i,v}(t) = F_{i,v}(w_i)\xi_{i,v}(t) + \Gamma_{i,v}\zeta_i(t), \quad (10)$$

where $F_{i,v}(w_i) = \begin{pmatrix} \Phi_{i,c} & \Gamma_{i,c}C_i(w_i) \\ B_{i,\varepsilon_i}(w_i)B_i^\top M^\top P_{i,3} & \bar{\Lambda}_{i,\varepsilon_i}(w_i) \end{pmatrix}$, $\bar{\Lambda}_{i,\varepsilon_i}(w_i) = \Lambda_{i,\varepsilon_i}(w_i) - B_{i,\varepsilon_i}(w_i)(G_{c,i} + B_i^\top M_i^\top P_{i,3}M_i\bar{E}_i)$, $\Gamma_{i,v} = \begin{pmatrix} -\Gamma_{i,c}C_0 \\ 0 \end{pmatrix}$ and $\Lambda_{i,\varepsilon_i}(w_i) = \begin{pmatrix} \Lambda_{i,11}(w_i) & \Lambda_{i,12}(w_i) \\ \frac{\Lambda_{i,21}(w_i)}{\varepsilon_i} & \frac{\Lambda_{i,22}(w_i)}{\varepsilon_i} \end{pmatrix}$ with $\Lambda_{i,mn}(w_i) = A_{i,mn}(w_i) + B_{i,m}(w_i)K_{i,n}$, $m, n = 1, 2$. For the stability of the TTSS (10), it is essential for $F_{i,v}(w_i)$ to be Hurwitz. The following lemma establishes that the Hurwitz condition for $F_{i,v}(w_i)$ can be satisfied under the proposed control design.

Lemma 2 ([16]) *Suppose Assumptions 4-5 hold. For i-th TTSS (1), when $\Lambda_{i,22}$ and $\Lambda_{i,0}$ are both Hurwitz, there exist $\bar{\varepsilon}_i > 0$ and an open neighborhood $\mathbb{W}_i$ of the origin, such that for $\varepsilon_i \in (0, \bar{\varepsilon}_i]$ and $w_i \in \mathbb{W}_i$, $F_{i,v}(w_i)$ is Hurwitz.*

Proof 1 *Similar to the proof of Lemma 4 in [16], for i-th TTSS (1), there exists $\bar{\varepsilon}_i > 0$, such that for $\varepsilon_i \in (0, \bar{\varepsilon}_i]$, $F_{i,v}(0)$ is hurwitz. Since $A_{i,mn}(w_i)$, $B_{i,m}(w_i)$, $C_{i,m}(w_i)$, $m, n = 1, 2$, are all continuous matrix functions of w_i, it is easily obtained that $F_{i,v}(w_i)$ is also a continuous matrix function of w_i. Thus, there exists an open neighborhood*

$\mathbb{W}_i$ *of the origin, such that for* $w_i \in \mathbb{W}_i$ *and* $\varepsilon_i \in (0, \bar{\varepsilon}_i]$, $F_{i,v}(w_i)$ *is Hurwitz. This completes the proof.*

With $A_{i,22}$ being non-singular, the model reduction technique has been utilized in [12,13,16,17] to design the desired control gains $K_{i,1}$, $K_{i,2}$. To relax such constraint and simplify the control design, we further propose a one step control design procedure.

Lemma 3 *If there exist positive definite matrices* $W_{i,1}$, $W_{i,2}$*, matrices* $Y_{i,1}$, $Y_{i,2}$*, a scalar* $\varsigma > 0$*, and the given matrix* $\mathcal{I}_i$, $i = 1, \ldots, N$*, such that*

$$\begin{pmatrix} He\{\Psi_{i,11} + \mathcal{I}_i \Psi_{i,12}^\top\} & \star \\ \Psi_{i,21} + \Psi_{i,22}\mathcal{I}_i^\top + \varsigma\Psi_{i,12}^\top & \varsigma He\{\Psi_{i,22}\} \end{pmatrix} < 0, \quad (11)$$

where $\Psi_{i,mn} = A_{i,mn}W_{i,n} + B_{i,m}Y_{i,n}$, $m, n = 1, 2$*, then* (6)-(7) *hold with* $P_{i,j} = W_{i,j}^{-1}$ *and* $K_{i,j} = Y_{i,j}W_{i,j}^{-1}$, $j = 1, 2$.

Proof 2 *Denote* $M_{i,0} = He\{\Psi_{i,11}\}$, $M_{i,1}^\top = -\Psi_{i,12}\Psi_{i,22}^{-1}$, $M_{i,2} = \Psi_{i,21}$, $Q_{i,1}^\top = -\Psi_{i,22}\mathcal{I}_i^\top$, $Q_{i,2} = -\varsigma\Psi_{i,22}^\top$*. Then, inequality* (11) *is rewritten as condition* (2) *of Lemma 1. From Lemma 1, inequality* (11) *is equivalent to*

$$M_{i,0} + He\{M_{i,1}^\top M_{i,2}\} = He\{\Psi_{i,11} - \Psi_{i,12}\Psi_{i,22}^{-1}\Psi_{i,21}\} < 0.$$

Since inequality (11) *holds and* $K_{i,j} = Y_{i,j}W_{i,j}^{-1}$*, it has*

$$He\{\Psi_{i,11} - \Psi_{i,12}\Psi_{i,22}^{-1}\Psi_{i,21}\} = He\{\Lambda_{i,0}W_{i,1}\} < 0. \quad (12)$$

Then, pre-multiplying and post-multiplying (12) *by* $P_{i,1}$*, one obtains that* (6) *holds. Finally, pre-multiplying and post-multiplying* (11) *by* $\begin{pmatrix} 0 & I & 0 \end{pmatrix}$*, it has* $\varsigma He\{\Psi_{i,22}\} = \varsigma He\{\Lambda_{i,22}W_{i,2}\} < 0$*. Since* $\varsigma > 0$*, it can also easily obtained that* (7) *holds. This completes the proof.*

Accordingly, the details of the controller parameter design for TTSS i as described in (5) are provided in the following three steps.

- S1: Select $\Gamma_{i,c} = \Gamma_i \otimes I_q$, $\Phi_{i,c} = \Phi_i \otimes I_q$ with the minimal polynomial of $\bar{\Phi}_i$ coinciding with that of S, and ensure that the pair (Φ_i, Γ_i) being controllable.
- S2: Solve LMI (11) to obtain matrices $W_{i,1}$, $W_{i,2}$, $Y_{i,1}$ and $Y_{i,2}$. Then calculate $P_{i,j} = W_{i,j}^{-1}$ and $K_{i,j} = Y_{i,j}W_{i,j}^{-1}$, $j = 1, 2$. Select $P_{i,3}$ satisfying (8).
- S3: Solve the equation (9) to obtain M_i, then compute $G_i(x_i, z_i, \eta) = -G_{i,c}\xi_i + B_i^\top M_i^\top P_{i,3}(\eta_i - M_i\bar{E}_i\xi_i)$ with $G_{i,c} = (B_{i,1}^\top P_{i,1} - B_{i,2}^\top(\Lambda_{i,12}\Lambda_{i,22}^{-1})^\top P_{i,1} + B_{i,2}^\top P_{i,2}\Lambda_{i,22}^{-1}\Lambda_{i,21}, B_{i,2}^\top P_{i,2})$.

Remark 1 *The control gain can be obtained by solving the linear matrix inequality (LMI)* (11) *using an LMI toolbox. It is noted that the two-layer hierarchical control design is independent of* ε_i.

3.2 Hybrid model

Since $F_{i,v}(w_i)$ is Hurwitz, the intersection of the spectrum of $F_{i,v}(w_i)$ and A_0 is empty. As in [27], there exist matrices Π_i and Σ_i uniquely defined such that

$$\begin{pmatrix} \Sigma_i \\ \Pi_i \end{pmatrix} A_0 = F_{v,i}(w_i) \begin{pmatrix} \Sigma_i \\ \Pi_i \end{pmatrix} + \Gamma_{i,v}. \quad (13)$$

As in [27], the first equation above implies $C_i(w_i)\Pi_i + C_0 = 0$. Denote $x_{i,v} = (\tilde{\eta}_i, \tilde{\xi}_i)$ where

$$\tilde{\eta}_i = \eta_i - \Sigma_i\zeta_i, \tilde{\xi}_i = \xi_i - \Pi_i\zeta_i. \quad (14)$$

Define $x_v = (x_{1,v}, \ldots, x_{N,v})$, $\psi = (\psi_1, \ldots, \psi_N)$. Then, the overall impulsive system can be rewritten as

$$\begin{cases} \dot{x}_v(t) = F_v(w)x_v(t), & t \in (t_k, t_{k+1}), \\ x_v(t_k) = x_v(t_k^-) + \Pi_v\psi(t_k^-), & t = t_k, k \in \mathbb{N}_+, \end{cases} \quad (15)$$

where $w = (w_1, ..., w_N)$, $F_v(w) = (F_{1,v}(w_1), ..., F_{N,v}(w_N))$, $\Pi_v = (\Pi_{1,v}, \ldots, \Pi_{N,v})$, $\Pi_{i,v} = \begin{pmatrix} \Sigma_i^\top & \Pi_i^\top \end{pmatrix}^\top$. Then, the TVOF problem of the interconnected TTSSs (1) is transform to the stabilization problem of (15).

To facilitate the stability analysis, we model the entire system using the hybrid formalism introduced in [30], where a jump corresponds to the injection of impulse signals. Define $\chi = (x_v, \tau) \in \bar{\mathbb{X}} = \mathbb{R}^{(r\times q+n_x+n_z)\times N} \times \mathbb{R}_{\geq 0}$, where τ is a time variable with $\tau(t_0) = t_0$. The hybrid model is given by

$$\dot{\chi} = \mathcal{F}(\chi) \;\; \chi \in \mathcal{C}, \qquad \chi^+ \in \mathcal{G}(\chi) \;\; \chi \in \mathcal{D}, \quad (16)$$

where $\mathcal{F}(\chi) := \begin{pmatrix} F_v(w)x_v \\ 1 \end{pmatrix}$, $\mathcal{G}(\chi) := (x_v + \Pi_v\psi, \tau)$, $\mathcal{C} = \mathbb{R}^{(r\times q+n_x+n_z)\times N} \times (\mathbb{R}_{\geq 0}\backslash\mathcal{T})$, $\mathcal{D} = \mathbb{R}^{(r\times q+n_x+n_z)\times N} \times \mathcal{T}$. Note that the sets C and D are respectively the flow and jump set, and defined according to the interaction sequence. When interactions occur, i-th system transmits ζ_i to neighboring systems, and this transmission generates an impulse control signal resulting in a jump in the state ζ, and also x_v. Then, the TVOF problem is transform to stabilization problem of hybrid system (16).

3.3 Asymptotical convergence

In the following, we present a sufficient condition for the asymptotic stability solving the TVOF problem of interconnected HTTSSs by ISS method of impulsive systems.

Definition 4 (ISS of impulsive system [24]) *The impulsive system* (15) *is input-to-state stable if there* *exist $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}_\infty$, such that for the initial condition $x_v(t_0)$ and any Lebesgue-measurable but bounded input ψ, there holds*

$$\|x_v(t)\| \leq \beta(\|x_v(t_0)\|, t-t_0)+\gamma(\|\psi\|_{[t_0,t]}), \forall t \geq t_0. \quad (17)$$

where $\|\cdot\|_{[t_0,t]}$ denotes the supremum norm over $[t_0, t]$.

Proposition 1 *Consider the hybrid system (16), and suppose that $F_v(w_i)$ is Hurwitz. For any locally bounded and Lebesgue-measurable input $\psi(t)$, (16) is uniformly ISS over impulse time sequence with smallest interval being greater than $\underline{\tau} > 0$.*

Proof 3 *Let us define a time-independent candidate ISS-Lyapunov function $W_1(\chi)$ of the form*

$$W_1(\chi) = x_v^T(t) P_v x_v(t) \quad (18)$$

where P_v and Q_v are two positive definite matrices satisfying Lyapunov equation $P_v F_v(w_i) + F_v^T(w_i) P_v = -Q_v$. Note that there always exists a unique solution P_v for any given positive definite Q_v. Therefore, for $\chi \in \mathcal{C}$,

$$\langle \nabla W_1(\chi), F(\chi)\rangle = x_v^T(P_v F_v + F_v^T P_v) x_v \leq -a W_1(x_v(t)),$$

where $a = \frac{\lambda_{\min}(Q_v)}{\lambda_{\max}(P_v)}$. When $\chi(t_k, k-1) \in D$, $\forall \varsigma > 0$,

$$\begin{aligned}
W_1(\chi(t_k,k)) &= x_v^T(t_k,k) P_v x_v(t_k,k) \\
&= [x_v(t_k,k-1)+\Pi_v\psi(t_k,k-1)]^T P_v \cdot \\
&\quad [x_v(t_k,k-1)+\Pi_v\psi(t_k,k-1)] \\
&\leq \frac{1+4\varsigma}{4\varsigma}[\Pi_v\psi(t_k,k-1)]^T P_v [\Pi_v\psi(t_k,k-1)] \\
&\quad +(1+\varsigma) x_v(t_k,k-1)^T P_v x_v(t_k,k-1) \\
&\leq (1+\varsigma) W_1(\chi(t_k,k-1)) + \gamma(\|\psi(t_k,k-1)\|) \\
&= e^{-b_v} W_1(\chi(t_k,k-1)) + \gamma(\|\psi(t_k,k-1)\|),
\end{aligned}$$

where $\gamma(\|\psi\|) = \frac{1+4\varsigma}{4\varsigma}\lambda_{\max}(P_v)\|\Pi_v\|^2\|\psi\|^2$ and $b_v = -\ln(1+\varsigma) < 0$. Obviously, $\gamma(\|\psi\|)$ is of class $\mathcal{K}_\infty$. Overall, $W_1(t)$ is almost differential everywhere, that is, it is differentiable except on a set of measure zero. By Corollary 1 in [24], we have that when $b_v < 0$ and $a > 0$, (16) is uniformly ISS for $\underline{\tau} > |b_v|/a = \ln(1+\varsigma)/a$. Note that, we can always find $\varsigma(\tau) > 0$ such that $\ln(1+\varsigma)/a < \tau \leq T_a$ holds. In other words, for any impulse time sequence with smallest interval being greater than $\underline{\tau}$, we have that (16) is uniformly ISS. This completes the proof.

Theorem 1 *Suppose Assumptions 1-3 hold, and apply the controller (3)-(5) on HTTSSs (1). Then, there exist $\bar{\varepsilon}_i > 0$ and an open neighborhood $\mathbb{W}_i$ of the origin, $i \in \mathcal{V}$, such that for all $\varepsilon_i \in (0, \bar{\varepsilon}_i]$ and $w_i \in \mathbb{W}_i$, it has:*

(1) When Assumption 4 holds, $\psi(t)$ converges to zero asymptotically if $\mu^T(1-\alpha^T) < 1$, where $\mu = \max_{t\in[\underline{\tau},\overline{\tau}]}\{\|e^{A_0 t}\|\}$;

(2) When Assumption 5 holds, $\psi(t)$ converges to zero asymptotically if $\mu^{N^2T}(1-\alpha^{N^2T}) < 1$;

(3) TVOF is achieved asymptotically if $\psi(t)$ converges to zero asymptotically.

Proof 4 *(1) Let $W(t) = diam\{\zeta_0(t), \zeta_1(t)-h_1(t), ..., \zeta_N(t)-h_N(t)\}$. Then the asymptotically convergence of $\psi(t)$ is equivalent to $\lim_{t\to\infty}\|W(t)\| = 0$. Since $\{\mathcal{G}_k\}_{k=rT+1}^{(r+1)T}$ is sequentially connected with node 0 as the root, suppose that $\mathcal{V}_{rT+k+1} \subseteq \mathcal{V}_k \cup \mathcal{N}(\mathcal{G}_k, \mathcal{V}_k) = \mathcal{N}(\mathcal{G}_{rT+k}, \mathcal{V}_{rT+k})$, where $\mathcal{V}_{rT+1} = \{0\}$ is a singleton and $\mathcal{V}_{(r+1)T} = \bar{\mathcal{V}}$. Denote*

$$\begin{aligned}
\mathcal{H}_{rT+k} &= conv(\zeta_i(t_{rT+k}) - h_i(t_{rT+k}))_{i\in\mathcal{V}_{rT+k}}, \\
\mathcal{H}^+_{rT+k} &= conv(\zeta_i(t^+_{rT+k}) - h_i(t^+_{rT+k}))_{i\in\mathcal{V}_{rT+k+1}}, \\
\hat{\mathcal{H}}_{rT+k} &= conv(\zeta_i(t_{rT+k}) - h_i(t_{rT+k}))_{i\in\bar{\mathcal{V}}}.
\end{aligned}$$

For any $i \in \mathcal{V}_{rT+k+1}$ and $i \neq 0$, it has

$$\begin{aligned}
\zeta_i(t^+_{rT+k}) - h_i(t^+_{rT+k}) = &\sum_{j\in\mathcal{V}_{rT+k}} a_{ij}(\zeta_j(t_{rT+k}) - h_j(t_{rT+k})) \\
&+ \sum_{j\notin\mathcal{V}_{rT+k}} a_{ij}(\zeta_j(t_{rT+k}) - h_j(t_{rT+k})) \\
\in &(\sum_{j\in\mathcal{V}_{rT+k}} a_{ij})\mathcal{H}_{rT+k} + (\sum_{j\notin\mathcal{V}_{rT+k}} a_{ij})\hat{\mathcal{H}}_{rT+k}.
\end{aligned}$$

Since $i \in \mathcal{V}_{rT+k+1}$, there is $j \in \mathcal{V}_{rT+k}$, thus $\sum_{j\in\mathcal{V}_{rT+k}} a_{ij} \geq \alpha$. Then, $\zeta_i(t^+_{rT+k}) - h_i(t^+_{rT+k}) \in \alpha\mathcal{H}_{rT+k} + (1-\alpha)\hat{\mathcal{H}}_{rT+k}$. Since node 0 is the root, we can also suppose that $0 \in \mathcal{V}_{rT+k}$ for any $t = 1, \ldots, T$. Then, $\zeta_0(t^+_{rT+k}) = \zeta_0(t_{rT+k}) \in \alpha\mathcal{H}_{rT+k} + (1-\alpha)\hat{\mathcal{H}}_{rT+k}$. Thus, $\mathcal{H}^+_{rT+k} \subseteq \alpha\mathcal{H}_{rT+k} + (1-\alpha)\hat{\mathcal{H}}_{rT+k}$. Consequently,

$$\begin{aligned}
diam(\mathcal{H}^+_{rT+k}) &\leq \alpha\, diam(\mathcal{H}_{rT+k}) + (1-\alpha)\, diam(\hat{\mathcal{H}}_{rT+k}) \\
&= \alpha\, diam(\mathcal{H}_{rT+k}) + (1-\alpha) W(t_{rT+k}).
\end{aligned}$$

Obviously, $W(t^+_{rT+k}) \leq W(t_{rT+k})$ and

$$\begin{aligned}
diam(\mathcal{H}_{rT+k}) &\leq \|e^{A_0(t_{rT+k}-t_{rT+k-1})}\| diam(\mathcal{H}_{rT+k-1}) \\
&\leq \mu\| diam(\mathcal{H}_{rT+k-1}), \\
W(t_{rT+k}) &\leq \|e^{A_0(t_{rT+k}-t_{rT+k-1})}\| W(t_{rT+k-1}) \\
&\leq \mu W(t_{rT+k-1}).
\end{aligned}$$

Thus, $W(t^+_{(r+1)T+1}) \leq \mu^T(1-\alpha^T) W(t_{rT})$. Since $\mu^T(1-\alpha^T) < 1$, it has $\lim_{r\to\infty}\|W(t_{rT+1})\| = 0$, thus $\lim_{t\to\infty}\|\psi(t)\| = 0$.

(2) Since Assumption 5 holds, it has $\{\mathcal{G}_k\}_{rT+1}^{(r+1)T}$ is jointly connected with node 0 as the root. Then, with a similar proof in [29], $\{\mathcal{G}_k\}_{rN^2T+1}^{(r+1)N^2T}$ is sequentially connected with

node 0 as the root. According to the statement (1), it is a direct result that $\lim_{t\to\infty} \|\psi(t)\| = 0$.

(3) By the definition of ISS and Proposition 1, there exist $\beta(\cdot,\cdot) \in \mathcal{KL}$ *and* $\gamma(\cdot) \in \mathcal{K}$ *such that*

$$\|x_v(t)\| \leq \beta(\|x_v(t_0)\|, t-t_0) + \gamma(\|\psi(t)\|_{[t_0,t]}), \quad \forall t \geq t_0.$$

where $\|\psi(t)\|_{[t_0,t]} = \max_{s\in[t_0,t]}(\|\psi(s)\|)$, *namely, the maximum norm of* $\psi(t)$ *over the selected interval. Note that* $\gamma(\cdot)$ *is continuous and strictly increasing with* $\gamma(0) = 0$. *If* $\psi(t)$ *converges to zero asymptotically, then for any* $\epsilon > 0$ *there exists* $T_0(\epsilon) > 0$, *such that* $\forall t > T_0(\epsilon)$,

$$\gamma(\|\psi(t)\|_{[T_0(\epsilon),t]}) < \epsilon. \tag{19}$$

In this way, for $t = T_0(\epsilon)$, *it holds that* $\|x_v(T_0(\epsilon))\| \leq \beta(\|x_v(t_0)\|, T_0(\epsilon) - t_0) + \gamma(\|\psi(t)\|_{[t_0,T_0(\epsilon)]})$.

Obviously, $\psi(t)$ *is bounded over* $t \in \mathbb{R}_{>0}$. *Let us recall the definition of the class* $\mathcal{K}$ *function,* $\gamma(\cdot)$ *is an strictly increasing function, here we have* $\gamma(\|\psi(t)\|_{[t_0,T_0(\epsilon)]})$ *is bounded and thus* $\|x_v(T_0(\epsilon))\|$ *is also bounded. By the property of the class* $\mathcal{KL}$, *for a fixed* $\|x_v(T_0(\epsilon))\|$, *the mapping* $\beta(\|x_v(T_0(\epsilon))\|, t)$ *is continuous and strictly decreasing to zero as* $t \to \infty$. *Then it can be obtained that for any* $\epsilon > 0$, *there exists* $T_f(\epsilon) > T_0(\epsilon) > 0$, *such that*

$$\beta(\|x_v(T_0)\|, T_f - T_0) < \epsilon. \tag{20}$$

Therefore, for any $\epsilon > 0$, *there exists* $t > T_f(\epsilon)$ *such that*

$$\|x_v(t)\| \leq \beta(\|x_v(T_0(\epsilon))\|, t - T_0(\epsilon)) + \gamma(\|\psi(t)\|_{[T_0(\epsilon),t]}) < 2\epsilon.$$

Then, it has $\lim_{t\to\infty} \|x_v(t)\| = 0$ *if* $\lim_{t\to\infty} \|\psi(t)\| = 0$.

Recalling the definition of $\tilde{\xi}_i(t)$ *and* $\lim_{t\to\infty} \|\psi(t)\| = 0$, *then it follows that* $\lim_{t\to\infty} \|\tilde{\xi}_i(t)\| = 0$, $\lim_{t\to\infty} \|C_i\tilde{\xi}_i(t)\| = 0$ *and* $\lim_{t\to\infty} \|C_0(\zeta_i(t) - h_i(t) - x_0(t))\| = 0$. *Therefore, we have*

$$\begin{aligned}
&\lim_{t\to\infty} \|y_i(t) - C_0 h_i(t) - y_0(t)\| \\
&\leq \lim_{t\to\infty} \|C_i\tilde{\xi}_i(t)\| + \lim_{t\to\infty} \|C_0(\xi_i(t) - h_i(t) - x_0(t))\| = 0.
\end{aligned}$$

Remark 2 *The designed two-layer control scheme facilitates dynamic formation of interconnected TTSSs with heterogeneous dynamics. It also demonstrates robustness to small structural uncertainties, and is applicable to directed topology that is sequentially or jointly connected. These features enhance flexibility of the control scheme and broaden its range of applications. Furthermore, the control scheme is designed via discrete-time communication, requiring fewer communication resources.*

Remark 3 *For each agent* i, *the approximate bound* $\bar{\varepsilon}_i$ *of* ε_i, *and the approximate available region* $\mathbb{W}_i$ *of* w_i *in Theorems 1 can be obtained by finding the available region such that* $\tilde{J}(0)$ *is Hurwitz, i.e., solving the following optimization problem inspired by [17]:*

$$\max_{Z_{i,1}=Z_{i,1}^\top, Z_{i,2j}=Z_{i,2j}^\top, Z_{i,3j}=Z_{i,3j}^\top, Z_{i,4j}=Z_{i,4j}^\top, j=1,\ldots,M-1} \bar{\varepsilon}_i \& \mathbb{W}_i,$$

s.t. $\forall w_i \in \mathbb{W}_i$,

$$\Omega_{i,1}(w_i) > 0, Z_{i,1} > 0, \Omega_{i,1}(w_i) + \sum_{n=1}^{m-1} \bar{\varepsilon}^i \Omega_{i,n+1}(w_i) > 0,$$

$$\begin{pmatrix} Z_1 & 0 \\ 0 & 0 \end{pmatrix} + \sum_{n=1}^{m-1} \bar{\varepsilon}^i \begin{pmatrix} Z_{i,2n} & Z_{i,3n}^\top \\ Z_{i,3n}^\top & Z_{i,4n} \end{pmatrix} > 0, m = 2, \ldots, M,$$

where $\Omega_{i,j}(w_i) = -(\tilde{E}_i F_{i,v}(w_i))^\top \mathcal{Z}_{i,j} - \mathcal{Z}_{i,j}^\top \tilde{E}_i F_{i,v}(w_i)$, $F_{i,v}(w_i)$ *is defined in* (10), $\tilde{E}_i = \{I_{n_{\eta_i}+n_{x_i}}, \varepsilon_i I_{n_{z_i}}\}$, $\mathcal{Z}_{i,1} = \begin{pmatrix} Z_{i,1} & 0 \\ Z_{i,31} & Z_{i,41} \end{pmatrix}$, $\mathcal{Z}_{i,j} = \begin{pmatrix} Z_{i,2(j-1)} & Z_{i,3(j-1)}^\top \\ Z_{i,3j} & Z_{i,4j} \end{pmatrix}$, $j = 2, \ldots, M-1$, $\mathcal{Z}_{i,M} = \begin{pmatrix} Z_{i,2M-1} & Z_{i,3M-1}^\top \\ 0 & 0 \end{pmatrix}$.

3.4 Output feedback control

Considering the case that the system's state is not measurable, an output feedback controller is further designed

$$\begin{cases}
u_i(t) = K_{i,1}\bar{x}_i(t) + K_{i,2}\bar{z}_i(t) + G_i(\bar{x}_i(t), \bar{z}_i(t), \eta_i(t)), \\
\dot{\zeta}_i(t) = A_0\zeta_i(t) - \sum_{k\in\mathbb{N}_+} \delta(t-t_k)\psi_i(t), \\
\dot{\eta}_i(t) = \Phi_{i,c}\eta_i(t) + \Gamma_{i,c}\hat{y}_i(t), \\
\dot{\bar{x}}_i(t) = A_{i,11}\bar{x}_i(t) + A_{i,12}\bar{z}_i(t) + B_{i,1}u_i(t) \\
\qquad + L_{i,1}(C_i\bar{\xi}_i(t) - y_i(t)), \\
\varepsilon_i\dot{\bar{z}}_i(t) = A_{i,21}\bar{x}_i(t) + A_{i,22}\bar{z}_i(t) + B_{i,2}u_i(t) \\
\qquad + L_{i,2}(C_i\bar{\xi}_i(t) - y_i(t)),
\end{cases} \tag{21}$$

where $K_{i,1}$, $K_{i,2}$, $G_i(\cdot)$, ψ_i, $\Phi_{i,c}$, $\Gamma_{i,c}$ have the same definition as in (3)-(5), $L_{i,1}$, $L_{i,2}$ are designed such that $\bar{\Lambda}_{i,22}$ and $\bar{\Lambda}_{i,0} = \bar{\Lambda}_{i,11} - \bar{\Lambda}_{i,12}\bar{\Lambda}_{i,22}^{-1}\bar{\Lambda}_{i,21}$ are both Hurwitz with $\bar{\Lambda}_{i,mn} = A_{i,mn} + L_{i,m}C_{i,n}$, $m, n = 1, 2$. Denote $\bar{\xi}_{i,v} = (\eta_i, \xi_i, \bar{\xi}_i)$. Then, it has

$$\dot{\bar{\xi}}_{i,v}(t) = \bar{F}_{i,v}(w_i)\bar{\xi}_{i,v}(t) + \bar{\Gamma}_{i,v}\zeta_i(t), \tag{22}$$

where $\bar{\Gamma}_{i,v} = \begin{pmatrix} -C_0^\top\Gamma_{i,c}^\top & 0 & 0 \end{pmatrix}^\top$, $L_{i,\varepsilon_i} = \begin{pmatrix} L_{i,1}^\top & \frac{L_{i,2}^\top}{\varepsilon_i} \end{pmatrix}^\top$ and

$$\bar{F}_{i,v}(w_i) = \begin{pmatrix} \Phi_{i,c} & \Gamma_{i,c}C_i(w_i) & 0 \\ B_{i,\varepsilon_i}(w_i)B_i^\top M^\top P_{i,3} & A_{i,\varepsilon_i}(w_i) & \bar{\Lambda}_{i,\varepsilon_i}(w_i) - A_{i,\varepsilon_i}(w_i) \\ B_{i,\varepsilon_i}B_i^\top M^\top P_{i,3} & -L_{i,\varepsilon_i}C_i & \bar{\Lambda}_{i,\varepsilon_i} + L_{i,\varepsilon_i}C_i \end{pmatrix}.$$

Lemma 4 *Suppose Assumptions 1-2 hold and* $\Lambda_{i,22}$,

$\Lambda_{i,0}$, $\bar{\Lambda}_{i,22}$ and $\bar{\Lambda}_{i,0}$ *are all Hurwitz. There exist* $\bar{\varepsilon}_i > 0$ *and an open neighborhood* $\mathbb{W}_i$ *of the origin, such that for* $\varepsilon_i \in (0, \bar{\varepsilon}_i]$ *and* $w_i \in \mathbb{W}_i$, $\bar{F}_{i,v}(w_i)$ *is Hurwitz.*

Proof 5 *Denote*

$$\tilde{F}_{i,v}(0) = \begin{pmatrix} I_{n_\eta} & 0 & 0 \\ 0 & I_{n_\xi} & I_{n_\xi} \\ 0 & 0 & I_{n_\xi} \end{pmatrix} \bar{F}_{i,v}(0) \begin{pmatrix} I_{n_\eta} & 0 & 0 \\ 0 & I_{n_\xi} & I_{n_\xi} \\ 0 & 0 & I_{n_\xi} \end{pmatrix}^{-1}$$
$$= \begin{pmatrix} \Phi_{i,c} & \Gamma_{i,c} C_i(0) & 0 \\ B_{i,\varepsilon_i}(0) B_i^\top M^\top P_{i,3} & \bar{\Lambda}_{i,\varepsilon_i}(0) & \bar{\Lambda}_{i,\varepsilon_i}(0) - A_{i,\varepsilon_i}(0) \\ 0 & 0 & A_{i,\varepsilon_i}(0) + L_{i,\varepsilon_i} C_i \end{pmatrix},$$

where $n_\eta = r \times q$ *and* $n_\xi = n_x + n_z$. *Clearly, the matrices* $\bar{F}_{i,v}(0)$ *and* $\tilde{F}_{i,v}(0)$ *are similar, implying* $\bar{F}_{i,v}(0)$ *is Hurwitz if and only if* $\tilde{F}_{i,v}(0)$ *is Hurwitz. Since* $\bar{\Lambda}_{i,22}$ *and* $\bar{\Lambda}_{i,0}$ *are both Hurwitz, there exists* $\bar{\varepsilon}_{i,1} > 0$, *such that for* $\varepsilon_i \in (0, \bar{\varepsilon}_{i,1}]$, $A_{i,\varepsilon_i}(0) + L_{i,\varepsilon_i} C_i$ *is Hurwitz. From Lemma 4, there exists* $0 < \bar{\varepsilon}_{i,2} \le \bar{\varepsilon}_{i,1}$, *such that for* $\varepsilon_i \in (0, \bar{\varepsilon}_{i,2}]$, $F_{i,v}(0) = \begin{pmatrix} \Phi_{i,c} & \Gamma_{i,c} C_i(0) \\ B_{i,\varepsilon_i}(0) B_i^\top M^\top P_{i,3} & \bar{\Lambda}_{i,\varepsilon_i}(0) \end{pmatrix}$ *is Hurwitz. Thus, for* $\varepsilon_i \in (0, \bar{\varepsilon}_{i,2}]$, $\tilde{F}_{i,v}(0)$ *is Hurwitz and so does* $\bar{F}_{i,v}(0)$. *Thus, there exists* $0 < \bar{\varepsilon} \le \bar{\varepsilon}_{i,2}$ *and an open neighborhood* $\mathbb{W}_i$ *of the origin, such that for* $\varepsilon_i \in (0, \bar{\varepsilon}]$, $i \in \mathcal{V}$ *and* $w_i \in \mathbb{W}_i$, $\bar{F}_{i,v}(w_i)$ *is Hurwitz.*

Remark 4 *Similar to Lemma 2, the control matrices is designed as* $L_{i,j} = \bar{W}_{i,j}^{-1} \bar{Y}_{i,j}$, $i \in \mathcal{V}$, $j = 1, 2$, *where the positive definite matrices* $\bar{W}_{i,j}^{-1}$ *and matrices* $\bar{Y}_{i,j}$, *satisfy*

$$\begin{pmatrix} He\{\bar{\Psi}_{i,11} + \mathcal{I}_i \bar{\Psi}_{i,12}\} & \star \\ \bar{\Psi}_{i,21} + \bar{\Psi}_{i,22} \bar{\mathcal{I}}_i^\top + \bar{\varsigma} \bar{\Psi}_{i,12}^\top & \bar{\varsigma} He\{\bar{\Psi}_{i,22}\} \end{pmatrix} < 0, \quad (23)$$

where $\bar{\Psi}_{i,mn} = \bar{W}_{i,m} A_{i,mn} + Y_{i,m} C_{i,n}$, $m, n = 1, 2$, $\bar{\varsigma} > 0$ *is a scale, and* $\bar{\mathcal{I}}_i$ *is a given matrix as in Lemma 3.*

Then, the next Theorem is obtained. The proof is similar to that of Theorem 1, thus is omitted here.

Theorem 2 *Suppose Assumptions 1-3 hold, and apply the output feedback controller* (21) *on HTTSSs* (1). *Then, there exist* $\bar{\varepsilon}_i > 0$ *and an open neighborhood* $\mathbb{W}_i$ *of the origin,* $i = 1, \ldots, N$, *such that for all* $\varepsilon_i \in (0, \bar{\varepsilon}_i]$ *and* $w_i \in \mathbb{W}_i$, *the following statements hold:*

(1) TVOF is achieved asymptotically if Assumption 4 holds and $\mu^T (1 - \alpha^T) < 1$;
(2) TVOF is achieved asymptotically if Assumption 5 holds and $\mu^{N^2 T} (1 - \alpha^{N^2 T}) < 1$..

3.5 Application to output consensus problem

In this subsection, the above result is extended to handle output consensus of HTTSSs (1), which is defined next.

Definition 5 *The interconnected TTSSs* (1) *are said to achieve output consensus asymptotically if for any initial states* $x_i(0)$, $z_i(0)$, $\lim_{t \to \infty} \|y_i(t) - y_0(t)\| = 0$, $i \in \mathcal{V}$.

The controller is designed in the form of (21) with $\psi_i(t) = \sum_{j=0}^{N} a_{ij}(t)(\zeta_i(t) - \zeta_j(t))$. Then, next theorem is obtained. The proof is similar, thus is omitted here.

Theorem 3 *Suppose Assumptions 1-3 hold, and apply the proposed controller on HTTSSs* (1). *Then, there exist* $\bar{\varepsilon}_i > 0$ *and an open neighborhood* $\mathbb{W}_i$ *of the origin,* $i = 1, \ldots, N$, *such that for all* $\varepsilon_i \in (0, \bar{\varepsilon}]$ *and* $w_i \in \mathbb{W}_i$, *the following statements hold:*

(1) Output consensus is achieved asymptotically if Assumption 4 holds and $\mu^T (1 - \alpha^T) < 1$;
(2) Output consensus is achieved asymptotically if Assumption 5 holds and $\mu^{N^2 T} (1 - \alpha^{N^2 T}) < 1$.

Remark 5 *Unlike the consensus results in [18–21], each TTSSs in this study has nonidentical dimensions and time-scale factors, better suited for practical applications. Moreover, the considered switching topology is directed and disconnected, resulting in a weaker assumption.*

4 Simulation

In this section, two examples are given to demonstrate the proposed results.

4.1 Example 1: TVOF of interconnected HTTSSs

Consider a group of four HTTSSs (1), with $x_i \in \mathbb{R}^2$, $i = 1, \ldots, 4$, $z_1, z_2 \in \mathbb{R}$, $z_3, z_4 \in \mathbb{R}^2$, $\varepsilon_1 = 0.06$, $\varepsilon_2 = 0.01$, $\varepsilon_3 = \varepsilon_4 = 0.1$, and

$$A_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ w_{1,1} - 0.5 & 1 + w_{1,2} & 0 \end{pmatrix} \otimes I_2, B_1 = \begin{pmatrix} 0 \\ 0 \\ 1 + w_{1,3} \end{pmatrix} \otimes I_2,$$

$$A_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 + w_{2,1} & w_{2,2} - 1 & 2 + w_{2,3} \end{pmatrix} \otimes I_2, B_2 = \begin{pmatrix} 0 \\ 0 \\ 1 + w_{2,4} \end{pmatrix} \otimes I_2,$$

$$C_1 = \begin{pmatrix} 1 + w_{1,4} & 0 & 2 + w_{1,5} \end{pmatrix} \otimes I_2, C_2 = \begin{pmatrix} 1 + w_{2,5} & 0 & 0 \end{pmatrix} \otimes I_2,$$

$$A_3 = \begin{pmatrix} 0 & 0.4 + w_{3,1} & 0 & 0 \\ 0 & 0 & 0.3 & 0 \\ w_{3,2} & w_{3,3} - 0.5 & 0.5 & 0.2 \\ 0 & 0 & 0 & -1 \end{pmatrix} \otimes I_2, B_3 = \begin{pmatrix} 1 \\ 0 \\ 1 + w_{3,4} \\ 2 \end{pmatrix} \otimes I_2,$$

$$C_3 = \begin{pmatrix} 1 + w_{3,5} & 0 & 1 + w_{3,6} & 1 \end{pmatrix} \otimes I_2,$$

$$A_4 = \begin{pmatrix} 0 & -0.5 + w_{4,1} & w_{4,2} - 0.4 & 0 \\ 0.5 & 0 & 0 & 0.5 \\ 0.5 + w_{4,3} & 0.4 & 0 & 0.5 \\ -0.5 & 0.5 & w_{4,4} - 0.5 & 0 \end{pmatrix} \otimes I_2,$$

$B_4 = \begin{pmatrix} -1 & 2 & 0 & 1+w_{4,5} \end{pmatrix}^\top \otimes I_2, C_4 = \begin{pmatrix} 1+w_{4,6} & -1 & 0 & 0 \end{pmatrix} \otimes I_2.$

The formation h_i, $i \in \{1,2,3,4\}$ is defined as $h_i(t) = \begin{pmatrix} 25\sin(t+\frac{2\pi\times i}{4}) & 25\cos(t+\frac{2\pi\times i}{5}) \end{pmatrix}$. The leader is with the form of (2), where $S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $C = I_2$ and $x_0(0) = (50, 0)$. The switching communication topology is given in Fig. 1, which switches with $0.1s < t_{k+1} - t_k \leq 0.5s$. Thus, Assumptions 2 and 3 are satisfied. The associated weighted adjacency matrix is obtained with Assumption 1 being satisfied.

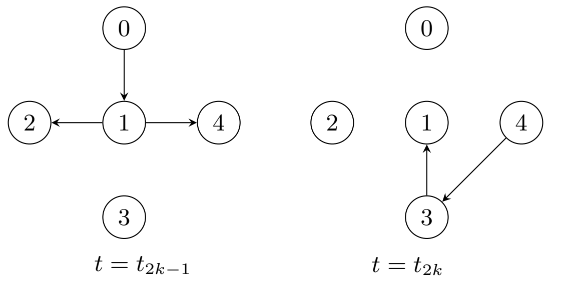

Fig. 1. The switching communication topology.

The simulation is presented with $w_{ij} \in (-0.01, 0.01)$, $\Phi_i = S$, and $\Gamma_i = \begin{pmatrix} 1 & 0 \end{pmatrix}^\top$ $i,j = 1,\dots,6$. Following the control parameter design procedure under Remark 1, the controller (5) can be obtained.

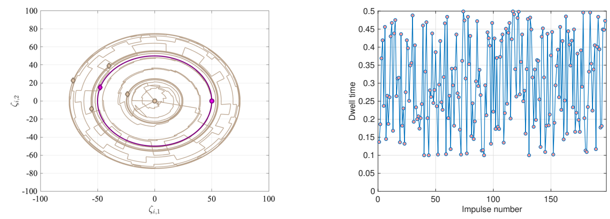

Fig. 2. Evolutions of virtual systems. (a) Trajectories of virtual states; (b) Dwell time of adjacent interactions.

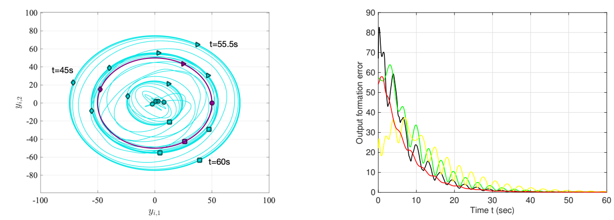

Fig. 3. Evolutions of HMASs. (a) Trajectories of output states; (b) Trajectories of output formation error.

The simulation results are shown in Figs. 2-3. Fig. 2(a) shows that the formation tracking of the leader is achieved for the interconnected virtual systems, which generates the reference signal for lower layer TTSSs. Fig. 2(b) shows that dwell time between two adjacent impulsives, which is aperiodic. Using impulsive control technique promotes to avoid persistently interacting, which reduces the burden of communication devises. Fig. 3 shows that the TVOF for interconnected HTTSSs (1) with switching topology in Fig. 1 is eventually achieved, and the output formation errors approach to zero along with time, which confirms the effectiveness of Theorem 1.

4.2 Example 2: Output consensus of interconnected motor systems

Consider a group of two permanent magnet synchronous motor(PMSM) systems as in [14] and two dual-PMSM systems as in [15], labelled with $\mathcal{I} = \{1,2,3,4\}$. The PMSM systems are of the form

$$\begin{aligned} J_m\dot{\omega}_i &= n_p\varphi_{i,f}I_{q,i} - B\omega_i, \\ L_{sq}\dot{I}_{q,i} &= -R_{i,s}I_{q,i} - n_p\varphi_{i,f}\omega_i + u_i, i = 1,2, \end{aligned}$$

the dual-PMSM systems are of the form

$$\begin{aligned} g_1\dot{\omega}_i &= 1.5n_{p_1}\varphi_{i,f_1}I_{p_1,i} + 1.5n_{p_2}\varphi_{i,f_2}I_{p_2,i} - g_2\omega_i, \\ L_{sq1}\dot{I}_{q1,i} &= -R_{i,s1}I_{q1,i} - n_{p1}\varphi_{i,f1}\omega_i + u_{q1,i}, \\ L_{sq2}\dot{I}_{q2,i} &= -R_{i,s2}I_{q2,i} - n_{p2}\varphi_{i,f2}\omega_i + u_{q2,i}, i = 3,4, \end{aligned}$$

where ω_i is the angular speed, $I_{q,i}$, $I_{q1,i}$, $I_{q2,i}$ are the q shaft armature current, u_i, $u_{q1,i}$, $u_{q2,i}$ are the control voltage, other system parameters have the same definition as in [14] and [15]. The goal is to achieve the consensus of angular speed, i.e., $y_i = \omega_i$. Then, the form of leader is the same as that in Example 1 with $x_0 = (50, 0)$. Set $J_m = 0.000021$, $B = 0.0004927$, $g_1 = 0.000063$, $g_2 = -.0014781$, $n_p = n_{p1} =_{p2}= 4$, $L_{sq} = L_{sq1} = L_{sq2} = 0.0098$, $\varphi_{i,f} = 0.0804 + 0.1w_{i,1}$, $\varphi_{i,f_1} = 0.0804 + 0.1w_{i,1}$, $\varphi_{i,f_2} = 0.0804 + 0.1w_{i,2}$, $R_{i,s} = 10.7 + w_{i,2}$, $R_{i,s1} = 10.7 + w_{i,3}$, $R_{i,s2} = 10.7 + w_{i,4}$.

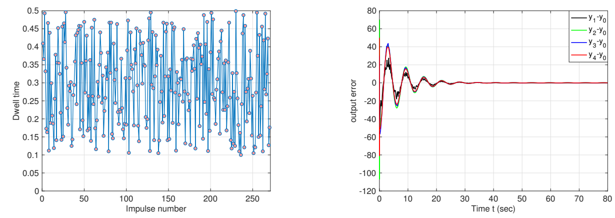

Fig. 4. Evolutions of motor systems. (a) Dewell time of adjecent interactions; (b) Trajectories of output consensus error.

The simulation is presented with $w_{ij} \in (-0.01, 0.01)$, $P_{i,3} = I_2$, $i,j = 1,\dots,4$. Let $\Phi_{i,c}$ and $\Gamma_{i,c}$ be the same as that in Example 1. Let $L_1 = L_2 = \begin{pmatrix} -0.001 & 0 & 0 \end{pmatrix}^\top$, $L_3 = L_4 = \begin{pmatrix} -0.001 & 0 \end{pmatrix}$. Then, following the control parameter design procedure under Remark 1, the output feedback controller (21) is obtained. The switching communication topology is the same as that in Example 1. The simulation results are shown in Figs. 4, which shows that the output consensus for interconnected TTSSs (1) with switching topology is achieved, which confirms the effectiveness of Theorem 2.

5 Conclusion

In this work, we presented new results on the hybrid two-layer hierarchical control protocol design for the TVOF of interconnected linear HTTSSs with model uncertainties. Each system can exhibit nonidentical dimensions and time-scaling factors, while the direct communication topology is switching and disconnected. On the top of that, the output formation is achieved, while systems can only interact with each other at discrete-time such that the communication burden is reduced. The results are also extended for the output consensus of interconnected HTTSSs. It would be interesting to extend the results to the hybrid cooperative control of the interconnected heterogeneous TTSSs in the presence of nonlinear dynamics and Byzantine agent [31] in future work.

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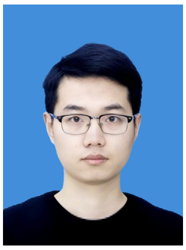

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