Comments on: "$\mathcal{H}_\infty$ control design for time-delay linear systems: a rational transfer function based approach"

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1 Discussion

The paper by R. H. Korogui, A. R. Fioravanti, and J. C. Geromel focuses on the $\mathcal{H}_\infty$ control design for time-delay linear systems. The results are based on the use of a comparison system obtained via Rekasius' substitution. As it is known from the delay systems' literature, this transformation allows to approximate the non-rational transfer functions by rational ones. Moreover the two transfer functions coincide on the imaginary axis which is essential to detect the exact crossings of the poles from the left-hand side to the right-hand side of the complex plane or reversed. As pointed out by the authors this property is instrumental for stability analysis of time-delay systems.

Based on the fact that the $\mathcal{H}_\infty$ norm of an analytic transfer function is defined as the supremum of its maximal singular value on the imaginary axis, the matching of the rational and non-rational transfer functions will help to deal with the $\mathcal{H}_\infty$ norm of time-delay linear systems.

By recalling Theorem 1 and Corollary 1 provided in their paper [4], the authors discuss how the $\mathcal{H}_\infty$ norm of the comparison function is a lower bound of the $\mathcal{H}_\infty$ norm of the original time-delay transfer function and in addition that there exists, under a few classical conditions, a subinterval ensuring the bounding (12) for the $\mathcal{H}_\infty$ norm of the time-delay transfer function. One great advantage provided by the use of Rekasius' substitution is the high accuracy in the determination of the relevant frequency values. It should be noted that the computation of $\mathcal{H}_\infty$ norm for linear time invariant (LTI) systems is based on the duality between singular values of the transfer function and the imaginary eigenvalues of an appropriately defined Hamiltonian matrix (see [2]). Therefore, we may expect a good estimation of the $\mathcal{H}_\infty$ norm when we use the Rekasius' substitution.

It is worth mentioning here that there exists in the literature at least one alternative method for the computation of the $\mathcal{H}_\infty$ norm for time-delay linear systems (the reader is referred to [5]). This approach is also based on an approximation due to the discretization of the Hamiltonian operator used to compute the singular values of the delay system. What is interesting in [5] is that the authors add a second step to their algorithm in order to locally correct the approximation done in the first step. Like this, they are able to compute with the desired precision the $\mathcal{H}_\infty$ norm of the original delay system.

Thanks to the discussions concerning the bounding (12), the authors propose in sections 3 and 4 design procedures of $\mathcal{H}_\infty$ norm controllers satisfying a given suboptimal level, respectively in the class of state feedbacks and output dynamic feedbacks. These procedures use Riccati equation and inequality, which are standard for $\mathcal{H}_\infty$ control (see [7, 1] for an overview and discussions) or $\mathcal{H}_2/\mathcal{H}_\infty$ (see for instance [6, 3]), applied on the comparison system. For the two kinds of feedbacks, the elegant refinement is here to consider the structure of the comparison system in order to specify the structure of the Lyapunov

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matrix and, doing so, simplify its computation.

What should be clearly emphasized is that the methodology proposed in the paper by R. H. Korogui, A. R. Fioravanti, and J. C. Geromel is computationally oriented since it is based on classical numerical routines for Ricatti equations and inequalities. Meanwhile, the alternative computation method is far to be standard in this moment since it requires the use of algorithms that compute the infinite spectrum of the Hamiltonian operator associated with the transcendent transfer function of a delay system.

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