Anticipating the Loss of Unknown Input Observability for Sampled LPV Systems

Gustave Bainier, Jean-Christophe Ponsart, Benoît Marx

Abstract Given a continuous-time Linear Parameter-Varying (LPV) system with a sampled scheduling parameter and subject to an unknown input, this paper provides – under some Lipschitz assumptions – an exact discretization of an extended system which translates the sampled-data unknown input estimation problem into a discrete-time LPV observer design problem with norm-bounded uncertainties. The bounds developed in this process account for the inter-sample behavior of the scheduling parameter, and allow for an estimation of some near-future observability Gramians, from which it is possible to lower bound the number of samples for which the unknown input is guaranteed to remain observable.

1 Introduction

The continuous-time Linear Parameter-Varying (LPV) representation is a powerful tool to study a large class of nonlinear systems using a linear-like framework, which generally facilitates controller and observer synthesis [18, 16]. It relies on the construction of a scheduling parameter θ typically accounting for the non-linearities of the initial system, and which is assumed to be continuously known or estimated in real time. However, in practice, the values of θ are generally recovered at a fixed sampling rate and are assumed to be held constant between two samples [20]. This zero-order hold (ZOH) assumption made on the values of θ creates a sampled-data problem which has already been discussed in the LPV literature [19, 15, 8].

The present paper considers a continuous-time LPV system subject to an unknown input. Taking into consideration the sampled-data problem described above, an exact discretization of an extended version of the system is given with consideration for the uncertainties due to the ZOH assumption made on θ. This discretization effectively

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translates the sampled-data unknown input estimation problem into a discrete-time LPV observer design problem with norm-bounded uncertainties. To the authors' knowledge, this is a new approach in the sampled-data literature. The exhibited bounds on the uncertainties rely on some Lipschitz assumptions, which are similar to the usual bounds put on the variations of θ that are typically found in the literature of LPV systems. These bounds constitute a generalization of the ones found in [1], the idea behind their proofs being similar.

Following the ideas of [1], the norm-bounded uncertainties of the exactly discretized extended-system allow for some real-time structural analysis. Specifically, from the last known value of θ, the observability Gramian of the extended discrete-time LPV system can be estimated between two future instants, as if the values of θ were known in advance. In practice, this Gramian-estimation can be used to evaluate in real time a number of samples for which the unknown input is guaranteed to remain observable. To the authors' knowledge, this is an uninvestigated question of the LPV literature. Some results on the robust observability of discrete time varying systems could arguably be leveraged to obtain similar bounds [17, 14], but the approach of this paper is more straightforward, the uncertainties being treated here as "yet to be known" values.

The paper is organised as follows: Section 2 introduces the notations used throughout the paper. Section 3 provides technical results, allowing for a smoother reading of the next two sections. Section 4 translates the sampled-data unknown input estimation problem into a discrete-time LPV observer design problem with norm-bounded uncertainties. Section 5 estimates the near-future observability Gramians of the previously constructed system in order to lower-bound the number of samples left until the unknown input observability of the initial system may be lost. Section 6 finally provides an illustrative example. Some perspectives are discussed in Section 7 to conclude the paper.

2 Notations

$\mathbb{N}$ is the set of natural numbers. $\mathbb{K}$ stands for $\mathbb{R}$ or $\mathbb{C}$, resp. the set of real and complex numbers. $\mathbb{K}^{p\times q}$ denotes the set of matrices with p rows, q columns and coefficients in $\mathbb{K}$. $\|\cdot\|$ stands for the spectral norm. $\mu : \mathbb{K}^{p\times p} \to \mathbb{R}$ denotes the logarithmic norm induced by the spectral norm. Given a matrix $E \in \mathbb{K}^{p\times p}$, E^* denotes its conjugate transpose, E positive-definite is denoted $E \succ 0$, and by definition $\mu(E) = \lim_{h\to 0^+} \frac{\|I_p+hE\|-1}{h}$ where $I_p \in \mathbb{K}^{p\times p}$ stands for the identity matrix [6]. Given the following continuous-time system $\dot{x}(t) = A(t)x(t)$ with $A : \mathbb{R} \to \mathbb{K}^{n\times n}$ a continuous function, the state transition matrix from t_1 to t_2 of such a system is often abstractly denoted $\Phi(t_2, t_1)$. However, this paper considers the state-transition matrix to be a product integral, which is denoted in the following way [21, 4, 1]:

$$\Phi(t_2, t_1) = \prod_{t_1}^{t_2} e^{A(s)ds} \tag{1}$$

3 Preliminary Results

Among the many properties of the logarithmic norm and of the product integral, several of them play a crucial role in this document.

Lemma 1 (Duhamel's Formula)

Let $E, F : \mathbb{R} \to \mathbb{K}^{p\times p}$ be two continuous functions. For all $t_1, t_2 \in \mathbb{R}$

$$\prod_{t_1}^{t_2} e^{E(s)ds} - \prod_{t_1}^{t_2} e^{F(s)ds} = \int_{t_1}^{t_2} \left(\prod_{u}^{t_2} e^{F(s)ds}\right) (E(u) - F(u)) \left(\prod_{t_1}^{u} e^{E(s)ds}\right) du \tag{2}$$

Proof See Theorem 5.1 of [4]. □

Lemma 2 *Let $E : \mathbb{R} \to \mathbb{K}^{p\times p}$ be a continuous function. For all $t_1, t_2 \in \mathbb{R}$ such that $t_1 \leq t_2$*

$$\left\|\prod_{t_1}^{t_2} e^{E(s)ds}\right\| \leq e^{\int_{t_1}^{t_2} \mu(E(s))ds} \tag{3}$$

Proof This result is a consequence of Grönwall's lemma [3]. □

Lemma 3 *If $E : \mathbb{R} \to \mathbb{K}^{p\times p}$ is a L_E-Lipschitz function, then $\mu(E(\cdot))$ is also a L_E-Lipschitz function.*

Proof For $t_1, t_2 \in \mathbb{R}$

$$\begin{aligned} \mu(E(t_1)) &= \mu(E(t_2) + E(t_1) - E(t_2)) \\ &\leq \mu(E(t_2)) + \mu(E(t_1) - E(t_2)) \\ \Rightarrow \mu(E(t_1)) - \mu(E(t_2)) &\leq \mu(E(t_1) - E(t_2)) \leq \|E(t_1) - E(t_2)\| \end{aligned} \tag{4}$$

interverting the role of t_1 and t_2 in the previous equations yields the same upper-bound. Since E is L_E-Lipschitz, combining (4) with its interverted counterpart provides

$$|\mu(E(t_1)) - \mu(E(t_2))| \leq \|E(t_1) - E(t_2)\| \leq L_E\, |t_1 - t_2| \tag{5}$$

which concludes the proof. □

Lemma 4 *Let $E : \mathbb{R} \to \mathbb{K}^{p\times p}$ be a L_E-Lipschitz function with for all $t \in \mathbb{R}$, $\mu(E(t)) \leq \sigma$ and let τ be defined by $\tau = t_0 + \frac{\sigma - \mu(E(t_0))}{L_E}$. For all $t_0, t_1, t_2 \in \mathbb{R}$ such that $t_0 \leq t_1 \leq t_2$*

$$\left\|\prod_{t_1}^{t_2} e^{E(s)ds}\right\| \leq \begin{cases} e^{(t_2-t_1)\sigma} & \text{if } \tau \leq t_1 \\ e^{t_2\sigma - t_1(\mu(E(t_0)) - \frac{L_E}{2}(2t_0 - t_1)) - \frac{1}{2L_E}(\sigma - \mu(E(t_0)) + L_E t_0)^2} & \text{if } \tau \in (t_1, t_2) \\ e^{(t_2-t_1)\mu(E(t_0)) + \frac{1}{2}L_E\left(t_2^2 - t_1^2 + 2t_0(t_1 - t_2)\right)} & \text{if } \tau \geq t_2 \end{cases} \tag{6}$$

Proof Lemma 3 provides

$$\int_{t_1}^{t_2} \mu(E(s))ds \leq \int_{t_1}^{t_2} \min(\mu(E(t_0) + L_E(s - t_0), \sigma)ds \tag{7}$$

and since $s \mapsto \mu(E(t_0)) + L_E(s - t_0)$ is increasing, it follows

$$\begin{aligned}(7) \leq & \min_{s\in(t_1,t_2)} (s - t_1)\mu(E(t_0)) + \frac{1}{2}L_E(s^2 - 2t_0 s - t_1^2 + 2t_0 t_1) + (t_2 - s)\sigma \\ = & \min_{s\in(t_1,t_2)} as^2 + bs + c\end{aligned} \tag{8}$$

The minimum of as^2+bs+c being reached for $\tau = -b/2a$, the sharpest upper-bound of (7) depends on whether τ belongs to (t_1, t_2) or not. The results are then applied to Lemma 2, providing (6) and thus concluding the proof. □

Remark 1 The first and last upper-bounds of (6) stay true for all values of τ, contrary to the second upper-bound which only holds when $\tau \in (t_1, t_2)$.

Lemma 5 *If $E : \mathbb{R} \to \mathbb{K}^{p\times q}$ and $F : \mathbb{R} \to \mathbb{K}^{p\times r}$ are L_E and L_F-Lipschitz functions respectively, then for all $k \in \mathbb{N}$, the following function is $(L_E + L_F)$-Lipschitz*

$$\begin{aligned} G_k : \mathbb{R} &\to \mathbb{K}^{(q+r+k)\times(q+r+k)} \\ t &\mapsto \begin{bmatrix} E(t) & F(t) & 0 \\ 0 & 0 & I_k \\ 0 & 0 & 0 \end{bmatrix}\end{aligned} \tag{9}$$

Proof Given $t_1, t_2 \in \mathbb{R}$, the following inequalities hold

$$\begin{aligned}&\|G_k(t_1) - G_k(t_2)\| = \left\|G_k^*(t_1) - G_k^*(t_2)\right\| = \sup_{\|x\|=1} \left\|\begin{bmatrix} E^*(t_1) - E^*(t_2) & 0 \\ F^*(t_1) - F^*(t_2) & 0 \\ 0 & 0 \end{bmatrix} x\right\| \\ &\leq \sup_{\|x_1\|=1} \left\|\begin{bmatrix} E^*(t_1) - E^*(t_2) & 0 \end{bmatrix} x_1\right\| + \sup_{\|x_2\|=1} \left\|\begin{bmatrix} F^*(t_1) - F^*(t_2) & 0 \end{bmatrix} x_2\right\| \\ &= \|E(t_1) - E(t_2)\| + \|F(t_1) - F(t_2)\| \leq (L_E + L_F) |t_1 - t_2|\end{aligned} \tag{10}$$

which concludes the proof. □

Lemma 6 *Let $E : \mathbb{R} \to \mathbb{K}^{p\times p}$ be a L_E-Lipschitz function with for all $t \in \mathbb{R}$, $\mu(E(t)) \leq \sigma$. For all $t_0, t_1, t_2 \in \mathbb{R}$ such that $t_0 \leq t_1 \leq t_2$.*

$$\left\|\prod_{t_1}^{t_2} e^{E(s)ds} - e^{(t_2-t_1)E(t_0)}\right\| \leq \min\left\{\begin{matrix} \frac{1}{2}L_E\left(t_2^2 - t_1^2 + 2t_0(t_1 - t_2)\right) e^{(t_2-t_1)\sigma}, \\ \left(e^{\frac{1}{2}L_E\left(t_2^2-t_1^2+2t_0(t_1-t_2)\right)} - 1\right) e^{(t_2-t_1)\mu(E(t_0))} \end{matrix}\right\} \tag{11}$$

Proof The proof of the first inequality can be found in [1]. The second inequality only relies on the Lipschitz asumption on E. Given $t_0, t_1, t_2 \in \mathbb{R}$ such that $t_0 \leq t_1 \leq t_2$, Lemma 1 provides

$$\begin{aligned}\prod_{t_1}^{t_2} e^{E(s)ds} - e^{(t_2-t_1)E(t_0)} &= \prod_{t_1}^{t_2} e^{E(s)ds} - \prod_{t_1}^{t_2} e^{E(t_0)ds} \\ &= \int_{t_1}^{t_2} \left(\prod_{u}^{t_2} e^{E(t_0)ds}\right)(E(u)-E(t_0))\left(\prod_{t_1}^{u} e^{E(s)ds}\right)du \\ &= \int_{t_1}^{t_2} e^{(t_2-u)E(t_0)}\,(E(u)-E(t_0))\left(\prod_{t_1}^{u} e^{E(s)ds}\right)du\end{aligned} \tag{12}$$

then, by submultiplicativity of the operator norm and the Lipschitz asumption on E

$$\|\,(12)\,\| \le L_E \int_{t_1}^{t_2} e^{(t_2-u)\mu(E(t_0))}(u-t_0)\left\|\prod_{t_1}^{u} e^{E(s)ds}\right\|du \tag{13}$$

the third upper-bound of Lemma 4 is finally applied, providing

$$\begin{aligned}\|\,(12)\,\| \le{}& L_E\left(\int_{t_1}^{t_2}(u-t_0)e^{\frac{1}{2}L_E(u^2-2t_0u)}du\right)e^{-\frac{1}{2}L_E(t_1^2-2t_0t_1)+(t_2-t_1)\mu(E(t_0))} \\ ={}& L_E\left(\frac{1}{L_E}e^{\frac{1}{2}L_E t_2(t_2-2t_0)} - \frac{1}{L_E}e^{\frac{1}{2}L_E t_1(t_1-2t_0)}\right)e^{-\frac{1}{2}L_E t_1(t_1-2t_0)+(t_2-t_1)\mu(E(t_0))} \\ ={}& \left(e^{\frac{1}{2}L_E\left(t_2^2-t_1^2+2t_0(t_1-t_2)\right)}-1\right)e^{(t_2-t_1)\mu(E(t_0))}\end{aligned} \tag{14}$$

which concludes the proof. □

Remark 2 If $t_1 < \tau < t_2$, with τ given in Lemma 4, a sharper (but very involved) upper-bound to (11) can be found.

4 An Exact Discretization of sampled LPV Systems subject to an Unknown Input

This section deals with the state and unknown input estimation of a sampled LPV system of the following form

$$\begin{aligned}\dot{x}(t) &= A(\theta(t))x(t) + F_1(\theta(t))f(t) \\ y(t) &= C(t)x(t) + F_2(t)f(t)\end{aligned} \tag{15}$$

where $x(t) \in \mathbb{K}^{n_x}$ is the state of the system, $y(t) \in \mathbb{K}^{n_y}$ is its output, $f(t) \in \mathbb{K}^{n_f}$ is its unknown input and $\theta(t) \in \mathbb{K}^{n_\theta}$ is a sampled scheduling parameter measured in real time. Note that the coefficients of $C(t)$ and $F_2(t)$ are assumed to be known at all time t, hence, these matrices are not θ-dependent. This section provides a discrete representation of this system which takes into account the intrinsic uncertainties that come with the sampling of θ.

Polynomial Unknown Input Assumption: f is a polynomial function of degree $q-1$ (hence for all t, the q-th derivative of $f(t)$ is null).

A common way [5, 9, 23] to reconstruct the unknown input $f(t)$ of (15) consists in finding an observer for the augmented LPV system

$$\begin{aligned} \dot{z}(t) &= \overline{A}(\theta(t))z(t) \\ y(t) &= \overline{C}(t)z(t) \end{aligned} \tag{16}$$

$$\text{with} \quad z(t) = \begin{bmatrix} x(t) \\ f(t) \\ f^{(1)}(t) \\ \vdots \\ f^{(q-1)}(t) \end{bmatrix} \quad \overline{A}(\theta(t)) = \begin{bmatrix} A(\theta(t)) & F_1(\theta(t)) & 0 & \dots & 0 \\ 0 & 0 & I_{n_f} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & I_{n_f} \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \tag{17}$$

$$\overline{C}(t) = \begin{bmatrix} C(t) & F_2(t) & 0 & \dots & 0 \end{bmatrix} \tag{18}$$

If the evolution of θ were exactly known between t_1 and t_2, it would be possible to perfectly find $z(t_2)$ based on the values of $z(t_1)$ and θ through the relation $z(t_2) = \left(\prod_{t_1}^{t_2} e^{\overline{A}(\theta(s))ds}\right) z(t_1)$.

Sampling Assumption: In practice, θ is measured at a sampling period T_s, and its values are not only unknown in the future, but also unknown in-between two samples.

For all $k, m, n \in \mathbb{N}$ with $n \geq m$, $\theta(kT_s)$ now represents the last known value of θ. The exact discretization of (16) accounting for these future and inter-sample uncertainties is given by

$$\begin{aligned} z_{k+n} &= \left(e^{(n-m)T_s\overline{A}(\theta_k)} + \Delta_{k,n,m}\right) z_{k+m} \\ y_{k+m} &= \overline{C}_{k+m} z_{k+m} \end{aligned} \tag{19}$$

where $\Delta_{k,n,m} \in \mathbb{K}^{n_z \times n_z}$ stands for a norm-bounded matrix of uncertainties (with $n_z = n_x + qn_f$), and where for all $p \in \mathbb{N}$, $z_p = z(pT_s)$, $y_p = y(pT_s)$, $\theta_p = \theta(pT_s)$ and $\overline{C}_p = \overline{C}(pT_s)$.

Remark 3 As a side note, notice how the peculiar structure of $\overline{A}(\theta_k)$ provides for all $s \in \mathbb{R}$

$$e^{s\overline{A}(\theta_k)} = \begin{bmatrix} e^{sA(\theta_k)} & \int_0^s e^{(s-u)A(\theta_k)} \begin{bmatrix} F_1(\theta_k) & 0 & \dots & 0 \end{bmatrix} e^{uN} du \\ 0 & e^{sN} \end{bmatrix} \tag{20}$$

with $N \in \mathbb{K}^{qn_f \times qn_f}$ the nilpotent matrix $N = \begin{bmatrix} 0 & I_{(q-1)n_f} \\ 0 & 0 \end{bmatrix}$.

If $A(\theta(\cdot))$ and $F_1(\theta(\cdot))$ are assumed to be L_A-Lipschitz and L_{F_1}-Lipschitz respectively – which is a similar assumption to the usual bounds put on the variations of θ that are typically found in the literature of LPV systems – then the following bounds are easily deduced from Lemmas 5 and 6

$$\left\|\Delta_{k,n,m}\right\| \leq \left(e^{\frac{1}{2}\left(L_A+L_{F_1}\right)\left(n^2-m^2+2k(m-n)\right)T_s^2}-1\right)e^{(n-m)T_s\mu(\overline{A}(\theta_k))} \tag{21}$$

Moreover, if the logarithmic norm of $\overline{A}(\theta(\cdot))$ is also upper-bounded by $\sigma \in \mathbb{R}$ – which is a similar assumption to the usual bounds put on the values of θ (see Lemma 1 of [10]) – then the following bounds are also verified

$$\left\|\Delta_{k,n,m}\right\| \leq \frac{1}{2}\left(L_A+L_{F_1}\right)\left(n^2-m^2+2k(m-n)\right)T_s^2 e^{(n-m)T_s\sigma} \tag{22}$$

Depending on the context, one of the previous upper-bounds might be sharper than the other. One may also bound $\Delta_{k,n,m}$ inside a polytope, or more specifically between two matrices in a component-wise approach, which can be achieved by shifting the Lipschitz assumption from $A(\theta(\cdot))$ and $F_1(\theta(\cdot))$ to $\theta(\cdot)$ directly.

Finally, to reconstruct the unknown input $f(t)$ while taking into account the intersample behavior of θ, it is possible to build a robust observer [22, 13, 12, 7, 11] for the discrete-time uncertain LPV system (19) taken with $n = 1$ and $m = 0$.

5 Anticipating the Loss of Unknown Input Observability

In this section, the possible loss of unknown input observability of the system (15) is anticipated using a sufficient condition of finite-time observability for (19) in a near future. This sufficient condition is based on upper-bounding, in terms of spectral norm, the difference between the observability Gramian of (19) and the observability Gramian of (19) taken without its uncertainties (i.e. with $\Delta_{k,n,m} = 0$). These Gramians are denoted W_k^Δ and W_k respectively, where θ_k still stands for the last known value of the scheduling parameter. On one hand, W_k^Δ evaluated between $(k+m)T_s$ and $(k+n)T_s$ can be expressed by:

$$\begin{aligned} W_{k,n,m}^\Delta &= \sum_{r=m}^{n-1}\left(e^{(r-m)T_s\overline{A}^*(\theta_k)}+\Delta_{k,r,m}^*\right)\overline{C}_{k+r}^*\overline{C}_{k+r}\left(e^{(r-m)T_s\overline{A}(\theta_k)}+\Delta_{k,r,m}\right) \\ &= \sum_{r=m}^{n-1}\left(\prod_{(k+m)T_s}^{(k+r)T_s} e^{\overline{A}(\theta(s))ds}\right)^*\overline{C}_{k+r}^*\overline{C}_{k+r}\left(\prod_{(k+m)T_s}^{(k+r)T_s} e^{\overline{A}(\theta(s))ds}\right) \end{aligned} \tag{23}$$

hence, $W_{k,n,m}^\Delta$ can only be fully computed if $\theta(t)$ is perfectly known for all $t \in [(k+m)T_s, (k+n)T_s]$, that is to say: in the future, and without sampling. On the other hand, W_k evaluated between $(k+m)T_s$ and $(k+n)T_s$ can be easily computed using:

$$W_{k,n,m} = \sum_{r=m}^{n-1} e^{(r-m)T_s\overline{A}^*(\theta_k)}\overline{C}_{k+r}^*\overline{C}_{k+r}e^{(r-m)T_s\overline{A}(\theta_k)} \tag{24}$$

The augmented system (19) is observable between $(k+m)T_s$ and $(k+n)T_s$ if and only if $W^{\Delta}_{k,n,m} \succ 0$ [2]. Moreover, following the ideas of [1], if the error $e_{k,n,m} = \left\|(W^{\Delta} - W)_{k,n,m}\right\|$ is upper-bounded by a constant $M_{k,n,m} \in \mathbb{R}$, then

$$W_{k,n,m} - M_{k,n,m} I_{n_z} \succ 0 \Rightarrow W^{\Delta}_{k,n,m} \succ 0 \tag{25}$$

Hence, $W_{k,n,m} - M_{k,n,m} I_{n_z} \succ 0$ is a sufficient condition of unknown input observability for (15) between $(k+m)T_s$ and $(k+n)T_s$. The upper-bound $M_{k,n,m}$ is easily deduced from Lemma 4, Lemma 6, and the following inequality [1]:

$$e_{k,n,m} \leq \sum_{r=m+1}^{n-1} \left(\left\| \prod_{(k+m)T_s}^{(k+r)T_s} e^{\overline{A}(\theta(s))ds} \right\| + e^{(r-m)T_s \mu(\overline{A}(\theta_k))} \right) \left\|\Delta_{k,r,m}\right\| \left\|\overline{C}_{k+r}\right\|^2 \tag{26}$$

This approach generalizes the upper-bound found in [1], and extends the resulting structural analysis [1] to unknown input observability of sampled LPV systems.

Following from the previous results, a lower-bound to the number of samples for which the unknown input is guatanteed to remain observable is given by

$$m^*(\theta_k) = \max\{m \in \mathbb{N} : \exists n > m, W_{k,n,m} - M_{k,n,m} I_{n_z} \succ 0\} \tag{27}$$

where θ_k is still assumed to be the last known value of θ.

6 Illustrative Example

The following second order unknown input LPV system is considered

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -10 & 0 \\ \theta_2(t) & -15 \end{bmatrix} x(t) + \begin{bmatrix} \theta_1(t) \\ 0 \end{bmatrix} f(t) \\ y(t) &= \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix} x(t) \end{aligned} \tag{28}$$

where $\theta \in [0, 60] \times [-60, 60]$ is assumed to be a scheduling vector (with $L_A = 2$, $L_{F_1} = 3$) sampled at $T_s = 0.03s$, and the unknown input is assumed to be constant, hence $f^{(1)}(t) = 0$. The values of $m^*(\theta_k)$ for (28) with $k = 0$ are plotted on Figure 1. Despite the conservativeness of the bounds used to compute $m^*(\theta_k)$, some values of θ guarantee the observability of the unknown input for at least the next 11 samples. These results are encouraging since they were computed without consideration for the structure of the extended system $\overline{A}(\theta_k)$, which tends to have a large logarithmic norm $\mu(\overline{A}(\theta_k))$.

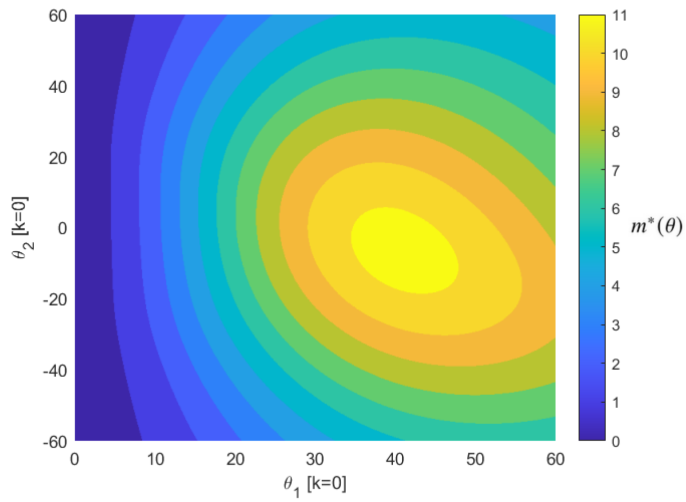

Fig. 1 $m^*(\theta)$, a lower-bound to the number of samples for which the unknown input observability of (28) is guatanteed

7 Conclusion and perspectives

In this paper, given a continuous-time LPV system with a sampled scheduling parameter and an unknown input, and under a Lipschitz assumption, the sampled-data unknown input estimation problem has been translated into a discrete-time LPV robust observer design problem. The bounds that were developed in the process allowed for an estimation of some near-future observability Gramians, from which a lower-bound to the number of samples for which the unknown input is guatanteed to remain observable was exhibited. The obtained results could be further enhanced by taking into consideration the structure of the extended system $\overline{A}(\theta_k)$, in particular by using Equation (20) in the computation of the Gramian estimation error. Moreover, the extension of this work to sampled systems with a non-constant sampling period remains to be investigated.

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